

The Curriculum's View of Knowledge Transferred to National Tests in Mathematics in Sweden

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Abstract: This article provides a report on the testing system in Sweden. The Swedish assessment system today is goal and knowledge related. Important for this system is the view of knowledge in the curriculum. In this article the authors give an overview of the view of knowledge in the curriculum and in the syllabi in mathematics and they describe how this influences the tasks in the national tests. The change from a norm-referenced to a goal and criteria referenced grading system influences how students' solutions of different tasks will be marked and assessed. In this article the authors also describe different assessment methods e.g. the assessment matrix, which focuses on different aspects of knowledge based on the syllabi in mathematics.

Kurzreferat: Dieser Beitrag berichtet über die Form des Testens von Mathematikleistungen im schwedischen Schulsystem. Das heutige System der Leistungsmessung ist wissens- und kriteriumsbezogen. Wesentliche Orientierung ist die Auffassung von Wissen, wie sie im Curriculum festgehalten ist. Die Autorinnen geben einen Überblick über diesen Wissensbegriff in den Mathematiklehrplänen und beschreiben seinen Einfluss auf die Aufgaben in den nationalen Tests. Der Wechsel von einer gruppenvergleichsbezogenen zu einer kriteriumsbezogenen Zensurengebung hat einen Einfluss darauf, wie Schülerlösungen bewertet werden. Die Autorinnen beschreiben ebenfalls verschiedene Bewertungsmethoden, wie z.B. die Bewertungsmatrix, die auf verschiedene Wissensaspekte, wie sie in den Lehrplänen festgelegt sind, fokussiert.

ZDM-Classification: B73, D63, D73

1. Historical Background

Before the year 1990, Swedish 9-year compulsory school was centralised and governed in detail on the national level. Since 1994, the school system in Sweden is decentralised and one of the least regulated school system in Europe.

From the middle of the 1940-s until the middle of the 1990-s, the system of grades was norm-referenced. Grades were based on students' performance in relation to each other and to the national average. The grades of students were principally assigned by reference to recommended distribution of grades in the whole country. Such a system is often called "grading to the curve" and essentially says that a certain percentage of the students taking a subject are getting the top grade, another percentage the next grade, and so on. Students' grades were mainly determined by the rank order in the class. At the beginning, there were norm-referenced tests in mathematics for grades 2, 4 and 6 in the compulsory school. Later there were compulsory achievement tests in mathematics only in school year 9. The aim of the tests was to get maximum uniformity in teachers' grading throughout the country. These achievement tests were used for a norm-

referenced grading system, i.e. the knowledge and achievement of all the students in the country who belonged to the same course was compared.

The teacher should use the results of the test to obtain information about the average achievement level of the class as well as the distribution of individual grades in the class, all in relation to classes across the country. The tests steered the grading since the teachers had to follow the distribution of grades in the test when they assigned the final grades in their classes. In this system, it was very important that the tests were objective in the sense that all teachers could assess the students' solutions in exactly the same way. The tasks of these tests had to provide variation in difficulty and they had to have a high discrimination index. The discrimination index measures to which extent a task differentiates between high-scoring and low-scoring students.

2. The New System

2.1 The Swedish school system

The Swedish school system is now a goal-based system with a high degree of local responsibility. The main responsibility for education activities lies within the municipalities and authorities responsible for independent schools. Various steering documents are used to steer activities, i. e. school curriculum, course syllabi, etc. which are drawn up at different levels within the school system.

The overall national goals are set out by Swedish Parliament and Government in The Education Act, Curriculum for the Compulsory School System and Course syllabi for compulsory school. The National Agency for Education draws up and takes decisions on General recommendations and Grading criteria for all types of Swedish schools.

The National Agency for Education is continually developing and revising syllabi, grading criteria and other steering documents to ensure that they help to improve equivalence and achievement of goals, as well as reflect the needs motivated by developments in society and the workplace and the needs of individuals. The National Agency for Education is also expected to evaluate, follow up and supervise the public school system in Sweden. One part of this evaluation is the national tests.

2.2 Background of the curriculum's view of knowledge

In the assignment given to the different university institutions working with the development of the tests the National Agency for Education emphasises that "the curriculum's view of knowledge and learning should permeate the tests". The view of knowledge and learning in the curriculum is based on the curriculum committee's report "Skola för bildning" (SOU 1992:94), especially on the chapter "Kunskap och Lärande" (Knowledge and Learning). The chapter discusses knowledge and learning based on recent research studies.

This chapter highlights three important aspects of knowledge: the constructive, the functional and the contextual aspect. The constructive aspect implies that knowledge is one way to make the world understandable.

“Knowledge develops through an interaction between what one wants to achieve, the knowledge one already has and the experiences one gains.” The functional aspect emphasises that “Knowledge works as a tool, solves problems or simplifies the work.” Finally the contextual aspect says that “Knowledge is always within a context – a practical, a social or a linguistic context” (SOU 1992:94, p. 79). The student must be able to understand the context where the knowledge is being developed (historical perspective). At the same time, the school must also offer a meaningful context where the student can obtain knowledge.

Four forms of knowledge are discussed in the report: facts, understanding, proficiency and familiarity. Factual knowledge is quantitative knowledge, knowledge as information that can be possessed in different quantities. Understanding is qualitative. To understand is to grasp, to perceive the meaning or the relevance of a phenomenon. Factual and understanding knowledge are intimately connected to each other. Understanding decides which facts we can see and perceive, but the still facts are the cornerstones of understanding. The proficiency aspect of knowledge is when we know how something should be handled or accomplished. While understanding is a theoretical form of knowledge, proficiency is a practical form. Familiarity is the silent dimension of knowledge. All four forms are found within all of the knowledge areas. However, the different forms can be seen differently in different areas and by different people. The purpose of discussing these four forms is to show the complexity when trying to express the definition of knowledge and to prevent one-sided reductions of the phenomenon of knowledge. “It is important that the different forms of knowledge are given space in all of the subjects. This means that the practical sides of the so called theoretical subjects must be acknowledged, as well as the practical subjects’ theoretical aspects” (SOU 1992:94, p 78).

The special task of the school is to provide the student with the type of knowledge that is unobtainable outside of the school. The student must be able to meet and be acquainted with the different knowledge within the different knowledge areas. Regarding the tasks of the school it is stated: “A student must be able to acquire the concepts and structures from the different subject areas in such a way that they can be used as intellectual tools in other contexts” (SOU 1992:94, p. 63).

2.3 The Curriculum

A new national curriculum for compulsory school came into effect in autumn 1994 (Utbildningsdepartementet, 1994).

The following competences are presented in the Swedish curriculum as being essential for students to achieve

Knowledge, which is durable over time; to be able to orient oneself in a complex reality with a huge flow of information; the ability to independently deal with sources of information and knowledge; to be able to critically examine facts and conditions and envision the consequences of different actions and decisions: to reflect upon problems and gradually develop an increasingly scientific attitude. (Utbildningsdepartementet, p. 7)

This curriculum is goal-related and it defines the underlying values and basic objectives and guidelines of the

school system. In addition there is a nationally defined syllabus for each individual subject. The compulsory school syllabi indicate the purpose, content and objectives for teaching in each individual subject. The goals both in the curriculum and in the syllabi are of two kinds: those that the schools should aim for and those which the schools have the responsibility of ensuring that all pupils attain. *Goals to aim for* specify the orientation of work in the schools. They express the direction the schools should take in terms of developing the pupil’s knowledge. *Goals to attain* define the minimum level of knowledge pupils shall have attained when leaving school. Both the schools and the principal organiser are responsible for ensuring that pupils are given the opportunity of attaining these goals.

Examples of goals to aim for in compulsory school:

The school should aim for that all pupils

- learn to listen, discuss, reason and use their knowledge as a tool to formulate and test assumptions as well as solve problems
- develop the ability to assess their results themselves and to place their own assessments as well as those of others in relation to their own achievements and circumstances.

Examples of goals to be attained by all pupils in compulsory school:

The school is responsible for ensuring that all pupils completing compulsory school

- have a mastery of Swedish and can actively listen and read as well as express ideas and thoughts in spoken and written language,.
- have a mastery of basic mathematical principles and can use these in everyday life.

2.4 The syllabi in mathematics

A school subject is not a natural phenomenon; it is a human and social construction that changes over time. Mathematics’ part of the overall view of knowledge is described in the syllabi as being a sound basis for the studying other subjects, for further education and for life-long learning. Furthermore the subject’s historical development, its importance and role in society is highlighted: “Mathematics should give students opportunity to practise and communicate mathematically in meaningful and relevant situations through actively and openly searching for understanding, by gaining new insights and finding solutions to different problems” (Skolverket, 2000).

The purpose of Mathematics is characterised in the syllabi as a way to investigate and structure theoretical and practical problems. Mathematics is also a way to think with part intuition and part logic. Mathematics is also a language that has symbols, which make it possible to condense and express with precision, as well as logically work with complicated ideas and assumptions. The supply of new technical resources partly changes the contents and methods of mathematics. A student must learn how to utilise the calculator and computer as a tool with familiarity and proper judgement.

In order to be successful in mathematics, a balance is required between on the one hand creative problem solving activities, and on the other hand knowledge about mathematical concepts, methods and ways of expression. Teaching in mathematics should according to the syllabi aim to develop the pupil's ability to use and critically examine simple mathematical models, to develop the pupil's creativity and persistence in mathematical problem solving, as well as the ability to draw conclusions and to generalise. The teaching should also develop the pupils' ability to present their work, both orally and in writing with help of mathematical language and expressions.

In the syllabi for compulsory schools in mathematics there are two kinds of goals to aim for. One kind describes more overarching mathematical competences and the other kind describes the mathematical content.

Examples of goals to aim for that are more overarching:

The school in its teaching of mathematics should aim to ensure that pupils

- develop an interest in mathematics, as well as confidence in their own thinking and their own ability to learn and use mathematics in different situations,
- develop their ability to understand, carry out and use logical reasoning, draw conclusions and generalise, as well as orally and in writing explain and provide the arguments for their thinking,
- develop their ability to formulate, represent and solve problems using mathematics, and to interpret, compare and evaluate solutions in relation to the original problem situation.

Examples of goals to aim for relating to mathematical content:

The aim should also be that pupils develop their numerical and spatial understanding, as well as their ability to understand and use.

- basic numerical concepts and calculations with real numbers, approximate values, proportionality and percentages,

Goals to aim for indicate the direction of teaching in mathematics and do not put a limit for the pupils' knowledge development. In addition to these goals there are goals that pupils should have attained by the end of school year 5 and by the end of school year 9. These goals relate to different areas of competence in mathematics, such as Sense of number, Geometry, Algebra and Statistics.

Examples of goals to be attained by the end of the fifth year in school are:

Pupils should have acquired the basic knowledge in mathematics that is needed to be able to describe and manage situations, and also to solve concrete problems in their immediate environment.

Within this framework, pupils should

- have a basic understanding of numbers covering natural numbers and simple numbers in fractions and decimal form,
- understand and be able to use addition, subtraction, multiplication and division, as well as be able to discover numerical patterns and determine unknown numbers in simple formulae,
- be able to calculate in natural numbers – mentally and by using written calculation methods and pocket calculators,

Examples of goals to be attained by the end of the ninth year in school:

Pupils should have acquired the knowledge in mathematics needed to be able to describe and manage situations, as well as to solve problems that occur regularly in the home and society, which is needed as a foundation for further education.

Within this framework, pupils should

- have developed their understanding of numbers to cover whole and rational numbers in fraction and decimal form,
- have good skills in and be able to make estimates and calculations of natural numbers, numbers in decimal form, as well as percentages and proportions, with the help of written calculation methods and technical aids.

The creativity-related aspects of knowledge are emphasized in the description of the mathematics subject's "goals to aim for". However, they are rarely mentioned in the "goals to be attained".

2.5 The grading system

Simultaneously with the introduction of the new curriculum and syllabi a new system of grades has come into effect. The grading system is goal-related, meaning that student achievement is assessed in relation to the goals stated in the syllabus. In this system grades are awarded on a three-grade scale from the eighth year of schooling onwards. Grades are set by the teacher and include one of the following possible grades:

- Pass (G),
- Pass with distinction (VG) and
- Pass with special distinction (MVG).

When assessing the student achievement level at the end of the year 9 teachers are expected to base their assessment on the goals and criteria stated in the syllabi. Basic achievement of goals to attain corresponds to the grade of "Pass". The awarding of the grades of "Pass with distinction" and "Pass with special distinction" follows nationally approved grading criteria (see below). In cases where a student fails to achieve a passing grade in a subject no grade is given.

According to the curriculum, when assigning grades the teacher should make use of all available information about the pupil's knowledge in relation to the require-

ments stated in the syllabus and make a comprehensive assessment of the knowledge acquired by the pupil.

The focus of assessment in mathematics is divided in three different qualities.

- The ability to use, develop and express mathematical knowledge.
- The ability to follow, understand and examine mathematical reasoning.
- The ability to reflect on the significance of mathematics in cultural and social life.

Examples of grading criteria in mathematics for the compulsory school are:

Criteria for “Pass with distinction”

- Pupils demonstrate accuracy in their problem-solving work and use different methods and procedures.
- Pupils follow and understand mathematical reasoning.

Criteria for “Pass with special distinction”

- Pupils develop problems and use general strategies for the planning and execution of the tasks and analyse and present their work in a structured way with correct mathematical language.
- Pupils participate in the arguments of others and based on these arguments propose mathematical ideas of their own.

2.6 The testing system

Also the new national tests work in a goal and knowledge context. It has in part other demands than that of a norm-referenced test. Grades are based on standards and are not related to the performance of other students. The new national tests must consist of more varying tasks and the students must have opportunities to show their competence in different ways (Pettersson, 2003). Mathematical competence is much more than merely knowing certain mathematical content and performing certain skills. It is also essential to communicate knowledge and to present mathematics in written, oral, visual and symbolic forms. In addition it is important to use mathematical strategies, models and methods within one's present knowledge and skills to create new skills and methods utilising a range of facts, concepts and processes. It is also essential to be able to communicate one's mathematical knowledge both in written and in oral forms.

The government writes in its plan for development:

The view of knowledge described in the curricula and expressed in various ways in the course syllabi and grading criteria provide entirely new premises for evaluating knowledge. Test tasks may no longer consist of “simple” measures of traditional style. They must also analyse what abilities those who have a particular education possess and not merely list achieved rote memory knowledge. They ought to encompass problem solving, applications and combinations of different areas of knowledge. Results of learning are shown better as comprehensive competence and attitudes rather than reporting of facts. This is especially valid for the broad goals and the moral value-system of the curriculum but also for the subject goals in the course syllabi. This requires new ways of constructing bases for evaluation and for analysing results of learning. (Regeringens skrivelse 1996/97:112, p. 106)

A consequence of the government's plan is that the students must be given opportunities to demonstrate their knowledge in various ways. When we develop the national tests in Sweden, we strive towards making the essentials assessable and not the most easily measured things the most essential (Pettersson & Kjellström, 1995).

Another important point of reference in the construction of national tests is of course the ongoing development that is taking place in mathematics teaching and in international and national research in this area. To an increasing degree countries like Australia and the Netherlands use more extensive tasks and authentic problems. There are different reasons for this (Wiliam, 1994). The authentic problems represent mathematical competence better than other types of problems. It is believed that better prediction of future success in higher studies is facilitated using authentic problems. Finally, an investigative studying approach is an important part of the subject of mathematics and authentic problems in tests to encourage teachers to work in a corresponding fashion (de Lange, 1992).

2.6.1 The construction of national tests

The starting point when constructing a test is the view of knowledge stated in the curriculum, the view of the subject in the syllabus and in the criteria for the different grades. The test paper consists of several different parts in order to give the students an opportunity to show as many sides of their competence in mathematics as possible. We try to make our tests as balanced as possible with various kinds of tasks in a variety of contexts and a range of response formats. The tasks in the testing material should be designed in such a way that the student is given opportunities to demonstrate different areas of competence and different qualitative levels in mathematics.

The following schematic schedule shows the different test materials in mathematics for compulsory school in Sweden.

<i>Diagnostic materials</i> Not compulsory	Pre-school up to Grade 6, and from Grade 6 up to Grade 9	Assessment scheme for analysis of mathematics A bank of tasks
<i>Subject test</i> Not compulsory	Grade 5	4–5 different parts + group-tasks and self-assessment
<i>Subject test</i> Compulsory	Grade 9	2–3 different parts + group-works or/and oral test

Table 1: Test materials for compulsory school

The *diagnostic materials in mathematics* have a formative purpose and they consist of two parts, one part for use in pre-school and up to Grade 6 and one part from Grade 6 to Grade 9. Each part consists of an assessment scheme for analysis and a bank of diagnostic tasks. The purpose of the materials for analysis is to help teachers to analyse and document the student's knowledge in mathematics. The teacher can use the same scheme for students of different ages. By using this scheme of analysis, the

development of a pupil’s knowledge can be traced over several years.

The *national subject tests in mathematics for Grade 5* are not yet compulsory. The tests are only one component in the entire assessment of the pupil’s knowledge done by the teacher. The test is intended to be integrated into the ordinary teaching and each pupil is given sufficient time to work on the various tasks in the test. The tests main purpose is to assist the teachers in assessing whether the pupils have reached the goals to attain for school year 5 in the syllabi, but the test also has a formative purpose. Therefore it is important to analyse how the pupils solve the problems and to examine the quality of their work in order to identify their strengths and weaknesses in mathematics. The teachers are supposed to assess the test holistically and not use scoring points. The intention is that both the way in which the pupil has solved the problem and the answer will be taken into consideration. To describe each pupil’s mathematical knowledge the teachers can use a Competency Profile. Teachers should then consider both their assessment of the pupil’s work on the subject test as well as their overall assessment of the pupil’s mathematical knowledge.

There are two possibilities for the teacher to document her/his assessment of the pupil’s performances in the Competency Profile. For each competency area, the teacher can indicate the assessment using an “X” on the arrow. The teacher can also describe the assessment in words making use of the opportunity to comment on or to describe verbally their assessment.

The following example shows only two out of ten competency areas.

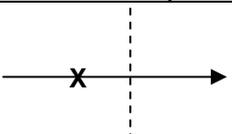
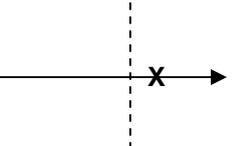
Competency areas	Teacher’s comment	Achievement of goals to attain for grade 5
– Basic spatial perceptions – shapes and patterns – scale	“Difficulty with the names of the shapes.”	satisfactory 
diagrams, tables, measures of central tendency	“Can handle simple tables and diagrams. Passes easy the goals for grade 5.”	

Table 2: Example for Competency Profile

The teachers can hopefully gain a more balanced picture of the pupil’s knowledge in mathematics, with the help of the profile.

The *national subject test in mathematics for Grade 9* is compulsory. The main purpose of the test is to help the teacher assess to what extent the pupils have reached the goals set up in the syllabus and to provide support for teachers in awarding grades. The test paper consists of several different parts in order to give the pupil an opportunity to show as many aspects as possible of her/his competence in mathematics. The pupil can show this by different ways of working (individually or in pairs or groups) and by different ways of answering (orally, by

written answers with or without explanations and calculations). The teacher should use different ways of marking and assessing (right/wrong, scoring rubric and analytic scoring). Two parts of the test are more traditional and in one of these the pupils are not allowed to use calculators. The third part consists of one more extensive task and the fourth part is often an oral test.

The national tests are not meant to steer teachers in their grading but rather to help them to assess whether and how well the individual pupil has reached the goals for the subject.

We can illustrate the comparison of working in the old and the new systems in Grade 9 in the following way:

The old system	The new system
1. Construction of the test	1. Construction of the test
2. The pupils work with the test	2. The limits for the three grades (“cut scores”) are established
3. The teachers assess and send results back for standardizing	3. The pupils work with the test
4. The intervals for different grades are established and sent to schools/teachers	4. The teachers assess the pupils work

Table 3: The old and the new system

The main difference between the two systems is that the teachers receive cut scores for the different grades *together* with the test rather than *after* the pupils have worked with the test.

2.6.2 Examples of tasks

In the old test system we asked things like: How many? How high? How much? When? Estimate... Compute...

In the new testing system we also have questions like: Why? Explain... Investigate... Describe...

An example from the national test for grade 5

On a plate in the classroom, there are apples, bananas and oranges.

- Half of the fruit consists of apples.
- A quarter of the fruit consists of bananas.
- The rest of the fruit consists of oranges.

Make a suggestion of how many apples, bananas and oranges there might be on the plate.

Feel free to draw a picture.

Feel free to write more suggestions.

Figure 1: Task from the test for the grade 5

An example from the national test for Grade 9

Lisa competes in archery. Each arrow can score at least 0 points and at most 10 points.

At a competition, Lisa shot 5 arrows. The mean score was 8 points and the median was 10 points. How might she have shot? Explain your choice and discuss various possibilities.

Figure 2: Task from the test for the grade 9

2.6.3 Examples of assessment directives

To be able to assess the pupil's knowledge in mathematics described in the different goals in the syllabi and in the grading criteria an assessment basis as broad as possible is needed.

An important element in qualitatively assessing pupils' knowledge is analysing, through various tasks and situations, how the pupils work with and master an area of mathematics. We examine whether the pupils have tried to find a solution to the task, how well they have understood the task, which concepts they have displayed and the quality of the strategies that the pupils have used. The teacher makes his judgement of the pupil's achievements according to the assessment directions existing for each sub test. A characteristic of the Swedish national tests is that the teachers assess and score the pupils' work themselves, using centrally predetermined scoring rubrics and cut-scores.

G-points and VG-points

To make clear the qualitative levels expressed in the grading criteria, we give G-points and VG-points in the assessment. G-points refer to knowledge, which can be connected to goals to attain for the school year 9 (G = Pass) and VG-points refer to knowledge, which can be connected to VG- and/or MVG-criteria (VG = Pass with distinction, MVG = Pass with special distinction). After each task in the test the maximum total number of points possible for the solution is shown. For example (2/1) indicates that the solution of the task can give a maximum of 2 G-points and 1 VG-point.

Tasks marked with the symbol ☒

At the construction of tests we take into account that certain tasks should invite to solutions and reasoning, which indicate qualities that are connected to the criteria for Pass with special distinction. Qualities like "using general strategies", "analysing one's results", "having a well-developed mathematical language" etc are more difficult to catch and summarize numerically in points. The judgement of the pupil's performance on the test as to the MVG category will therefore not be reflected in a scale of points. To get the highest mark on the test the pupil must have demonstrated both width and depth in her/his mathematical knowledge. The width is shown when the pupil more than enough has satisfied the requirements for Pass with distinction. The depth is judged when the teacher particularly studies the pupil's work with certain, especially marked tasks in the test. These are problems, which as such do not need to be very complicated. It is rather so, that these tasks might be solved in several ways, which makes it possible for the pupils to use a more or less general method and a more or less developed mode of mathematical expression and language.

Example of a task where the pupils' solution can show different quality levels

Andreas and Lisa both got a rise in salary, each with equally many crowns. Andreas' rise was 5 % and Lisa's was 2.5 %. Investigate with calculations and reasoning for which salaries this can be possible. (1/1) ☒

Figure 3: Task with different quality levels of the solution

Assessment directives for this task

E.g. "Lisa has a twice as high salary as Andreas"	(Max 1/1) ☒
An attempt of a solution e.g.: Lisa has higher salary	1 g
Example with numerical or general calculations	+ 1 vg
<i>Copied authentic pupil working</i>	
1/0 They do not have the same salary from the beginning. Lisa had higher salary since she got smaller increase. Andreas got larger increase i.e. he had lower salary from the beginning.	
1/1 Let us suppose that Lisa from the beginning had 200 kr/h and Andreas had 100 kr/h. When Lisa's salary rise with 2.5 % ($0.025 \cdot 200$) it increases with 5 kr/h. Andreas' salary rise with 5 % ($0.05 \cdot 100$) that is 5 kr/h. Both salaries rise with equal amount since Lisa got twice as much in salary from the beginning compared with Andreas.	
1/1 ☒ Lisa's old salary: x kr. Lisa's salary increase $0.025 \cdot x$ kr. Andreas' old salary: y kr. Andreas' salary increase $0.05 \cdot y$ kr. If the increase is the same then $0.025 \cdot x = 0.05 \cdot y$; $x = 2y$ i.e. Lisa's salary from the beginning is twice as big as Andreas'.	
The last pupil's work shows MVG-quality	

Figure 4: Assessment directives for the task in Figure 3

2.6.4 Assessing an extensive task

One part of the test contains tasks, which shall be answered orally and another part contains one task which is of a more investigating, extensive and open character. A requirement for these tasks is that they give all pupils the opportunity to start at a solution, while at the same time the task should be sufficiently challenging that a very good solution can show qualities at the highest-grade level.

A distinguishing feature of more extensive tasks, often placed in authentic contexts is that they are evaluated and assessed using methods different from those used for ordinary tasks. The teachers assess/evaluate these tasks with the support of assessment matrices. The purpose is to show the teacher and the pupil the different aspects of knowledge that can be assessed and to describe the different qualitative levels within each aspect of knowledge. These aspects and descriptions are taken from the curriculum and grading criteria. The result of the assessment

of each one of these sub tests gives a number of G-points and VG-points and demonstrated MVG-quality.

One example of a more extensive task from the national test for Grade 9

Assembly halls

- I In the new school building there will be an assembly hall where the first row has 10 seats and the second row has 13 seats. Row 3 will have 16 seats and after this, each row will increase with 3 seats until the back row, which has 31 seats.
- How many seats will row 6 have?
 - How many rows will there be in the hall?
 - Describe in words or by using a formula how to work out the number of seats in row n .
- II In another assembly hall, the number of seats in row n can be worked out using the formula $12 + 5n$. Explain how this hall is arranged.
- III Kalle claims that it is possible to work out the total number of seats in an assembly hall, similarly arranged, by multiplying the number of seats in the middle row by the total number of rows. Investigate if Kalle is right.

Your teacher will take into account the following when assessing your work

- what mathematical skills you have shown
- how well you explain your work
- which descriptions and conclusions you have reached.

Figure 5: Extensive Task for Grade 9

This kind of task was first used in the national test in 1998 and in the assessment guidelines, the teachers were advised to make holistic assessments of the pupils' work. To help the teacher in assessing and grading there were descriptions of pupils' work at different achievement levels as well as authentic pupils' work, graded by a group of teachers and researchers (Kjellström & Olofsson, 2001).

2.6.5 Assessment matrix

Sweden has a long tradition of teacher-assessed national tests and teachers are accustomed to having detailed assessment/scoring guidelines. The holistic approach in assessing pupils' performance turned out to be difficult for the teachers and some of them found it to be subjective and unfair to their pupils. Research carried out concerning examinations has shown that essay-type questions are less reliably marked than structured, analytically marked questions (Murphy, 1982). The greater the amount of structure in the marking scheme the more likely it is that two people will agree on the marking. This is also supported by research which shows that even for performance-based tasks, when assessors are trained and scoring rubrics are provided, inter-coder reliability can be high (Brown, 1992; Shavelson et al., 1992).

We started looking for other assessment models than holistic ones and found among others different ways of analytic scoring. In the book "Mathematics Assessment and Evaluation" edited by Thomas Romberg, we found a way of assessing problem solving in Vermont's work with assessing portfolios using a matrix (Romberg, 1992). This matrix was the starting point for the development of our first general assessment matrix. We organised the differ-

ent aspects of the problem-solving process according to our syllabus in mathematics and the grading criteria. The quality levels were described in three levels according to the criteria for different grades. One of the purposes of the assessment matrix was also that it would be possible to use it independently of mathematical content and in different courses.

After the first test, for which the assessment matrix was used, we evaluated teachers' opinions in a teacher questionnaire. Half of the teachers answered that the matrix facilitates assessment and that it was fairer to the pupils than holistic assessment. Many teachers thought, however, that the assessment was time-consuming. Half of the teachers answered that it was worth the effort but they wanted more detailed matrices. For the tests in 2000/2001 and onwards we have therefore developed task-specific matrices for each more extensive task.

The first step in the process of developing a matrix for a task involves an in-depth analysis of the pupils' work. We always pre-test all tasks for our national tests. We start with the general assessment matrix and with the help of this analysis we make task-specific descriptions in the matrix. Together with the matrix, we also publish authentic pupil works assessed and graded by a group of teachers and researchers. From the teacher questionnaire, we now know that almost all teachers think that assessing/marking with a task-specific matrix is worth the effort and that it facilitated the marking. The pupils as well as their teachers can find examples of more extensive tasks as well as assessment matrices at our website (www.lhs.se/prim).

2.6.6 Assessing oral communication

It is quite a challenge to construct problems and make assessment directives for the oral part. The aim is to construct problems that are more suitable for oral communication than for written solutions. The problems are quite different from the ordinary math problems you can find in Swedish textbooks. The oral test for Grade 9 is carried out in groups of 3–4 pupils, selected by the teacher, who work well together. Our intention with this is that there should be a conversation between the pupils with as little interference from the teacher as possible. The oral test consists of two sub-sections. First, every pupil gives an account of "her/his problem" to the other pupils in the group. Then the whole group discusses all the problems.

One of the models we have used in the oral test

This task assesses whether the pupil knows how to describe and compare geometrical figures and to find similarities and differences between the figures, e.g. concerning perimeter and area. The task also tests to what extent the pupil can use mathematical language, see relations, argue and accept arguments from others.

Task: Each one of you will get a sheet of paper with two geometrical figures. Your task is to describe equalities and differences between the figures e.g. concerning perimeter and area.

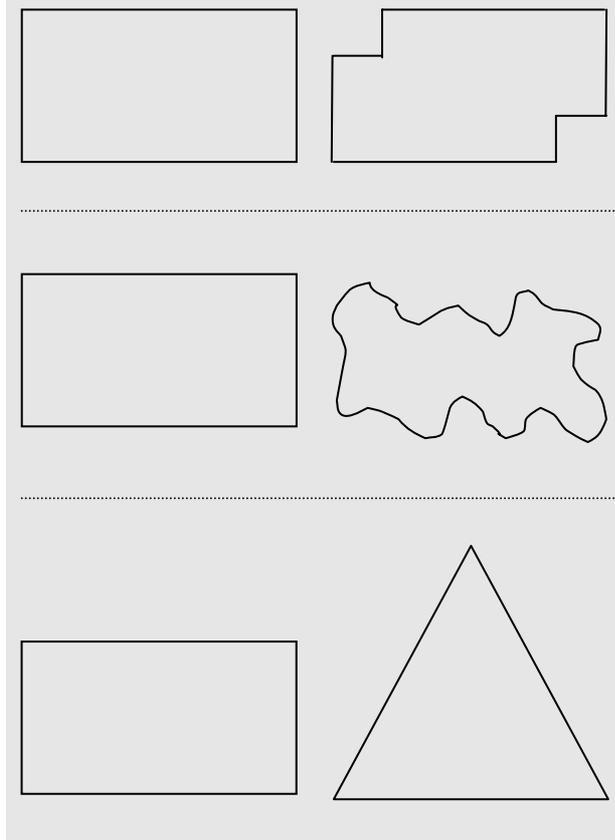


Figure 6: Geometric figures (three examples out of six)

Every pupil gets a sheet of paper with *one pair* of figures and is then given approximately five minutes for preparation. The different pairs of figures are of varying difficulty to compare and can be chosen according to the pupil's capacity. The pupils report in turn to the other pupils in the group. To help the pupil to advance the teacher might insert short questions. After each report, the other pupils in the group are invited to ask questions, make comments and additions. Finally all figures are put on the table and the pupils are invited to discuss together similarities and differences between as many figures as possible.

In assessing the pupil's performance one should take into account what pair of figures the pupil has compared. The teacher should make a holistic assessment of a pupil's performance with the support of a task-specific assessment matrix (see next page).

3. Concluding Remarks

In Sweden, there are no external examinations when students leave secondary or upper secondary school. The teachers do all assessing and grading. This makes the situation concerning assessment rather unique in Sweden in comparison with many other countries. The national tests are not intended to steer the teachers in their grading but to help them to assess whether and to what extent the

individual student has attained the stated goals for the subject. The purpose of having national tests is also to create a basis for assessment that is as uniform as possible across the country.

Of course, there are both disadvantages and advantages to the Swedish system. For the teachers assessment of their students' knowledge takes a lot of time to perform and when it comes to assessment and grading the Swedish students depend on their teachers. It is important that the teachers have sufficient competence to carry out the assessment satisfactorily. It is necessary to provide more time for teachers for discussions about assessment. It has been shown that the quality of assessment is enhanced if teachers discuss assessment in concrete terms with each other (Gipps, 1994). Our contacts with teachers at several lectures and courses and in thousands of questionnaires have given us the impression that most teachers really want to do a good job concerning assessment of their students' knowledge.

Teachers are generally pleased with the structure and content of the national tests. Most of the comments on the various parts of the tests are positive. The teachers feel that the tests give them support in assessing the performances of the students and that the assessment guidelines provide sufficient help. One purpose of the national tests is to reflect the values and ideas described in the national curriculum and syllabus. In this way the national tests in mathematics can stimulate discussion about assessment in general and about the teaching of mathematics in particular.

<i>The assessment concerns</i>	<i>Qualitative levels</i>		
	<i>Lower</i>	—————→	<i>Higher</i>
<i>Understanding</i> – To what extent the student shows an understanding for the task and motivates his/her conclusions – To what extent the student uses relations and generalisations	Shows understanding for either of the concepts perimeter and area, i.e. by describing how you can calculate perimeter or area for some of the figures.	Shows a good understanding for both the concepts perimeter and area, i.e. by explaining how you can see that perimeter and area are the same/ not the same for the two figures.	Shows a good understanding for both the concepts perimeter and area and relations between them. I.e. by carrying out general reasoning and extending the problem.
<i>Language</i> – How clear and distinct the student's account is. – To what extent the student uses relevant mathematical language and terminology.	Comprehensible and possible to follow.	Easy to follow. Uses acceptable mathematical language and terminology.	Well-structured and clear with relevant mathematical language and terminology.
<i>Participation</i> – To what extent the student participates in discussions with mathematically based ideas and explanations	Accounts only for one's own figures.	Contributes with his/her own ideas and explanations after explanations of others or in the final discussion.	Is involved in the arguments of others. Carries the discussion forward.

Table 4: Task-specific assessment matrix for “Geometric figures”

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¹ PRIM-gruppen at Stockholm Institute of Education, where the authors are project leaders, has constructed standardized achievement tests in mathematics since 1984. PRIM-gruppen is a research group which conducts its research within the area of assessment of knowledge and competencies. We develop, within this area, assessment methods in collaboration with teachers, teacher trainers, subject experts and researchers. Student knowledge progress in mathematics over time and analysis of how students solve problems in mathematics are examples of research that are continuously focused by the group. Since 1994 we have the responsibility to develop and construct diagnostic material and national tests in mathematics for the compulsory schools. PRIM-gruppen is also involved in both national and international evaluation of knowledge in mathematics.