Mathematical Modelling in classroom: a socio-critical and discursive perspective

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Abstract: *In this paper, I outline a socio-critical perspective of modelling in mathematics education and discuss implications for analysis of students' activities at the micro level. In particular, a discursive perspective is presented with contributions from discursive psychology. Recent studies and classroom examples are taken into consideration.*

ZDM-Classification: C50, M10

1. Introduction

Mathematical modelling[1] has been one of the foci of Mathematics Education, especially following the critiques of modern mathematics that began in the 1960's (Niss, 1987). Since 1983, *The International Community of Teachers of Mathematical Modelling and Applications* (ICTMA) have organized biennial conferences to promote modelling in schools and universities. The work of Niss (1987), Houston (2003) and Burkhardt (2006) provide important contributions for understanding the development of this international movement.

In Brazil, the modelling movement is closely associated with ethnomathematics, drawing strongly from the field of anthropology (Bassanezi, 1994; D'Ambrósio, 2002; Fiorentini, 1996). Like the international community, teachers and researchers in Brazil have organized biennial national conferences on modelling in Mathematics Education since 1999 (Caldeira et al, 2005; Marafon, 2003).

Many research papers in the ICTMA – for example Matos et al (2001), Qi-xiao Ye et al (2003) and Lamon et al (2003) – and the recent ICMI Study (Blum et al, 2006) are indications of the growth of the modelling community in Mathematics Education at the international level.

To date, little research has focused on students' activities in modelling. Borromeo Ferri (2006) points out that aspects of students' cognition at the micro level have received little attention. The aim of this paper is to join her in contributing to this debate. Here I outline a framework for understanding students' practices in modelling activities, drawing on the work of other authors.

First I will describe a modelling perspective with a critical emphasis in order to clarify how my point of view influences the following discussion. Next I argue that discursive cultural psychology can be used as a kind of lens to analyse aspects of students' cognition in modelling. The notions of mathematical, technological and reflexive discussions are proposed and linked to a critical perspective.

2. A modelling perspective

My intention is not to restrict the modelling debate to the level of argumentation, but to consider how, by attributing different emphases in the arguments, different ways to organize and conduct the activities, from which will emerge (Barbosa, 2001; Lingefjärd, 2006). Analogously, different perspectives result in different theoretical frameworks and research agendas.

Julie (2002) distinguishes "modelling as content" from "modelling as vehicle". The former emphasizes the development of the competencies needed to model real situations, while the latter views modelling as a way to teach mathematical concepts. These metaphors are related to a previous classification in the so-called pragmatic and scientific school of thought pointed out by Kaiser (1995) and in the new classification of Kaiser & Sriraman (2006). The difference appears to be subtle, but the perspective has influence on what is and is not considered to be legitimate in modelling-based teaching.

In Barbosa (2003), I propose a third possibility, reflection on the role of mathematics in society, drawing on studies about the sociocultural dimensions of mathematics (Atweh et al, 2001; D'Ambrósio, 1986, 1999), and in particular, the critical nature of mathematical models in society (Borba & Skovsmose, 1997; Keitel, 1993; Skovsmose, 1994).

[1] I am only going to use the term modelling in order to avoid repetition.

These studies point out that mathematical models are not neutral descriptions about an independent reality, but that the modelling process has devices that are usually concealed to the general public. Since arguments and decisions in society are based on mathematical models, it is important that the students have the opportunity to discuss the nature and role of mathematical models. I adopt a critical mathematics education perspective.

> We can characterise a critical epistemology as a theory of developing or constructing knowledge, where critique of what is learned is seen as part of the learning process (AlrØ & Skovsmose, 2002, p. 256).

The learning of mathematical concepts and the development of "modelling competencies" are viewed as vehicles to criticize mathematical models. In this way, the "modelling as critic" metaphor is appropriate. This emphasis is related to the idea that mathematics education must take part in efforts to educate students be critical, engaged citizens. D'Ambrosio (1999) and AlrØ and Skovsmose (2002) have proposed the notion of *matheracy* to denote this wider role of school mathematics. Thus, I want to propose complementing Kaiser's (1995) earlier classification with a third school of thought in modelling, called socio-critical.

Having outlined the perspective, I will now demarcate a concept of modelling in mathematics education, attempting to clarify its boundaries. The expression "Modelling and Applications" has been used as an umbrella term to denote all activities that link mathematics to everyday and/or other sciences (Blum & Niss, 1991). Thus, word problems, problem posing, open problems and project work have been recognised as modelling and applications.

Another aspect of modelling is that it is usually characterised by diagrammatic representations like those proposed by the Open University (Galbraith & Stillman, 2006). A number of similar examples can be found in the literature. Users of diagrammatic representations intend to describe the modelling process through well-defined stages. In mathematics education, researchers have borrowed these stages from applied mathematics to analyse students' modelling.

However, I would outline another point of view, defining the boundaries of a modelling activity with two main features:

- the activity has to be a problem (not an exercise) for the students;
- the activity has to be extracted from the everyday or other sciences that are not pure mathematics.

In specific terms, I have established the boundaries of modelling as a learning milieu where students are invited to take a problem and investigate it with reference to reality via mathematics (Barbosa, 2003). This notion is quite removed from the characterization of modelling as involving diagrammatic representations. It refers to modelling as a school activity, which may be informed by a pragmatic, scientific or socio-critical perspective. However, as discussed above, here this definition is taken with the last option.

3. Illustrating with an example

To illustrate the above ideas, I present a classroom modelling experience. It occurred in a 7th grade class in a public school in the rural region of Feira de Santana (interior of Brazil). In that time, the local government announced a program to distribute bean and corn seeds to farmers. Since the families of many of the students were beneficiaries of the program, the teacher thought the situation would be interesting for them. She clipped an excerpt from the newspaper about the program and invited the students to discuss it. A key part of the news clipping is translated below.

> "The bean and corn seeds donated by the Government begin to be distributed in yesterday afternoon. There are 37.5 tons, 25 tons of bean and 12.5 tons of corn seed. About 8000 subsistence farmers will be benefited. According to the mayor, each farmer will receive 3 kg of bean and 2 kg of corn."

This news referred to the students' everyday lives outside of school and directly involved their families. The situation was neither artificial nor fictional. The intention was to generate, through mathematics, discussions about a case that touched the students' lives.

After reading the news clipping, the teacher posed some questions to the students: What do you see in this news report? What is it about? Many points were raised in the discussion. Students soon detected a mistake, either in the news report or in the program: 37.5 tons would not be sufficient for 8000 farmers if each received 5 kg of seeds. After that, the teacher questioned the students regarding the criteria used by the government to distribute the seeds, and they appeared to be uncomfortable with it. Arguments were based on the hypothesis that the families had different needs, and should therefore receive different quantities.

Now the teacher was able to ask the students to concentrate on the specific question: What criteria would be fairer? It was a problem for them, because they knew of no ready strategies to use. Thus, I understand this activity as being a modelling activity, because it is a problem without artificial or pure mathematical features, as discussed in the previous section.

The students were arranged in groups, while the teacher followed their activities. Since the students did not agree with the earlier criteria for the distribution of the seeds, they developed another based on the number of members in the farmer family. Thus, the amount of seeds would be proportional to the number of people in each family. When the teacher perceived the criteria and the strategy used by the students, she asked one group to explain it on the blackboard. All the students shared their points of view, coordinated by the teacher. The criteria they developed are illustrated in figure 1, with p being the number of persons per family and S the amount of seeds in kg.

p	S
2	1,6
3	2,3
4	3,1
5	3,9
6	4,7
7	5,5
8	6,2
9	7,0
10	7,8

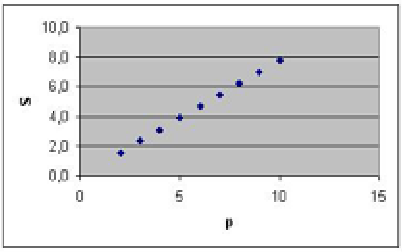

Figure 1 – The table and graph relating p and S.

In this activity, the students had the opportunity to observe and discuss how different criteria generate different mathematical results. Thus, the students discussed how different hypotheses and/or interests produce different mathematical models and decisions. It was a good opportunity to perceive the non-neutral nature of mathematical descriptions of real situations.

This episode illustrates what I have called a socio-critical perspective to modelling in mathematics education because it addressed the role of the mathematical models in society. Certainly students dealt with mathematical ideas and "modelling competencies", but these mostly provided support for understanding a current social situation. This example follows the same line as others in literature, for example in: AlrØ & Skovsmose, 2002; Borba & Villareal, 2005; Jacobini & Wodewotzki, 2006; Julie, 1994.

As I build my argument in favour of a social-critical perspective in modelling, I will address the micro level of the classroom and attempt to highlight implications for teachers' actions as well as for research. My aim is to support the argument that a wider perspective must guide the organization and conduction of modelling activities in the school and the research agenda.

4. Students in modelling activities

In a significant part of the literature on modelling, the diagrammatic representation view of modelling is used as a parameter to analyse students' modelling activities. Usually the intention is to bring the students closer to professional modelling. For example, Haines et al (2003) conducted a comparative study between novices and expert modellers. Lamon (2003) used the notion of "enculturation" to denote modelling in the schools as type of initiation into the modelling community.

Two other types of studies are presented in the literature: those that focus on students' competencies at each stage of the modelling process, and those that analyse the transition from one stage to the next. Houston and Neill (2003) and Maaβ (2006) are examples of the first case, as they investigate students' skills and competencies and compare them to some normative descriptions of modelling. Borromeo Ferri (2006) and Galbraith and Stillman (2006) investigated the transition among the modelling stages by the students.

However, there is evidence in some of these studies suggesting that normative models may be inappropriate for describing students' actions, such as those expressed by diagrammatic representations. Borromeo Ferri (2006) concludes that the differentiation among modelling stages is only theoretical, and that it is difficult to distinguish between them empirically. The author raises the question of a possible need to have one kind of modelling diagram for school and another for research, putting in doubt the efficiency of normative diagrammatic representations in modelling to describe students' activities.

Busse (2005) and Maaβ (2006) used empirical data as the basis for the identification of four ideal types of students in modelling: those who neglect mathematical aspects; those who neglect real aspects; those who strike a balance between the two; and those who consider both, but disassociated from each other.

The students' different styles are a reflection of the subjectivity inherent in the interpretation of the real situation. In Busse & Kaiser (2003) and Busse (2005), there is evidence that the problem context may be reconstructed in different ways by students, having diverse effects on them, since each has his/her own experiences and beliefs.

In addition, the use of modelling in the school opens up single features. Christiansen (2001) discusses a case in which students were solving a population problem, ignoring references to the out-of-school reality. The meaning produced was based on previous school experiences that legitimized this procedure. Araújo and Barbosa (2005) present a similar case in which the students were instructed by the teacher to create and solve a real problem. In response, the students first thought of a mathematical content, and then developed a fictional situation around it. The authors refer to this as an *inverse strategy*. Analysis of this case led to the hypothesis that students were influenced by the school culture to adopt such a strategy. These findings provide clues regarding the role of the social context in students' cognition.

Therefore, the evidence suggests that students' modelling routes, using the term coined by Borromeo Ferri (2006), are not linear and they do not fit well in normative stages. Many elements of these stages may appear, but not as described in diagrammatic representations. Variables related to the school context impose other conditions for the students' and teachers' work, such as time, teaching objectives, the school programme, and so on.

The incompatibility with the diagrammatic representation approach to modelling is not a result of students' deficient skills, but rather of using an inappropriate lens to examine their practice. Diagrammatic representations have origins in applied mathematics and, within the field of mathematics education, are associated with students' work as an object of research. In an earlier paper (Barbosa, 2003), I point to the need to:

> … carry out a systematic reflection on modelling from the locus of mathematics education. I do not suggest a separation of applied mathematics from mathematics education (Indeed, I recognize the intersection of these fields), but rather, the *singularisation* of the object in the field of mathematics education (p. 228).

Since students and professional modellers share different conditions and interests, the practices conducted by them are different. Studies in social perspectives (Lave, 1988; Wenger, 1999) have strongly stressed that school mathematics is not a mere transposition of scientific knowledge, but rather represents another kind of knowledge. Thus, I propose considering school modelling as a practice that differs from that of professional modelling.

This argumentation has political motivations, as well. Critiquing modelling is part of the learning that takes place in the process of doing modelling, and one of the aims is to produce critical, politically engaged citizens. The use of normative stages overlooks these dimensions because the focus is to bring the students closer to the community of professional modellers. According AlrØ and Skovsmose (2002), *an educational process cannot be interpreted as a straightforward process of enculturation* (p. 255).

Thus, implications are generated for research regarding students' cognition in modelling. Rather than measuring how closely students approximate normative descriptions of modelling, their practices can be viewed as fertile ground for formulating theory about modelling in the schools, the conditions of its production, and the possibilities for constituting socio-critical practices.

5. Focus on students' discourse in modelling activities

There are many ways to understand the aspects of students' cognition in modelling. Some studies focus on their actions and internal plans, using notions such as competencies (Maaβ, 2006), modelling routes (Borromeo Ferri, 2006), blockages (Galbraith & Stillman, 2006), styles and ideal types (Busse, 2005; Maull & Berry, 2001; Maaβ, 2006), reasoning (English, 2003), metaphor (Carreira, 2001) and so on.

From the methodological point of view, the discourses of the students are the data that are considered for analysis. Discourse tells us something about internal processes. However, it is more than a translation of what is going on inside their heads. People don't really think with clear explanations; the socially-shared language regulates and guides what we think. Discourse and cognition are inseparable (Chronaki & Christiansen, 2005; Lerman, 2001). Lerman (2001), drawing on a cultural discursive perspective, claims that discourse constitutes consciousness, as opposed to other theories which consider it as background.

I propose understanding discourse not only as data, but as an object of research. Discourse refers to all types of language, including signs, gestures, artefacts, mimics and so on (Lerman, 2001). Following, I will describe a modelling-based lesson as a discursive formation.

Recently, I used the notion of interaction spaces to denote the moments when students and/or teachers interact verbally about a modelling activity (Barbosa, 2006a). Modelling activities usually involve group work, seminars and other types of interactions. Empirical evidence points to the key role of these moments (Araújo & Salvador, 2001; Christiansen, 2001; Ikeda & Stephens, 2001). Through shared discourse, people negotiate meanings, and the discourse has a regulatory function.

In the modelling milieu, students produce many discourses, but my interest is in the verbal ones. I do not want to reduce the modelling practice to verbal forms, but to clarify an object of research. Students may discuss something when they are modelling real situations. Skovsmose (1990) proposed the notions of mathematical, technological and reflexive knowledge to refer to the modelling process. However, the term knowledge denotes something happening in the internal plan. Although it is beyond the scope of this paper, these ideas provide inspiration for defining students' discussions in modelling. In Barbosa (2006b), I suggest considering these types of discussions:

- mathematical: refers to the ideas belonging to the pure mathematics field.
- technological: refers to the techniques of building the mathematical model.
- reflexive: refers to the nature of the mathematical model, the criteria used in its building and its consequences.

To illustrate these different types of discussions, I present an episode that took place during a modelling course proffered to future mathematics teachers in the city of Salvador, in Brazil. In the beginning of the course, they were invited to formulate and solve problems in groups as a long-term project. This activity took four months. The teacher monitored the students' work through two partial written reports and occasional meetings.

During second semester, in 2004, one team – João, Maria, Vânia and Carlos – chose the theme "heart disease". In their first report, they included information regarding the influence of eight variables in the development of heart disease. For each variable, they attributed a numerical value to indicate the healthiness of the lifestyle. For example, for the cigarette smoking variable, zero represented people who had never smoked, one represented ex-smokers, and so on. In their final report, they presented an equation to determine the risk of heart disease.

Taking the report, the teacher had a meeting with the students. Below is a part of the discussion:

Maria: It's hard for us because we know how to deal with...
Teacher: 2 variables?
Maria: Sure…
Maria: Now we have 8 variables… we need 8 axes.
Teacher: Isn't there a risk variable, too?

Maria: Yep, it is, so there are 9 variables.
Maria: Now, how to plot the graph?
Teacher: Let's think about… in the case of 2 variables?
João: We could use the Cartesian plan.
Teacher: And…
Maria: With 3 variables, we use the space XYZ.
Teacher: And with 9 variables?
(…)

Faced with the impossibility of graphing the function with 9 variables, and they decided to abandon some of them.

João: So let's eliminate variables and work with three… after that we'll make the graph.
Carlos: Perhaps we could analyze the influence of a variable in the development of heart disease.
Maria: Keeping it simple, we can apply the correlation coefficient such as levels of cholesterol and obesity.

In this part, the discussion in bold between the student and the teacher refers to the building of the model, and it is called technological discussions. The excerpts in italics refer to mathematical procedures, and they illustrate mathematical discussions. In terms of the framework adopted, there is an interaction space.

Other questions could be formulated in this interaction space: What is the effect on the results of choosing only three variables? What criteria were used to select them? Are these results good for anyone? These questions could lead to reflections about the nature of the modelling process, like those described previously in the seed distribution case. This is what I call reflexive discussions. They function as the entrance to thinking about the role of mathematics in society.

The interest in discussions taking place in the interaction spaces is that they not only provide information about what the students say, but also about the perspective used, shared meanings, and their nature.

A preliminary study (Barbosa, 2006b) on the dynamics of discussions suggests that transitions between them occur by impasses. In the above episode, the intention to do a graphical representation with 9 variables, which is a technological discussion, produced a mathematical discussion when Maria asked about the plotting of the graph. Other transitions may take place according to the questions raised in discussions (see fig. 2).

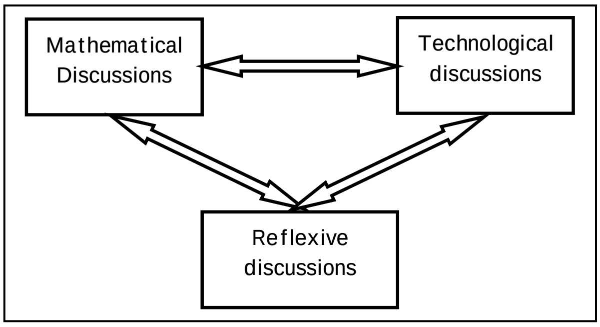

fig.2: types of students discussions

The place of these discussions in the organization of modelling activities in school changes according to the weight attributed to them by the teacher. In the pragmatic school of thought, the technological discussions are privileged; the mathematical discussions are given priority from a scientific perspective; and according to a socio-critical school of thought, the interest is on the reflexive discussions. Emphasizing one discussion type does not eliminate the others; often the other types are included as vehicles to support the predominant type. Depending on the teacher's perspective, he or she may place more weight on one type of discussions than another.

So the possibility of constituting a socio-critical approach in modelling is associated with the presence of reflexive discussions. It is difficult to imagine this perspective being put into practice without the questions raised in such discussions. However, their mere appearance in interaction spaces is not enough; stimulating reflexive discussion should be a primary goal, drawing on the other types of discussion for support.

6. Final remarks

In this paper, I revisit the debate about students modelling, putting together evidence and arguments from previous studies and illustrating examples. Initially, a perspective was described to orient the reader, emphasizing critical aspects. Normative diagrammatic representation approaches to modelling were questioned with respect to their applicability to modelling as viewed from the perspective of mathematics education. Maybe it is a polemic question that demands more discussion.

After that, the discursive and cultural psychology perspective is used as a lens to look at students' modelling. The notions of mathematical, technological and reflexive discussions were presented as a powerful way to describe the students' practice. Partial results of current research were discussed here; other aspects are currently under study.

To improve the research agenda on this theme, researchers may gain insights from the broader field of Psychology in Mathematics Education (PME). Other studies which authors have considered studies in PME has developed important analytic tools, such as situated cognition, used by Busse (2005). Approaches based on grounded theory, like Borromeo Ferri (2006), are important too, because they allow the generation of new ideas.

Supporting the demand for more attention about students' modelling from the locus of mathematics education in modelling community, this paper is a contribution for the debate.

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