Sensitivity to Complexity – an Important Prerequisite of Problem Solving Mathematics Teaching

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Abstract: *Teaching is deciding and acting in a complex system. If a teacher attempts to fulfil demands to teach mathematics with a stronger problem solving orientation, it becomes even more complex. This complexity must not be reduced arbitrarily. Instead, a sufficient degree of sensitivity is necessary to competently and flexibly deal with emerging demands on the teacher.*
In this article I provide an introduction to the concept of sensitivity to complexity of mathematics teaching and report on specific realistic and interactive diagnostic instruments. A particular focus is placed on a diagnostic interview about decision-making situations which could occur in a mathematics lesson. A first pilot study with student teachers from different German universities – briefly outlined in the last part of this article – suggests its suitability for gaining important indications of the agent's degree of sensitivity to complexity of problem solving mathematics teaching.

ZDM Classification: C30, D50

1. Introduction

For many decades, individual didacts of mathematics as well as research groups and associations of scientists and teachers, have demanded a stronger problem solving orientation in teaching of mathematics (Burman, 2006; Southwell, 2004; Zimmermann, 2003; see also e.g. standards of the NCTM). However, more recent empirical studies (e.g. TIMSS, TIMS Video Study) show a certain stability of in this regard dissatisfactory classroom reality; for example, the questioning-developing teaching style is still a major characteristic of mathematics lessons in Germany (Krainer in Schlöglmann, 2005). There are several different reasons for this that partly have their roots outside schools. But the reasons frequently stated by school practitioners can partly also be seen as excuses.

One reason I see in special demands that more problem solving mathematics teaching makes on the teacher. These demands result from the high complexity of this kind of teaching especially with regard to mathematical-cognitive aspects which cannot be reduced arbitrarily.

Not only during classroom teaching, but already during the training and education periods of furture teachers Schlöglmann (2005) sees existing teaching practice becoming solidified. According to him, an important reason are routines which are already acquired before actually beginning studies. On the one hand, these are considered a prerequisite of tackling complex classroom situations, on the other hand they significantly contribute to the self-stabilisation of the dissatisfying status quo.

If you take the core ideas of problem solving mathematics teaching seriously (e.g. Fritzlar, 2004; Zimmermann, 1991), it is especially the traditional routines that are often acquired or enhanced during own classroom teaching and university studies that must be broken and overcome. However this does in no way make teaching easier.

2. Theoretical framework

"Teaching is complex." Similar statements can often be heard or read (Jaworski & Gellert, 2003; see Fritzlar, 2004, for more references), but what exactly can it mean? To get a more concrete understanding, international efforts are being made to transfer ideas, methods and results of a complexity theory that is based on cybernetics and systems theory to the area of classroom teaching (e.g. Davis, Phelps, & Wells, 2004; Davis & Simmt, 2003 or see the Working Session at PME 30–Conference held 2006 in Prague). In this context, studies of cognitive psychology on deciding and acting in complex constellations seem more fertile to me at the present state. These studies not only attempt to show the complexity of specific systems, but also describe resulting requirements of an agent and his behaviour in the respective constellation (e.g. Dörner, Kreuzig, Reither, & Stäudel, 1983; Frensch & Funke, 1995; Funke, 1998).
There are however several different approaches within this line of research (see e.g. *Psychologische Rundschau 50*(4)), that cannot be elaborated further in this paper. I would like to restrict myself to the brief discussion of characterizing attributes of complex systems and the important requirements that result from them.

Comprehensiveness: A vast amount of information has to be considered to adequately deal with complex systems. The degree of comprehensiveness however also depends on the agent and her modelling of the system. If the degree is too high, the agent must attempt an *adequate reduction of information abundance.*

Linkage: In complex systems, elements are usually connected by a close network of interdependencies. Thus, the agent can generally not do only one thing without simultaneously doing many others. Consequently, she must also *consider remote- and side effects* alongside the intended chief effect of her interventions.

Dynamics: Complex systems change their state of being also without external impact. They do not „wait“, which frequently puts the agent *under time pressure when planning and executing actions*. Furthermore she should not only regard the current state of the system, but must also *consider its development over time.*

Intransparency: In complex systems, the variable states and their interconnections between them are partly inaccessible. But, often enough, data must also remain ignored due to the low capacity of the working memory and the limitations of human information processing. The agent generally has to complement her lack of knowledge by *actively acquiring suitable information.*

Additional requirements can result of the agent by pursuing several (often partly contradictory) goals at the same time when dealing with a complex system.

Teaching in this sense can undoubtedly be understood as deciding and acting in a complex system. More problem solving oriented lessons of mathematics, in which students work as independently as possible on comprehensive mathematical problems, are furthermore characterised by an additional complexity with regard to mathematical-cognitive aspects and by resulting requirements of the teacher: Typical characteristics of such a lesson are a particularly high mathematical comprehensiveness and students' numerous different working processes. These however often run parallel to each other and are highly dynamic. They should be accompanied by the teacher and supported if necessary, while they are only in few cases predictable, surprising and only difficult to see through. They are differentially influenced by a variety of classroom conditions and characteristics of lesson arrangement and certainly also retroact on them.
Overall, problem solving mathematics lessons possess a fundamental ambiguity and can only be prepared and planned to a limited degree, which potentially leads to enormous insecurity. The latter is enhanced by the fact that the teacher loses his status as the sole and authoritarian source of knowledge.

Hopefully these elaborations not only suggest the special complexity of more problem solving oriented mathematics teaching, but beyond that make it clear that this complexity can be reduced neither in an arbitrary extent nor in an arbitrary orientation. For successful teaching on the long term, the teacher must rather bear with a certain minimum of complexity and be able to deal with it successfully (also Fritzlar, 2003). A fundamental prerequisite for this is a sufficient sensitivity to this complexity. In the sense of a preliminary definition the agent should be sensitive to the (specific) complexity of (problem oriented) mathematics teaching, if she is aware of the characteristics of this scope of reality, resulting requirements, but also of the resulting limitations of her own decision and acting ability.
Sensitivity to complexity thus seems a fundamental component of a teacher's professional competency (on this concept see e.g. Krauss et al., 2004) that has been neglected up until now.

Students already have a fixed idea of what classroom learning is like at the beginning of their studies (Brookhart & Freeman, 1992). This idea is formed during their (high) school education over many years and has a proven influence on what will be learned later on during tertiary education (Calderhead & Robson, 1991). For this reason it seems especially relevant to break inadequately simplifying ideas as early as possible and sensitise students to the complexity of teaching.
However, also internationally, there is no experience regarding appropriate supplements of university teacher education. Thus, one must first of all carry out some preliminary work. Among other things it seems necessary to develop an instrument with which indications of the development of sensitivity to complexity can be determined. It can help to clarify the existing starting position, as well as evaluate specific elements of education later on.

3. Diagnostic instruments

Real situations however are not suitable for gaining diagnostic information. On the one hand they are not repeatable, not systematically variable and partly only available under large efforts. For these reasons, interactive computer scenarios with at least superficial reference to reality (face validity) have already been applied in cognitive psychology research. An additional factor in classroom situations is the so-called pressure of performance – especially for beginners – i.e. actually being able to realise the decisions that have been made. Also, these decisions would often have to be made under extreme time pressure. Both would most likely limit an agent's possibilities of demonstrating her sensitivity through according decisions as well as her decision behaviour. In this context one must also consider that a sensitive agent does not *have to* act successfully in a complex situation, rather is an according sensitivity a necessary, though not a sufficient, precondition for this.

3.1 A realistic interactive computer scenario

For this reason I have developed an interactive computer scenario in which the user can design a virtual lesson in the role of the teacher for three modelled Year-5- school classes (students aged 10-11 years) on the following folding problem (see also Fritzlar, 2006; Kießwetter & Nolte, 1996):
A sheet of normal rectangular typing paper is halved by folding it parallel to the shorter edge. The resulting double sheet can be halved again parallel to the shorter edge and so on. After n folding procedures the corners of the paper stack are cut off. Find out and explain a connection between the number of folding procedures and the number of holes in the reopened paper. (Formulation not for students)

In the scenario, the user respectively makes her decisions by choosing from given alternatives to which the computer programme reacts, traces selected aspects of the course a lesson could take and in doing so confronts the user with the following decision situations. In this way, the complete lesson can be "played through" and at the end of it the user receives a summarised evaluation of her decisions that is made according to her chosen or weighted goals. Thereby the programme user can work without time pressure, revise her decisions later on and explore the modelled situations.

The empirical foundation of the scenario are above all close to 50 lessons on the folding problem that were mainly conducted in primary and lower secondary schools[1] in several German federal states. The video taped lessons were analysed especially regarding the decisions made during class, the noticeable or reconstructable processes of problem solving and –results as well as possible correlations between these factors. In addition, surveys were conducted among students, teachers and didacts about important decisions and their probable consequences. The data gained in this way provided the basis for an extensive, densely ramified network (only manageable by computer) which is the core of the computer scenario. You can find more details about the computer scenario and its development in Fritzlar (2004).

If a sensitive agent deals with the computer scenario, one can expect

- that she tries to understand the unknown system more deeply and in doing so is also willing to give up own perceptions, critically question decisions and correct them if necessary,
- that, when making decisions, she particularly also considers mathematical-cognitive aspects and in many situations aspects from different areas simultaneously,
- that she is able to act consistently with the characteristics of problem solving processes and classroom situations shown in the scenario,
- that she tries to develop an adequate mental model of interrelations between the elements of the programme due to her dealing with the scenario.

For this reason, dealing with the scenario is described by the following dimensions which altogether – in the sense of a four-dimensional value – characterise important aspects of the agent's sensitivity to the complexity of problem solving mathematics teaching.

Explorative behaviour describes quantitative (number of returns and programme cycles) as well as qualitative aspects (difference in selected options) of the scenario exploration.

Context sensitivity describes in how far characteristics of students' problem solving processes,

[1] Realschule of the German educational system.

mathematical, motivational and behavioural aspects are considered when making decisions.

Inconsistency describes qualitative aspects of decisions.

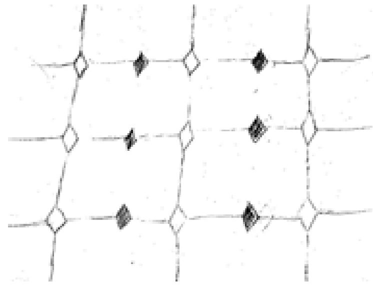
Reflectiveness describes in how far modelled interrelations are deliberated upon and put in question, in how far there are approaches to metacognition and a search for additional alternatives (see also Fritzlar, in press).

3.2 An interview

First trials of the computer scenario showed that the students' verbalisation skills or willingness to communicate can absolutely vary. However, the test persons' descriptions on "what is going on in their head" can provide very important clues regarding their degree of sensitivity to complexity. For this reason, an interview is being developed as a supporting diagnostic instrument. This interview, based on the compiled teaching experiences (see above), confronts the interviewee once more with classroom situations regarding the folding problem. She is asked to interpret students' problem solving results and –processes, evaluate alternatives for the further design of the lesson and develop own proposals. It was above all these *evaluations of options* that proved fertile during a test run of the preliminary version of the interview. In the computer scenario, there are mere *decision making situations* often handled without verbalisations. This is why the interview not only provide more, but also different types of information (Rieskamp & Hoffrage, 1999), from which further indications regarding sensitivity to complexity can be drawn.
The following example of a teaching situation has been processed in the interview with presentation software.

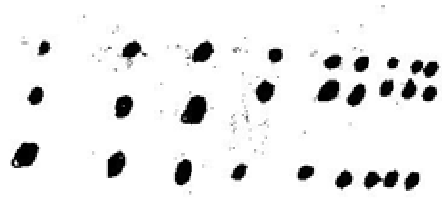
During a lesson, a student teacher first of all carried out some folding procedures together with the students of a Year-5-class[2]*. Afterwards, the students were asked to sketch what the piece of folding paper looked like after the following, fifth folding.*
Please comment on the students' suppositions.

[2] Further information on the class is given in the interview.

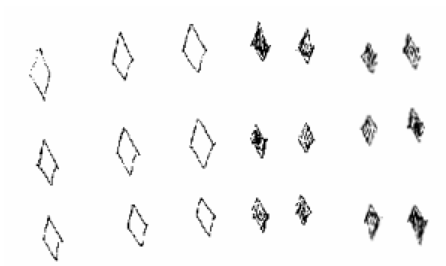


Anne: "I think the holes always show up on the edges."

John: "A hole always appears in between. So then it's 21 holes."



Kate: "I got 27 holes. Just like that."



Susan: "I thought we'll just get this once more."

…

Please name possible consequences as well as possible advantages and disadvantages of the following imaginable continuations:

- *For the sixth folding, the students also sketch the assumed appearance of the reopened piece of folding paper. In doing so,*
  - *they again sketch on a piece of drawing paper.*
  - *they sketch on the piece of folding paper that has been used so far.*
- *Existing results (especially number of holes) are briefly summarised. Afterwards, students are asked about their assumptions regarding the number of holes after the next folding.*

How would you continue the problem solving process? Give reasons for your decision.

In a situation such as this, one can expect of a sensitive agent that she will first of all make efforts to appropriately interpret her students' problem solving processes and -results, even though this may partly not be easy due to the students' frequently limited verbalisation skills and possible differences to the agent's own way of problem solving. In addition, a sensitive agent will consider numerous aspects from different areas when evaluating alternatives. In doing so, she not only keeps her goals and assumptions, but also the depicted classroom situation in mind. Due to this multidimensional situation there is often no "best solution" for the agent.

Statements made by the questioned persons are therefore analysed regarding the following dimensions which further describe the sensitivity to complexity:

The dimension *interpretation* characterises on a scale from zero to three (0-3) in how far the agent succeeds in giving an appropriate interpretation of the students' problem solving process.

The *level of differentiation* describes how many and which aspects are considered when evaluating optional actions while – also with respect to the currently existing empirical data – aspects of mathematical content, of students' problem solving processes, motivational and behavioural aspects as well as "other" aspects are differentiated. It is also recorded whether evaluations are made rather with regard to intentions (forwards) or with reference to the current situation, previous working processes, experiences etc. (backwards). The dimension characterising the *level of integration* describes on a scale from zero to three (0-3) in how far different aspects, and with it generally at the same time advantages and disadvantages, are considered when evaluating individual options. The two extremes of the scale are the "absolute decision" which is one-sided and disregards the context; and the "strongly weighing decision" that finds different advantages and disadvantages for each option. To assessments with regard to this dimension the quality of individual arguments is less important; significant is rather the way arguments are structured for evaluating the given alternatives.[3]

The focus on mathematical-cognitive aspects – in the computer scenario as well as the interview – can be justified on the one hand by their special significance to problem solving mathematics teaching, the complexity which results from them and the demands on the teacher as a consequence. On the other hand, personal experience, pre-tests, but ultimately also results of the explorative study depicted in the following, have shown especially that this area is often neglected by teacher students.

4. An explorative study

The aim of a first explorative study was to find out how students work with the computer scenario described above. The main goal of the study was to construct characteristics which on the one hand differentially describe the way the computer scenario is dealt with, and on the other hand provide theoretically founded indications to the user's sensitivity to the complexity of problem solving mathematics teaching.

Additionally, a preliminary version of the interview, in which teaching situations and optional actions are interpreted and evaluated, was tested. A major point to be examined was in how far supplementing dimensions describing the interviewee's degree of sensitivity to complexity can be constructed from an analysis of the interviews. Twenty students participated in the study. They were enrolled as teacher students for primary and secondary schools[4] at the universities of the cities of Erfurt, Jena, Bielefeld and Braunschweig in Germany. The relatively small number of test persons is to be explained by the large-scale analysis involved. However, by interviewing teacher students for different school types and from different universities, an attempt was made to include a broad spectrum of students from the target group.

Some results

On the basis of existing empirical data, the handling of the computer scenario can be characterised by the dimensions e*xplorative behaviour*, *context sensitivity, consistency* and *reflectiveness,* which at the same time describe the programme user's sensitivity.

[3] Point of orientation for my development of the dimensions characteristing the levels of differentiation and integration are, among others, thoughts on "cognitive complexity" (e.g. Mandl & Huber, 1978).

[4] Grundschule (primary school), Hauptschule, Realschule or Gymnasium (secondary schools) of the German eduactional system.

In this paper we can summarise some results on the computer scenario only: In general, merely a *low sensitivity to the complexity of problem solving mathematics teaching* was observable with regard to the above named dimensions among the participating students. The computer scenario was frequently only marginally explored. The possibilities of systematically testing different options also under varying conditions were only used to a low extent. Mathematical aspects and particularly characteristics of students' problem solving processes were only considered to a very low extent and partly superficially. Reflectiveness when dealing with the scenario was generally low: Possible interrelations between classroom teaching conditions, decisions in class and students' problem solving processes were hardly contemplated on, and it was partly because of this that only few reasons to explore the scenario emerged. Furthermore, the multidimensional character of numerous decisions seldom became an issue, the test persons' meta-cognitions regarding their made decisions or their own decision behaviour were hardly noticeable. For a comprehensive and detailed depiction of the study please refer to Fritzlar (2004).

Additionally I would like to give some examples of the supplementing interviews. Considering the limited extent of the tested version and its preliminary character, I will restrict myself to describing extracts from two interviews in more detail.

Nicole is 21 years old and a teacher student for primary and lower secondary schools in her first year of studies. So far, Nicole has only been able to give one lesson, though not in mathematics. Her comments on the above described classroom situation were as follows:[5]

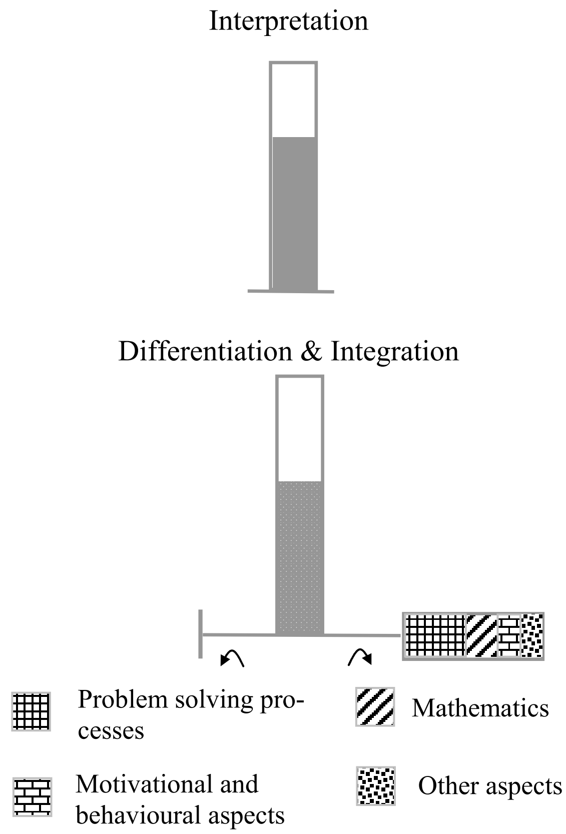
**Yes I think Anne's is very well structured and shows some good observations. "I think the holes always show up on the edges": I guess she'll mean the corners. With the lines, ...in the middle... so at least she's identified some kind of structure that has something to do with the points of intersection.*
John at least realised that everything stays in one line, even though he doesn't yet see the connection with these points of intersection.
I don't think there's any particular idea behind Kate's comment. I'd think she has difficulties getting all points in one row. I don't think the points moved up deliberately. She has a purely motoric problem. "I got 27 holes just like that." Perhaps the whole thing is too difficult for the girl. Maybe she's so concentrated on drawing dots or cutting the paper that she forgets everything else.
Susan's comment is in line with the theory that the fields always double. But in that case I don't really know how she made seven rows out of three.
Anyway I reckon Anne's has a real thought behind it.

…The advantage [of the first option] for myself as the teacher alone would be to realise whether the child that has the correct sketching after the fifth folding has perhaps done this with a certain idea in mind. Or just ordered them in a way that looks pretty. The only disadvantage I see is that the children are not allowed to cut again.
The problem [with the second option] is that it [the piece of folding paper] probably already looks quite damaged. The advantage is that they don't have to copy the points again and don't lose time this way, though some mistakes might occur when they shift the holes.
*The disadvantage [of the third option]: I'm back to complete frontal teaching. Advantage: I really take the kids in the direction I want to go, which is finding this correlation. …**

Nicole obviously has great problems interpreting the students' results and comments. While she notices the many correct elements in Anne's result, she does not initially realise that the second student's sketching is absolutely correct. The possibility that Kate works in a purely arithmetic way with patterns in the sequence of hole numbers does not enter Nicole's mind. She understands basic elements of Susan's comment: Nicole sees the doubling but relates this to the number of „fields", i.e. the spaces between the folding lines. Overall, Nicole's interpretation performance was allocated to the second-lowest rank in this situation. In her evaluation of the offered options to continue the lesson, Nicole puts a strong focus on problem solving processes, but she also deals with other aspects such as time available for class and general views on classroom teaching. She finds numerous advantages and disadvantages in the given options. However, she hardly considers the current situation – e.g. the question whether an option fits students' ideas and interim results. Overall Nicole's integration performance was allocated to the second-highest rank.

If we now condense the data of the five situations that Nicole interpreted and evaluated, we receive the following image:

[5] The asterisks indicate that the quotes have been polished.



The columns illustrate Nicole's overall achieved level of interpretation and integration ability.[6] She received medium ratings here. The width of the patterned boxes in the second part of the figure shows the number of considered aspects, while a differentiation takes place between thoughts with backward- and forward orientation respectively. An overall intensive consideration of students' problem solving processes becomes evident. However, the current classroom situation or previous solving processes are not considered.

Nadin is a teacher student for primary schools, already in her third year of studies and like Nicole 21 years of age. She has already done two internships. Her remarks were as follows:

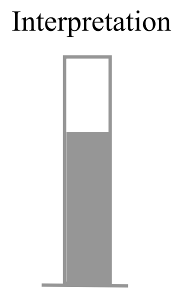
**Anne got the idea, but she didn't think far enough to realise that foldings also occur here [on the outside]. … But I mean otherwise she said "Holes always show up on the edges." She knows that edges appear here, but she hasn't thought far enough to consider the edges here [on the outside] but only on the inside.*
And John has 21 holes; he's understood the principle for sure. I'm not sure whether the fourth ... (Nadin takes a piece of folding paper to help her illustrate her point.) His comment was also quite good. "A hole always appears in between." Yes, he understood it too. Perhaps Kate could have sketched it a little differently. I think by sketching so messily she perhaps got some problems. … If she'd done it like the others, like sketching a piece of folding paper, it may not have happened. "I got 27 holes just like that." I would think that she didn't understand it. If she did, she would have sketched the basic model like John and then thought more about where a hole can appear. It looks a bit confusing to me like that, as if it didn't come to her mind.
"I thought we'll just get this once more." Well, Susan also didn't get the idea. I'd think this on the left side were the holes she got from the fourth folding procedure, and by taking the other ones to the other side she didn't understand where the holes appear, like John for example. Yes, but at least she has the right number of holes. Maybe she copied from someone else.

*Well I would … get the students to sketch on the folding paper that has been cut already. Because I – well they get good illustration that way and I think that's best.**

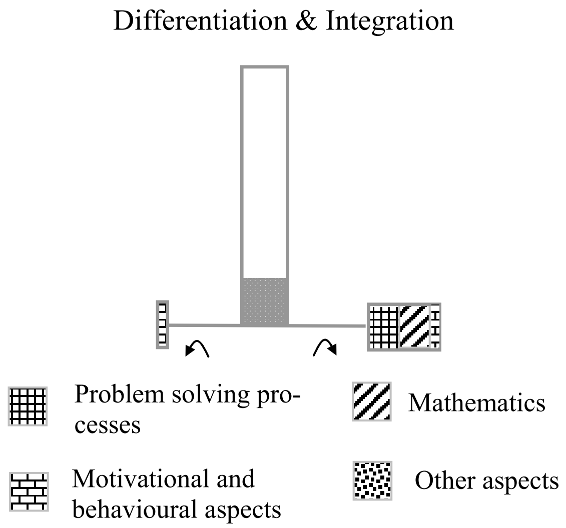
Nadin sees the potential of Anne's proposal as well as the correct solution in John's drawing. Like Nicole, Nadin does not realise that Kate has most probably worked with arithmetic patterns and that this is why the shape of the holes in the paper was not important to her. Neither does Nadin see the core of a very elegant solution in Susan's proposal. Overall, her interpretation performance was just allocated to the second highest rank.

Nadin decides on an option on the basis of a single argument without even mentioning the others and without taking the depicted situation into consideration. Thus, her integration performance is ranked on the lowest level.

Overall we receive the following picture for Nadin:



[6] Due to the small number of situations that was dealt with, the overall assessment is illustrated by the sum, not the median.



Overall, testing this version of the interview has shown that further important indications about the agent's sensitivity to complexity of problem solving mathematics teaching can be gained with help of the used interpretation- and evaluation situations. With regard to the described dimensions, also the interviews suggest a low sensitivity among the group of test persons. For example, only one student managed to reach the highest ranking in all interpretation situations. In general, only few aspects are considered when evaluating options of action. Striking is that very little attention is paid to the current classroom situation and the students' previous problem solving processes. The necessity of making adaptive decisions in class is perhaps not understood. This could also be in line with results of expertise research according to which experts generally work in the sense of a forward search, novices on the other hand rather work in the sense of a backwards search and take sought-after values, aims and goals as a starting point. (Waldmann, 1996).

5. Outlook

A next step in further development of the described diagnostic instruments could be a more comprehensive empirical testing of the computer scenario. It seems especially important in this respect that teachers that have extensive experience with problem solving mathematics teaching are involved. To test the validity of the theoretically founded descriptive dimensions, one should look for the differences between experienced teachers and novice teachers that deal with the computer scenario.

The preliminary version of the interview should be enhanced above all with further interpretation- and evaluation situations to provide a broader base for the assessment of the interviewees. A further dimension to characterise the sensitivity to complexity could be the ability to independently develop appropriate options of action in the described classroom situations. Empirical data on this is already available (to a limited extent); however it has not yet been possible to complete the design of a relevant analytical method.

Due to the small number of test persons and the partly preliminary character of the applied analytical instruments, results of the pilot study can of course not be generalised. However, they do suggest to some degree that intensified efforts to sensitise students to the complexity of problem solving mathematics teaching could be necessary in teacher training at universities if these educational institutions want to make a long term contribution to an improvement in mathematics teaching.

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