# Outcome standards and core curricula: a new orientation for mathematics teachers in Germany

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**Abstract:** One of the pivotal innovations in Germany after publication and discussion of the results of international large scale assessment studies is the introduction of national educational standards for major subjects. The 16 German federal states have committed themselves to implement these standards by developing core curricula for the use in schools and by regularly testing the achievement of students. This development is expected to have considerable impact on the outcome of education. In this article we give a picture of the current development in the German education system by describing the design of the core curriculum in one of the federal states (NRW) and discuss its function in bridging the gap between national educational standards and teaching practice.[1]

**Kurzreferat:** Eine Schlüsselstellung unter den Innovationen im deutschen Bildungssystem nach der Veröffentlichung und Diskussion der Ergebnisse internationaler Leistungsstudien nimmt die Einführung nationaler Bildungsstandards für die Kernfächer ein. Die 16 Bundesländer haben zugesagt, diese Standards zu implementieren, indem sie Kerncurricula für die Schulen entwickeln und regelmäßig die Schülerleistungen erheben. Von dieser Entwicklung wird eine beträchtliche Wirkung auf die Bildungsergebnisse erwartet. In diesem Beitrag geben wir einen Einblick in die aktuellen Entwicklungen im deutschen Bildungssystem, indem wir die Anlage eines solchen Kerncurriculums in einem Bundesland (NRW) darlegen und seine Funktion als Brückenschlag zwischen nationalen Bildungsstandards und Lehrpraxis diskutieren.

**ZDM-Classification**: B73, D33

## 1. Introduction

Germany's recent reorientation to steering the school system by the outcome resulted in a multi-faceted development:

- the creation of outcome standards
- the implementation of centrally designed instruments of measuring students performance
- the redefinition of the role of school-inspectorates

In this article we concentrate on the first aspect of this process, the creation of outcome standards. Section 2 begins with a brief description of the political background and situation in Germany as a basis to explicate the actual development. Section 3 gives an account of the (partially incongruent) concepts and the conditions at the outset of creating the standards in the different federal states. In Section 4 we will concentrate on giving the picture that resulted from this process as it is perceived by teachers in the largest of the sixteen federal states, North-Rhine-Westphalia (NRW). The perspective of a single state seems not only adequate but decisive when it comes to the effects palpable inside schools, since teachers comply with the *regional* regulations, indicated in the federal states curricula. They acknowledge *national* standards (if at all) only as a central instrument of harmonization but not as an instrument for organising everyday teaching. Finally section 5 summarizes aspects of the future development – esp. open questions and problems.

## 2. Current situation in Germany causing a curricular shift of paradigm

To understand the curricular development which we try to depict on the following pages one has to take into account the characteristics of the German school system. Its most important trait is the parallelism of different secondary schools, which is visualised in fig. 1.

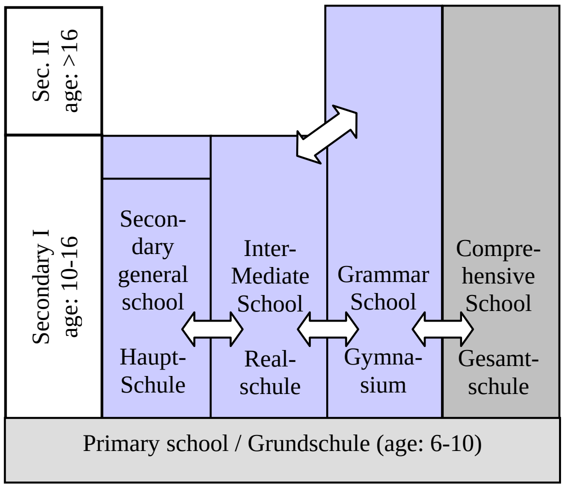


Fig.1: Basic structure of the German education system

Students are already distributed among the different secondary school types at the age of 10 and a horizontal change between school types takes place almost exclusively in the direction of the "lower" type.

Depending on the school type the orientation and the aims of teaching differ. *Secondary general schools* provide general education as a basis for practical vocational training. *Intermediate schools* provide extended general education. The final certificate provides the basis for training in all types of medium-level occupations and qualifies holders for attendance at several specialised schools. *Grammar schools* are secondary schools that cover 8-9 years and lead to academic study at universities. *Comprehensive schools* combine those different types of secondary schools and let students decide later about the type of certificates which they aim to obtain. The idea of comprehensive schools represents a different paradigm of education than the division in three different secondary schools but has not been successful in competition with the other types.

This system in combination with the federal structure of Germany leads to an extremely diversified curricular situation. Every state has its own curriculum for its primary and for every of the different secondary schools, which amounts to more than 100 mathematics curricula

[1] The authors of this paper were members of the curriculum-commission.

in Germany. In this situation national standards appear to provide a new unifying approach. But since federal states' curricula are not dispensable within the federal structure a curricular standardisation can only be obtained by coordinating these curricula.

Certificates have been given to students mostly on the basis of their grades which are obtained by comparing students' performances within a class and – in some of the federal states – on the basis of their performance in central examinations. There has been no frame for nationally derived norms and criteria as a basis for certificates. In this respect standards are connected with the hope of increasing the objectivity and equivalence of certificates within and between the federal states.

Quite different from many other nations such as England or the US, in Germany the impact of recent international and national large-scale studies, especially that of TIMSS (cf. Baumert et al. 1997) and PISA (cf. Baumert et al. 2001, Prenzel et al. 2004) was considerable. It has set in motion a development whose effects cannot be foreseen clearly yet. Part of this process, seen from the national perspective, is described in the article by Blum in this issue. According to the strictly federal structure of the educational system in Germany the 16 federal states are responsible for any legal or curricular changes with regard to the school system. Hence a complex process unfolded simultaneously at different levels:

- The governments of each of the federal states tried to find their individual way to deal with the situation politically and announced reforms of quite different quality and range.
- The federal states tried to reach a common perspective in their common standing conference for education and cultural affairs ("Kultusministerkonferenz" – KMK).
- The federal ministry for education and research (BMBF) – having only few constitutional responsibilities in that matter – tried to give recommendations and gain influence in the process.

The reform ideas which the federal states put forward after PISA 2000 went into several directions. Often they tried to base their argumentation on PISA results, although this reasoning proved unsustainable more often than not (cf. Prenzel & Drechsel 2004). The options for reform roughly moved within the following frame:

i) Generally increasing the investment in the educational system
ii) Increasing the quality by intensive teacher training
iii) Reorganising the school system (cf. section 2 of this article)
iv) Revising the curriculum
v) Introducing increased accountability by new ways of evaluation

While approach i) and ii) seemed economically unfeasible, approach iii) was politically problematic: Although examples of comparable OECD countries showed that a system with only one single type of secondary school is capable of producing good results, politicians in most of the federal states were reluctant to discuss a reform of the system, since they did not see wide support in society for such ideas. Regarding approach iv) it was generally acknowledged that a mere change in curriculum could not effectively lead to any changes - this is evident from preceding experience and from curriculum research (cf. Künzli & Hopmann 1998).

Thus the line of reform that resulted in most federal states can be characterized as follows:

- Reduce legal regulations and give schools more freedom to work out their professional and organizational development.
- Increase the amount of accountability of schools by introducing different forms of external evaluation, especially by redefining the role of school inspectorates and by implementing and testing certain standards.

This shift of paradigm towards "freedom and accountability" can be identified with the aforementioned approaches iv) and v) ("Defining and testing standards") and affects school in all areas.

What these standards should look like and how their attainment should be evaluated was still unclear, when some of the federal states' decisions were made to commence the shift towards outcome orientation.

## 3. What kind of standards? – Implicit decisions in a federal system

How should reasonable standards for usage in schools look like? In what way should they be used? Who should define them? In 2001 Baden-Wuerttemberg and North Rhine-Westphalia began as the first two federal states to develop a concept for standards rather independently (for a brief comparison cf. Barzel, Hußmann & Leuders 2004). Although the tendency to create standard oriented curricula seems similar in the federal states at first sight, the pace and the actual realisation turns out to be quite different. There are considerable differences in curricula and evaluation instruments, which cannot be elaborated in depth in this article. For example Baden-Wuerttemberg has developed quite independent standards for the different school types whereas North Rhine-Westfalia tried to consort this process by having one commission for all school types of the lower secondary level. At this moment the development is still unfolding in the other federal states.

In view of the imminent divergence of standard development it appears politically sensible and economically necessary for such a reform process to be coordinated centrally. Thus on a national level the aforementioned institutions (KMK, BMBF) commissioned an expert group to write a counselling paper, an "expertise" that described how standards could be developed and used (Klieme et al. 2003). Simultaneously – expecting the results of this expertise – a group of representatives of all federal states began to write national educational standards for the end of lower secondary education (cf. the article by Blum in this issue).

How could these processes on the national level be reconciled with the already evolving process of developing standards in some of the federal states? Due to the asynchronicity there was no simple deduction of concepts: Neither did the national educational standards follow the *expertise* in all aspects nor were the national educational standards the reference frame for developing the standards of the federal states. Eventually agreement and coherence was reached by mutual inspiration in an informal exchange between the commissions working on the different levels.

Still some incongruence with respect to intention and terminology remains. We will briefly address these aspects of incongruence, since they are important for understanding the specific structure of the federal states curriculum.

*Incongruence (i) "output vs. input":*
The aforementioned counselling paper (Klieme et al. 2003) pointed out that there are several ways of defining standards (loc. cit. p.29). For the sake of briefness we want to mention only three:

- *Content standards* define what should be taught in terms of concepts and methods of the subject.
- *Opportunity to learn standards* specify appropriate learning situations (like e.g. part of the NCTM Standards, cf. NCTM 2000).
- *Outcome standards* describe what students should know and be able to do at the end of a certain period of schooling.

Traditional German curricula mainly consist of *content standards* and advice for organising teaching and assessment (*teaching* and *assessment standards*). Of these texts only the chronological distribution of content is actually regarded important by teachers. The recommendations of the counselling paper (Klieme et al. 2003, p.81ff) instead put forward a new curricular concept - one may say a "new curricular paradigm":

- On a national level *outcome standards* should define the goals of education in terms of cognitive competencies at the end of grade 10.
- On the level of the sixteen federal states so called *core curricula* should describe the content and its organisation in lesson sequences.

In short, federal states and single schools should be guided by concise national standards while receiving comparatively large freedom to arrange and create their individual curriculum.

It should be clearly stated that these recommendations were not followed by the first federal states that created core curricular and dubbed them "Bildungspläne" (education plans, Baden-Wuerttemberg) and "Kernlehrpläne" (core curricula, NRW) respectively. Irrespective of these termini both federal states restricted themselves to fixing only *outcome standards*, leaving questions of content and teaching organisation to their schools.

At first glance, this decision seems reasonable regarding the federal structure in Germany: If the federal states want to install a system relying on outcome evaluation as depicted, they have to publish standards that schools can refer to and legally rely on. In the federal structure these texts are represented exclusively by the curricula issued by the federal states governments. At a second glance the federal states could have republished the *national standards* as their federal curricula. This step would have been too progressive though: Firstly the educational policies of the federal states presumably were to divergent and secondly the schools were regarded in need of more detailed standards than only a concise list at the end of 6 years of secondary education.

Hence the development described here has (unintentionally) lead to the situation that the meaning of the term "standards" in Germany almost always reduced to "*outcome standards*". Nearly all newly created curricula tend to follow this specific interpretation of "standard orientation", deleting in their curricula any hint to the learning process.

*Incongruence (ii) "performance vs. education":*
Many curricula in the past also referred to educational goals such as the formation of personality or the capability of working cooperatively. Since outcome standards focus on cognitive and subject-specific competencies that can be operationalised and tested ( cf. Klieme et al. 2003), these more general goals of education such as personal competence or social competence are missing or marginalised in new "standard-oriented" curricula - with the use of the term "standard" in the outcome sense as explained above. Though being called "education standards" they actually should be called "performance standards".

This shift of perspective may also be ascribed to the idea of literacy which is underlying the standards and which is influenced by the PISA-framework (cf. OECD 1999). Student's capabilities are seen here from the perspective of later life performance in a free market economy. For German purposes this concept of "mathematical literacy" is a hitherto neglected perspective. Nevertheless it needs to be complemented by the more general goals of personal and social formation and of enculturation. These are part of the more general concept of "mathematical general education" (as a product not as a process) which is aptly described by Winter (1995) and has created a wide consensus in German mathematics and mathematics education. According to this concept mathematics education has to provide three *fundamental experiences*:

(1) Mathematics as a way to perceive and understand the phenomena in our surrounding world
(2) Mathematics representing and structuring a world of mental objects of its own kind and
(3) Mathematics as a realm of acquiring and exercising heuristic abilities

For some educators these aspects still neglect more general educational goals of mathematics as a school subject, such as enculturation, critical thinking, responsibility, cooperation etc. (cf. Heymann 1996). Clearly these more general aspects of mathematics education can hardly be represented in performance standards.

*Incongruence (iii) "minimum vs. average":*
Not following the recommendation to create standards by describing *minimum expectations* the national standards and federal states curricula have decided to focus on *average expectations* that describe the competencies of a hypothetical average student. This often is criticised since it considerably changes the character of the standards as an instrument of steering the educational system. The formulation of minimum expectations in mathematics could have been used as a tool to identify and support students with serious problems. The formulation of average expectations on the other hand could result in perpetuating a problematic trait of German mathematics teaching, namely the lack of differentiation and support. This may be remedied when, based on the standards, valid empirical instruments will be developed that help to differentiate between performance levels.

Further aspects of incongruence between national and federal states in the process of standard development with regard to specific competence models for mathematics will be discussed later. In the following section we describe the development of standards in NRW, discuss its benefits but also its limitations and drawbacks and thus hope to give recommendations for future standard-oriented curricula.

## 4. The mathematics "core curriculum" in NRW

The term "core curriculum" (Kernlehrplan) hints to a point of view that is older and somewhat different from the "standard movement" depicted earlier. Especially for primary education there have already been demands for a reduction of content (cf. Böttcher 2000) and a concentration on central goals. This was also the original idea when in NRW the ministry of education launched the development of core curricula in the three main subjects: German as mother tongue, English as first foreign language and mathematics. This simple curricular model in view was still content oriented ("what should students learn?") and only gradually changed its direction to outcome orientation ("what competencies are students expected to have?").

In this first phase of development some central requirements for a new standard-oriented curriculum emerged. The core curriculum should

(i) identify and specify the expectations towards students at the end of grades 6, 8, 10,
(ii) rely on a concept of "mathematical competence" that is in congruence with recent didactical insights,
(iii) be concrete and comprehensible for teachers, parents (and if possible even for students),
(iv) restrict itself to a core and create freedom for teachers in choosing content and teaching methods,
(v) give a coherent picture of mathematics as a subject,
(vi) give a coherent picture of mathematical literacy throughout the school types,
(vii) set up the stage for a reasonable system of standard-based school evaluation.

Although these requirements indicate a considerable reorientation towards a genuine concept of "standards" the term "core curriculum" remained, even though no recommendations are given as to a chronological sequence ("curriculum") of content or to arranging learning environments that enable students to acquire the expected competencies.

In the following we discuss in detail how the assigned commission, of which the authors of this article were members, tried to meet these requirements.

### *4.1 Requirement (i): The core curriculum should identify and specify the expectations towards students at the end of grade 6, 8, 10.*

Following a consequent definition of outcome standards a description of expectations at the end of grade 10 would be sufficient for giving an orientation for schools. This would indeed be a giant leap forward from a detailed specification of content in units of several weeks of teaching periods as it was common before. The national standards actually do take this perspective: they only specify the expectations at the end of compulsory secondary education ("Mittlerer Bildungsabschluss"). These expectations are intended to be the common goal for all students in all federal states and in all secondary schools.

Nevertheless the core curriculum in NRW was meant to be a tool for the use in schools for planning and arranging teaching sequences and so it was regarded necessary to give a more specific picture of what is expected from students. Using a coherent system of competence areas for all schools and grades the core curriculum thus describes a progression in two-year-steps from grade 5 to grade 10. This is illustrated by the example from the competence area "reasoning".

|  *Reasoning* | |
|---|---|
| End of grade 6 | Students use different intuitive kinds of reasoning (describe observations, use plausible considerations, give examples and counter-examples) |
| End of grade 8 | Students use mathematical knowledge for reasoning, also in argumentations with several steps |
| End of grade 10 | Students combine mathematical knowledge for reasoning and proving and use formal and symbolic elements and procedures |

Fig. 2: Competence area "reasoning"

It has to be conceded that the steps chosen for the curriculum cannot in general be justified by psychological findings. They represent a more or less arbitrary but pedagogically plausible model for increasing expectations. This increase is described differently throughout the competence areas, e.g. in terms of additional content or strategies, of higher level of abstraction or reflection or of increased use of terminology.

### *4.2 Requirement (ii): The core curriculum relies on a concept of "mathematical competence" that is in congruence with recent didactical insights*

The concept of "competence" is central to all new curricula. It was used by Chomsky (1965) as a response to Skinner´s behaviourism and points to the tension between the cognitive abilities and their execution, between competence and performance. In education it was first used in vocational training since there the applicability of knowledge was a central goal (cf. Klieme et al. 2003, p. 17). Later the concept of competence was used and adapted by literacy-based large scale assessment studies which lead to the abundant use of the term in educational policies nowadays. This popularity presumably stems from the fact that it describes precisely the central aim of education: To enable the individual to cope with the demands of life. You can find subtle but decisive nuances in what *competence* means (cf. Klieme, p.66ff) but in the context of the discussion about educational standards a definition from pedagogical psychology is widely adapted:

...competencies are cognitive abilities and skills possessed by or able to be learned by individuals that enable them to solve particular problems, as well as the motivational, volitional and social readiness and capacity to utilise the solutions successfully and responsibly in variable situations.“ (Weinert 2001, p.27f)

In this interpretation the term “competence” must be distinguished from other connotations such as “social competence” or “personal competence”, where it is more or less understood as performance disposition in specific domains. Klieme et al. (2003, p.67) suggest (corresponding to Weinert 2001) the following criteria to determine individual degrees of competence: ability, knowledge, understanding, skill, action, experience and motivation.

What implications does this have for a mathematics curriculum? First of all it puts the emphasis on the questions: “What is the *use* of certain skills and pieces of knowledge? In what way should they be *applied*?” These were the key questions when the curriculum commission tried to formulate specific competencies. Revisiting the traditional curriculum with such a perspective made evident that a large part of the traditional content had been taken for granted and not reflected in terms of functionality. The key question for writing down competencies thus became: “What for?”

This criterion promoted the deletion of many pieces of content from the former curriculum, e.g. excessive algebraic work with power laws. Moreover this perspective lead to a clarification of connections between pieces of mathematical content and mathematical activities, e.g.:

“Students interpret parameters from linear, quadratic and exponential functions in their graphic representation and use this in applications.” or

“Students combine expressions by expanding and factorising with a simple factor; they use the binomial formulas as a strategy of counting.”

Keywords such as “use”, “apply”, “solve”, “handle” emerged helpful in this context, because they point to the *application* of knowledge and skills and thus to a “functional” perspective in the formulation of standards.

The role of counselling the commission was also to make sure that the description of competencies takes into account current didactical insights. We cannot mention all aspects in detail but instead want to give a few examples:

“Students represent simple fractions in different ways: by acting in situations, by drawing pictures, by using number symbols and locating them on the number line; they interpret fractions as measures, operators and ratios and use the principle of cancellation and expansion as a refining and coarsening of a partition.”

This competency formulation emphasizes the active use of fundamental mathematical concepts and their different aspects (“Grundvorstellungen”, cf. e.g. vom Hofe 1995) instead of just executing procedures.

“Students solve exponential equations of the type $b^x=c$ approximately by trial and use logarithms on a calculator as inverting operations for exponentiation”

This competency formulation reduces the role of logarithms from (the formerly emphasized) symbolic manipulation to their use as a tool for solving certain equations (if only approximately).

“Students explain the steps of mathematical procedures (constructions, calculations, algorithms) in their own words and use adequate mathematical terms.”

Here the complementary function of everyday language and technical language is emphasised.

Concluding one can say that the “competence perspective” leads to a fruitful re-evaluation of curricular content. This experience is consistent with that of other countries.

### *4.3 Requirement (iii): The core curriculum should be concrete and comprehensible for teachers, parents (and if possible even for students),*

It can be considered as a hermeneutic challenge to formulate competencies in a way that they can be understood and used in everyday teaching contexts. On one hand they have to be kept simple and concrete, on the other hand they have to be sufficiently abstract to encompass all aspects of a complex expectation. This is a difficult task and some formulations in standard-oriented curricula do not meet this demand successfully, which can be seen by the following examples[2]:

“Students understand the concept of probability“
“Students grasp ordered situations and structures“
“Students connect mathematics as a mental construct with the perceptible or symbolic reality by mathematical modelling“

Phrases like these are difficult to interpret by teachers and do not fulfil the requirement of being precise and being “described in such specific terms that they can be translated into particular tasks and, in principle, assessed by tests.” (Klieme et al. 2003, p.15) This recommendation must not be mistaken as an invitation to reduce expectations to simple goals that can be reached within a single lesson, which indeed would not describe a competence.

Many competence formulations in the NRW core curriculum remain to be judged critically, since they do not clearly represent a “competence view”:

“Students solve simple quadratic equations.”
“Students find the divisors and multiples of integer numbers and apply divisibility rules for 2, 3, 5 and 10”

Other competence formulations do indeed reflect the idea of functional literacy:

„Students find – in simple problem situations – mathematical questions.”
„Students use elementary mathematical rules and procedures (measuring, calculating, reasoning) to solve easily comprehensible everyday problems.”

[2] cf. http://www.bildungsstandards-bw.de/, choose “Realschule”

Here one can see another limitation of describing competencies. Any such formulation can only describe the *type* of expectation but not its actual *level* - let alone a *precise* and *measurable* level. This can only be done by a group of tasks or situations and a description of the accepted solutions. Often, when the curriculum needs to differentiate between levels of performance to account for differences between school types or to describe a progression throughout the grades, it has to rely on relatively imprecise attributes such as "simple", "medium", "complex" and so on.

Another important aspect concerning the language used in standards is the polarity between being comprehensible in terms of an established practice and the need to introduce innovative ideas. This holds for example for aspects of the curriculum such as "*validating a model*" or "*using heuristic strategies*". Here the policy was to avoid an abundance of new terms and only introduce the terms that are considered central for a necessary change in practice.

As a last aspect we would like to stress the intention of the curriculum to be understood by everybody involved in school-based education - not only teachers but parents and students as well. On the level of secondary education it would be a benefit if central goals were transparent and could serve as a basis to speak and reflect about what is going on in mathematics lessons. In fact this aim has not been reached yet. Still many pieces of didactic language are left which are hard to understand by everybody.

Mathematical standards that consistently describe the goals of mathematics learning as functional mathematical literacy are still an ideal and remain a challenge for mathematics education. This ideal seems more easily reachable in other subjects such as foreign languages, where the central aim is much clearer, namely coping with communication situations.

### *4.4 Requirement (iv): The core curriculum should restrict itself to a core and create freedom for teachers in choosing content and teaching methods*

The view on the role of curricula that has been dominant in Germany was that they should prescribe all content requirements in detail and a teaching process that is regarded as adequate. The result was that teachers often felt under pressure to include all the compulsory content in their lessons. Hence, defining a curricular core should be seen as a chance for concentrating on central aspects and leaving freedom to choose from and to focus on. These ideas are by no means new. They can be retraced to historical examples as the "Meraner Reform", in which Felix Klein in 1905 put down the compulsory content to be taught at school for secondary I and II on three pages (cf. Gutzmer 1908).

When the commission first tried to reduce content the first impression was that the former curriculum was a tightly woven system in which almost everything seemed to be necessary for something else. The fear of cutting important things away impeded the work of the commission and a lot of discussion was needed to see that many things that seemed indispensable were so only in theory. After further consideration it became clear that certain concepts or procedures were only in the curriculum for reasons of completeness or formal rigidity (e.g. irrational exponents). Some were meant as a preparation for later use but in practice they did not prove to be efficiently recalled when needed since they were not dealt with in relevant contexts (e.g. functions of the type $a{\cdot}x^n$ with $n<0$).

After "deleting" more than a quarter of the content that was formerly regarded as mandatory and after publishing a first draft there were many reactions of teachers opposing to cutting away certain topics, since they felt a strong necessity for almost everything. After publication of the standards this tendency was also reflected by many curriculum conferences in schools that had difficulties in perceiving and using the freedom brought by deletion of many topics. This experience seems to paradoxically contradict the content-pressure often criticised by the same teachers and indicates that there is still a long way to go. Even the published core-curriculum is still containing many aspects and details that could be left out in favour of a few central ideas.

### *4.5 Requirement (v): The core curriculum should give a coherent picture of mathematics as a subject.*

Mathematics is a product and a process as well. According to that dualism learning mathematics should not be reduced to acquiring knowledge as a kind of inventory for later life but should always reflect on the opportunities to experience how this knowledge is actually being generated and applied. So the aim is to focus on mathematical content and on mathematical processes at the same time. The dialectic relation between mathematical content and mathematical processes should be reflected in a curriculum for three mutually related reasons:

- The aspects of product and process are central traits of mathematics as a discipline and hence should also underlie the school subject.
- They are also a central trait of *learning* mathematics, since mathematical concepts emerge from doing mathematics in certain situations (cf. the concept of *realistic math education* in the article by Panhuizen & Wijers in this issue).
- They also give a perspective for describing and measuring performance: every competence relies on certain content knowledge and refers to a certain way of dealing with it.

Tall & Gray (1994) also refer to this dualism. They call the use of a symbol to evoke either a process or a concept a "procept":

> "The ambiguity of notation allows the successful thinker the flexibility in thought to move between the process to carry out a mathematical task and the concept to be mentally manipulated as part of a wider mental schema. We hypothesise that the successful mathematical thinker uses a mental structure which is an amalgam of process and concept which we call a procept" (loc. cit. p 115)

To account for this dualism and to integrate both aspects the core curriculum is structured in eight "competence areas" whereof four are content-related and four are process-related.

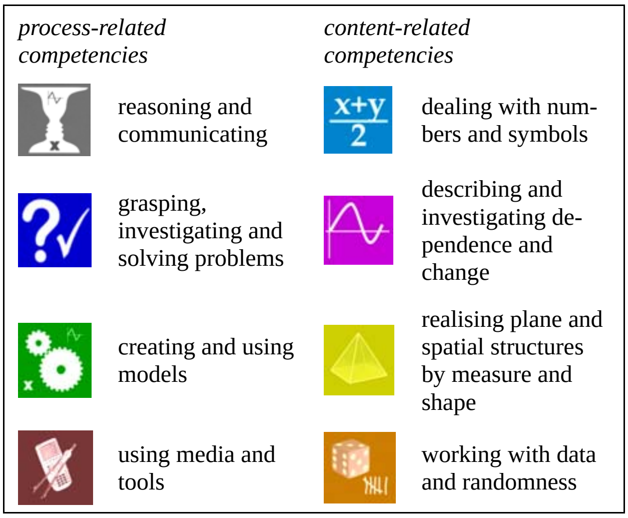


Fig. 3: The competence areas of the NRW-core-curriculum

One has to concede that this categorisation is a plausible but contingent one, many others are possible. One of the main considerations deals with the inseparability of content and process. This becomes clearest on the level of *indicators* that specify what students are expected to do. Most of the text of the central part of the curriculum consists of such indicators, many were cited above ("Students ...."). Indicators serve as a way of roughly describing competencies.

„Students regard mathematical models (especially linear and exponential functions) and find situations in reality to which the model is applicable"
„Students apply linear, quadratic and exponential functions to solve mathematical and realistic problems"

The first competence can be found under the process-related "finding and using models" and the second under the content related "describing and investigating dependence and change". The formulation of the two competences clearly shows the interrelation between content and process:

- A mathematical process has to rely on content.
- If a piece of content is presented as a part of a competence description, it must be specified in which way students deal with this content, and thus a process is involved.
- Students' activities can be seen complementarily from the content and from the process perspective. Either of the two aspects can be emphasized when it is necessary to explain a certain competency.

So one could ask whether it would be more adequate to take this relation into account by defining standards in a two dimensional way. Thus any competence can be seen in its relation to a content aspect and to a process aspect (or several of them) at the same time. This construction was indeed discussed but abandoned, since it would make the presentation of standards much too complicated to the reader.

Instead it was decided to juxtapose the process and the context aspect and thus deliberately give the same weight to both of them. It has to be emphasized that this is a major innovation in German curricular tradition. Aspects of "doing mathematics" that have hitherto been moved to the abstract framework texts of the curriculum now take a central position. Furthermore they are described with (almost) the same degree of detail and concreteness.
The dual content/process-structure of the core curriculum can thus be seen as an analytic approach and conveys the

| NRW core curriculum (2004) | National educational standards (2003) | NCTM Standards (2000) |
|---|---|---|
| **Process-related standards** | **General competencies** | **Process standards** |
| Reasoning and communicating | Mathematical reasoning | Reasoning and proof |
| | Communicating | Communication |
| Problem solving (investigating and solving problems) | Mathematical problem solving | Problem solving |
| Modelling (finding and using models) | Mathematical modelling | Representation, Connections |
| | Using mathematical representations | |
| | Dealing with symbolic, formal and technical elements of mathematics | |
| Using media and tools | | |
| **Content-related standards** | **Competencies related to overarching ideas** | **Content standards** |
| Arithmetic/Algebra (dealing with numbers and symbols) | Number | Number & Operations |
| | | Algebra |
| Functions (describing and investigating dependence and change) | Functional connection | |
| Geometry (grasping plane and spatial structures by measure and shape) | Space and Form | Geometry |
| | Measurement | Measurement |
| Stochastic (working with data and chance) | Data and Chance | Data analysis and probability |

Fig. 4: Comparison of competence areas in NRW core curriculum, German national educational standards and NTCM Standards

intention (and hope) that the process-characteristic of learning and doing mathematics is carried into the classroom, either directly by the text or via teacher training.

Another way to stress the process aspect is that although traditional content categories are chosen (arithmetic/algebra, functions, geometry, probability) those areas are described by processes that indicate competence. So instead of "knowing about statistics and probability" teachers are supposed to focus on "working with data and randomness". Even subcategories like "presenting", "analyzing", "judging" that are introduced to further articulate the competence areas refer to processes.

The plausibility of this analytic model of competence areas also relies on many years of experience that American authors collected in a long-term process of formulating the NCTM-Standards (NCTM 2000). The competence areas used in those standards were an inspiration, since they show a coherent picture of mathematical concepts and processes from pre-kindergarten to college mathematics. Still the categories used in the core curriculum have been slightly adapted to the German perspective as fig. 4 shows.

In this juxtaposition some discrepancies emerge (even between the NRW core curriculum and the national educational standards), that deserve a brief explanation.

i) "Reasoning" and "Communication" has been subsumed to one area in the NRW core curriculum for mere reasons of conciseness. It would be plausible too, to consider the difference between finding a plausible mathematical argument ("prove") and communicating an idea or an argument to others ("presenting").

ii) Regarding the category "problem solving" the core curriculum concentrates mainly on two aspects: "applying strategies" and "reflecting the results and the process of problem solving". By no means are the strategies introduced in the curriculum text exhaustive nor is there a psychological necessity to distribute them over the grades as it was done here. The idea of choosing such a detailed progression is to define a plausible point in education when students should for example be able to tackle problems by consciously investigating examples. Nothing is said about how problem solving should be taught in class. This example shows that fixing certain expectations in standards does not imply a prescribed way of teaching.

*Problem solving* (at the end of grade 8)

Students

- plan and describe their approach to solve a problem
- use algorithms to solve simple mathematical tasks and evaluate their practicability
- check the possibility of different solutions and approaches in the context of a single problem
- use the problem solving strategies „tracing back to already known cases“ (construction of extra lines, computation of intermediate results), „finding special cases“ und „generalising“
- use different representations (tables, sketches, equations) for solving a problem

Fig. 5: "Problem solving" at the end of grade 8

iii) "Modelling" is a difficult category for several reasons. In the core curriculum it is made explicit as "using mathematics to describe real situations" including the processes of "mathematising", "interpreting", "validating a model" and "realizing a model". This coincides with ideas of Pollak (1979) and Schupp (1988).

The national educational standards on the other hand implicitly refer to the PISA-framework (cf. OECD 1999) where modelling encompasses mathematical activities related to situations both *outside* and *inside* mathematics. This more general concept of problem solving thus also contains "problem solving".

The NCTM standards (NCTM 2000) on the other hand do not differentiate a category for dealing with realistic problems but they refer to it in the categories "connections" and "representation".

The core curriculum tries to avoid this difficulty by focussing on the simple concept of modelling as doing mathematics in real world situations. Still problem solving and modelling stay closely related in teaching practise. They are merely separated in the core curriculum for analytic purposes. It has to be admitted though that the difference between a "model" (e.g. a linear function) and a mere "representation" (the graph, term or table) is not clearly defined in the core curriculum.

iv) A category "connection" has also been discussed for the core curriculum. The main reason why it has not made its way into the final competence area model is that it describes opportunities to learn rather than competencies students are expected to possess at the end of grade 10. Here the distinction between *outcome* and *process* standards emerges quite obviously and can be made explicit by the activity that the NCTM standards regard as a part of "connections":

> "Students understand how mathematical ideas interconnect and build on one another to produce a coherent whole."

This certainly is a perspective for the learning process but not a competence that can be expected after grade 10.

v) "Using media and tools" is a competence area that is rather artificially separated in the core curriculum from the other categories, while in the other standards texts it is integrated as an aspect in every competence category. The decision for this structure was mainly a political one and had the intent to promote the use of computer-based tools by making them compulsory:

> "Students use mathematical tools (spreadsheets, geometry software, function plotters) to investigate and solve problems."

The national standards on the other hand do not go that far since they do not want to prescribe the media development for the 16 federal states:

> "Students use geometry software to construct geometric shapes"

In fact, standards like these are a political compromise and do not reflect the usefulness of media in doing mathematics.

vi) The competence area "functions" receives much attention in German standards. This fact has two reasons that have to be judged differently: Firstly it can be ascribed to a curricular tradition emphasising the manipulation of different types of functions. This has unfortunately often been done on a very algebraic level with the aim of preparing students for calculus. This aspect has been reduced in the new standards. Secondly the core curriculum stresses the necessity to promote functional thinking at a very early stage, long before algebraic expressions for functions are introduced. Thus calculating with percentages, working with scaling procedures or investigating number patterns is regarded already as a part of functional thinking.

Comparing the competence areas in the NRW core curriculum with other standard texts one can conclude that apart from slight differences the consideration of process-related competence areas (sometimes called "general competencies") is a main common trait. Besides that the content-related competence areas tend to be structured by overarching ideas ("fundamental ideas" cf. Schweiger 1992, Heymann 1996, "basic ideas" cf. Atiyah 1977). This concept is supposed to emphasise the fundamentals of the subject and stress the close connection between content areas.

Concluding one can say that although the models show certain differences and even discrepancies the overall impression is that they deliver a coherent picture of the central aspects of mathematics. That can be used as a firm basis for further development. Specifically in the development of the core curriculum didactic support was able to introduce central ideas – old and new – into the text that hopefully contribute to promote innovations in classroom teaching – via teacher training or via new school books.

Still typical questions of teachers show how unfamiliar they are with some concepts: "How can one include problem solving in such detail throughout the grades? Isn't math always problem solving?" and "Modelling is a too complex task for students and should be restricted to university." This is especially revealing since the concepts of problem solving, modelling and reasoning have always been present in curriculum texts, albeit in the preamble and general recommendations. Those parts have (in contrasts to the catalogues of content) never been perceived intensively. So from the point of view of promoting innovation and initiating development one has to conclude that the analytic categorization of content-related and process-related standards has already proved successful. It has focused the activities in teacher training to questions like: "What is modelling? What significance does it have for learning math? How and with which goals can we increase problem solving activities in the classroom?" and so on.

### *4.6 Requirement (vi) The core curriculum should give a coherent picture of mathematical literacy throughout the school types*

One of the main results of PISA 2003 was that at least 25 percent of the 15-year-olds tested did not reach the level that curricular experts believe is necessary for successful completion of lower secondary schooling (cf. Klieme et al. 2003, p.11). Therefore it was recommended as one of the necessary steps to overcome the problem to define a core of competencies which is mandatory for all students. This requires that a curriculum really elaborates the core regarded as central to mathematics in all school types.

This was indeed a difficult step since curriculum development in Germany was a highly differentiated process. The pre-existing curricula for the four school types in NRW had been 2 to 15 years old. They were written independently by school-type-specific commissions and had developed individual traditions and even different mathematical terminology. For the first time in curriculum development the representatives of all four school types sat together in a joint commission. The commission consisted of two teachers and an inspector for each school-type, members of the curriculum institute (Landesinstitut für Schule/Qualitätsagentur, Soest), representatives of the ministry of education and of scientific didactic consultants.

To reach a common product the process was structured like this: First the four school-types agreed on a common competence area model (the process-related competence areas were not accepted in the beginning and had to be exemplified first by the experts). Then every school type filled the model with their expectations – which naturally lead to a very heterogenic collection. The task for the scientific consultants then was:

- to rearrange the expectations to form a coherent picture,
- to eliminate arbitrary differences in content and terminology,
- to modify the formulations with respect to competence orientation (cf. requirement ii) and
- to create a plausible progression throughout the standards for the end of grades 6, 8 and 10.

The result of this work was returned to the commission and discussed in the plenary. Then representatives of every school-type checked and modified the list independently which lead again to a diversification. This cycle of working on the standards was repeated several times until a maximum agreement was reached. The remaining differences amount only to a small percentage, the (somewhat arbitrary) differences between school types could be entirely removed. Only when a school type deliberately includes or excludes aspects there is a difference in the curricula.

This coherence throughout school types caused some irritation when the first draft was published and publicly discussed: How could the expectations be so similar when we see the huge difference in capability between the students in the school types? Here again one has to distinguish between the *type* of expectation and its *level*. The commission tried to clarify this argument by adding tasks that reflect the difference of expectations. We give example from the core curriculum:

*Version 1: Secondary general school (lower level)*

*Game of dice*

Rules of the game: You can throw the dice until a number appears a second time: e.g. 1 – 3 – 4 – 3 – stop! You score as many points as how often you have thrown, in this example four points.

a) Play the game several times and write down the scores.

b) You have thrown the following numbers:
1 – 3 – 4 – 4
2 – 3 – 3
6 – 3 – 4 – 2 – 6
2 – 2
4 – 3 – 1 – 2 – 6 – 5 – 1
How many points did you get on average?

c) Write down a game in which the player wins 4 points (7 points).

d) A game begins with 2 – 4. Write down all possible games in which the player wins 3 points.

e) Why can't a player win 8 points?

Fig. 6: "Game of dice" version 1

*Version 2: Grammar school (higher level)*

*Game of dice*

Rules of the game: You can throw the dice until a number appears a second time: e.g. 1 – 3 – 4 – 3 – stop! You score as many points as how often you have thrown, in this example four points.

a) You have the following result: 2 – 1 – 5. Which numbers would end the game?

b) What is the minimum or maximum result in one game?

c) How many games are possible in which you win 3 points?

d) You want to know how many points you get on average in one game. How do you play? Which strategy would you use?

Fig. 7: "Game of dice" version 2

Unfortunately the commission did – for lack of time – not succeed in creating similar differentiating tasks for all areas. As a long term goal such tasks should be developed and field tested to represent standards with empirically reliable difficulty levels.

### *4.7 Requirement (vii): The core curriculum should set up the stage for a reasonable system of standard-based school evaluation*

A thorough specification of competencies allows a translation into particular tasks for teaching and for testing (cf. requirement ii). In an education system relying on standards and accountability the publishing of standards has to be complemented by an evaluation of the attainment of these standards that is consistent with the expectations.

Evaluating the outcome is not an entirely new task for the German school system, (e.g. in states with central examinations at the end of grammar school). But until now there has never been such a deliberate development

*Remarks on how to use this task*

The task does not focus on the concept of probability (even though basic ideas of probability can come up) but on collecting data, counting systematically and reasoning. Students must have the possibility to really play the game and to write down several results. It depends on the learning group how much time will be spent on every part of the task.

Both versions focus on exactly the same significant competencies from the curriculum:

| | Students |
|---|---|
| reasoning/ communicating | • use different intuitive kinds of reasoning (describing observations and plausible considerations, giving examples and counter-examples)<br>• present ideas and results in short contributions |
| problem solving | • apply the problem solving strategies „finding examples“, „checking by trying “ |
| arithmetic/ algebra | • determine quantities in a systematic way |
| statistics | • collect data and represent them in tally sheets<br>• determine relative frequency, arithmetic average and median |

Fig. 8: Remarks on "Game of dice"

of standards that were intended to serve as an explicit reference point for central examinations. Often it was the examinations that implicitly set the actual standards.

In NRW there were no central examinations or tests that compared students' performance before. Only in the first course of the last three years of upper secondary a central test in mathematics was offered on a voluntary basis. The already mentioned counselling paper gave recommendations as to the quality of an independent evaluation:

"…strongly advises against using standards-based tests for purposes of grading and certification. In our view, the development and implementation of educational standards can serve to improve the quality of individual schools and of the education system as a whole, it may also help to further individual students, but it does NOT serve the purpose of centralised examination." (Klieme et al. 2003, p.76)

NRW tried to follow this recommendation by establishing a centralised test ("Lernstandserhebung") for grade 9 which is compulsory for all students in all secondary schools and which serves as an orientation for teachers (cf. the article by Büchter & Leuders in this issue). To communicate the "new" demands according to the standards it was vital to connect the test items tightly to the formulations in the core curriculum. Moreover the test concentrates, in an annual rotating system, every year on a single process-related competence, the first one being *modelling*. In this way the attention is closely drawn towards the innovative messages of the curriculum.

However, it is not yet clear to what extent future central examinations will use the curriculum as orientation. The danger remains that "feasible" and "legally secure" central examinations will reduce the intentions of the curriculum to a certain collection of skills and pieces of knowledge that can aptly been tested within the limitations of central tests. To avoid this tendency we recommend a wide spectrum of examination forms, including group work, presentations, portfolios etc.

## 5. The future of standards and core curricula: open questions and problems

Publishing sensible standards, measuring students' achievement and giving feedback to teachers and schools are currently regarded as necessary steps to install a so called 'data-driven school improvement' (cf. Weinert 2001). Nobody would raise doubts that this aspect was missing in the German system.

But it is self-evident that these steps are not sufficient to remedy the drawbacks of German mathematics teaching as they have been diagnosed (cf. BLK 1997, Stigler et al. 1999, Borneleit et al. 2001), such as the predominance of low level calculation activities or of teacher direction. These drawbacks will not change by just looking at the outcome. To restructure their teaching practice teachers do not only need standards and tests but further support.

On the contrary, looking at the learning process only from the perspective of performance can result in neglecting the learning process. The standards as they are issued at the moment give almost no advice on how to create adequate and effective learning environments for acquiring the expected competencies. One may argue that this cannot be done effectively by curricular texts anyway. The consequence is that there need to be extensive and intensive teacher training programmes. The key questions for training modules could be:

- how to create learning arrangements in which students can develop self-regulation competencies (learn how to learn),
- how to integrate problem solving activities and develop problem solving competencies,
- how to create authentic modelling activities, e.g. as a part of project work,
- how to systematically include the use of computer-based tools or
- how to work adequately and specifically with mathematical tasks (e.g. open-ended problems for differentiated learning opportunities, competence-oriented tasks for testing).

The list may be continued, the central goal being to support teachers in creating adequate course-programmes in their schools. This activity could for example be organised in development groups doing "lesson study" (Fernandez & Yoshida 2004).

The new autonomy in deciding about school curricula and lesson planning is not only seen as a chance but a lot of teachers regard it as an excessive and unreasonable demand. Hopefully the few remaining resources in teacher time do not flow into formal tasks like doing curricular work but in teacher training.

Finally we must recall that standard-oriented curricula and education standards as the recently developed ones should be regarded only as subject specific "performance standards". They should not make other goals of education seem less important. Standards and tests can only look at a certain, but still important part of the education process.

We conclude with a simple but important caveat from the counselling paper:

> "The innovations must thus be implemented with utmost care to ensure that work done with the best of intentions does not turn out to be counterproductive." (Klieme et al. 2003, p.86)

This danger seems imminent to us unless there are no substantial supporting systems such as a widespread and thorough teacher training. Otherwise the innovative impulse new standards and their assessment can create in schools is not directed in a productive direction or idly disappears.

The innovations at hand that we have described in this article refer to the organisation of the educational system and to the intentions and the structure of the subject mathematics as well. It is a big challenge for all those who are involved in the education system – not only teachers but also school administration, politicians and educational scientists who have to find their role in the process.

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