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Adding It Up: Helping Children Learn Mathematics

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This fine book is a report by the Study Committee on Mathematics Learning, established in 1998 by the National Research Council. The Council is organised by the National Academies of the Sciences and Engineering and the Institute of Medicine in the USA to provide services to government, the public and to scientific and engineering communities. The task of the Committee was to synthesis research on pre-kindergarten to Year 8 mathematics learning, and to provide advice for educators, researchers, publishers, policy makers and parents, including research-based recommendations for teaching, teacher education and curriculum. The committee of 16 eminent people from diverse backgrounds, including several internationally known mathematics educators, psychologists and mathematicians, was chaired by Jeremy Kilpatrick, with input from a "special oversight commission" of ten, a host of reviewers and presentations and commissioned reports. Jane Swafford was the Study Director. The study was funded by the US Department of Education and the National Science Foundation.

There is a saying in English that a camel is a horse designed by a committee. Since this is a book produced by a committee, one immediately wonders whether it will have a clear message and balance competing agendas in an intellectually coherent way. Here it succeeds. The book is characterised by wisdom and a balanced approach to controversial issues. The intention is to select research that is relevant, sound and generalisable (these characteristics are discussed) and to look for multiple lines of research that converge on a particular point. Importantly, the report identifies carefully where the gaps in research findings have been filled by professional judgement and reasoned argument. The writing is impressive because it is persuasive, factual and always clear. The use of endnotes for notes and references assists in making the text very readable.

"Adding It Up" begins with the premise that "all young Americans must learn to think mathematically, and they must think mathematically to learn" (p. 1). It identifies two central problems: achievement and equity. Firstly too few students in elementary and middle schools can successfully use the mathematics they have learned. Secondly, certain segments of the U.S. populations are underrepresented amongst those who do succeed. The major sources of data cited to confirm that these are indeed major problems and to explain the dimensions of these problems are: NAEP, the US national achievement tests and TIMSS, the Third International Mathematics and Science Study, conducted in the mid 1990's, along with a very large number of other studies. The point of view underlying all the recommendations for improvement is that improvement requires a thorough, methodical overhaul which is co-ordinated across the many players (state and federal governments, education authorities (school districts), textbook and test publishers, professional organisations, teachers, parent groups etc): tinkering with one factor at a time will not be satisfactory.

The report is based on an analysis of mathematical proficiency as composed of five strands which are interwoven and interdependent: *conceptual understanding*, *procedural fluency*, *strategic competence*, *adaptive reasoning* (the capacity for logical thought, reflection, explanation and justification) and *productive disposition*. This structure is similar to others which have been used in the USA and elsewhere. For example, it is quite similar to the five components of the Singapore Mathematics Framework of concepts, skills, metacognition, processes (i.e. reasoning) and attitudes. Drawing principally on NAEP data, the report concludes that traditional curricula and teaching methods in the US produce students who are most proficient in procedural fluency and less proficient in the other four strands. It calls for a change in the mindset of many people in the USA who consider procedural fluency to be the heart of the elementary school mathematics curriculum (p 144). A major broad-scale conclusion of the report is that students are unlikely to develop mathematical proficiency in any one strand unless all the strands receive attention. Whilst praising the NCTM's efforts since 1989 in introducing standards for school mathematics, the report finds these fragmented and drawing on limited conceptual frameworks. It proposes that its work, including the five strand definition of mathematical proficiency, can provide a conceptual framework which could be used in developing "a coherent, well-articulated, widely accepted set of learning goals […] that would detail what students *at each grade* should know and be able to do" (p. 36). The specification at each grade level is intended to reduce the width and increase the depth of curriculum studied at any time. This is one example of the many well-argued evaluations of mathematics teaching and proposals for future action in the report. These make the report an important statement for those in a position to influence directions for school mathematics.

Four major chapters in the book survey the curriculum strand of Number. The first of these chapters sets out to provide a "panoramic view of the territory on which the numerical part of the school curriculum is built" (p. 71). Ideas such as the commutative properties of multiplication and addition, the meanings of operations, the reasons why a negative multiplied by a negative is a positive, place value and algorithms are discussed. The second of these chapters, extensively referenced, surveys children's early mathematical knowledge including topics such as learning the names of numbers in English. The third chapter deals principally with the development and teaching of whole number single digit arithmetic and multi-digit algorithms, and the fourth extends this work to rational numbers and decimals, proportional reasoning

and integers, through an integrated discussion of mathematical content and research on teaching and learning. A later chapter in this style surveys algebra and, very briefly, measurement, geometric thinking and statistics and probability. These five chapters, constituting about half of the book, provide a summary of research that could be used to inform curriculum development and policy. They are not a statement of what the curriculum might be for children, or indeed for teacher education. At times, I was unsure of the intended audience since the material is in places very detailed, yet would be familiar to an expert in the field. The overall evaluations are, however, still thought provoking.

Chapter 9 describes teaching for mathematical proficiency and is a particularly strong chapter. It views teaching as a product of interactions between the teacher, the students and mathematical content in a context and illustrates these with some classroom vignettes. The research findings are organised around " teachers and content", "teachers and students" and "students and content". The last section, for example, covers topics including the role of practice, homework, manipulatives and calculators. A set of suggestions for needed research concludes the chapter.

Chapter 10 moves on to developing proficiency in teaching mathematics. This is analysed in terms of teachers' knowledge of mathematics, knowledge of students and knowledge of instructional practices. The chapter also proposes an extension of the five strands of mathematical proficiency to describing components of mathematical teaching proficiency. My impression is that this is not a useful move, as it blurs the meanings of the phrases which have been carefully established in earlier chapters and creates a false impression that there is a close parallel between the components of ability to do mathematics and the components of the ability to teach it. For example, the report labels as fluency firstly ability in carrying out mathematical procedures, and then in this chapter ability to conduct basic instructional routines, such as dealing with collecting homework. In my opinion, these abilities are of quite a different nature and it does not help to confound them. Otherwise, the chapter continues in the pattern of making sensible recommendations based on careful review of the research literature in controversial and uncontroversial areas. Amongst other things, it recommends that mathematics specialists be available in all elementary schools and that programs of professional development should usually be sustained and multi-year with teachers given substantial financial and other support. The recommendations from all chapters are usefully collected together in the final chapter.

Early in the book, the committee justifies its decision to concentrate the book on the teaching of Number. The justification is in terms of the centrality of Number in the mathematics curriculum, the fact that it is the most controversial area and the abundance of good research on it. The report also identifies the possible negative consequence of this decision: that it may be taken to imply that Number should dominate the mathematics curriculum in K to 8 grades. My own preference would be that they had not made this decision. I would have liked to see their attempt at advising us on the development of a curriculum which is really balanced across areas. Moreover, the approach to Number that the book conveys, along with much of the US research which is reviewed, is a very "naked number" version of a Number curriculum, very much focussed on within-school algorithmic mathematics. It does not draw into the classroom the everyday situations which can enrich mathematics teaching, but instead seems implicitly to focus on instruction based around a textbook. A good deal of attention is given to the potential of the study of multi-digit algorithms to strengthen all strands of mathematical proficiency, whereas little attention is given to the potential of more open mathematical problem solving based on everyday or mathematical situations to contribute. Perhaps a future report with the same good qualities as this one will help us explore a different kind of mathematics classroom that puts applications of mathematics and links to the world around the students on an equal footing with abstract investigations; investigations that remain abstract even with manipulatives.

There is a great deal in this book that will be useful to practitioners involved in all parts of the mathematics education enterprise. I thoroughly recommend it, as a useful compendium of research findings and for the wisdom and clarity of its recommendations. One does not have to agree with all the recommendations to benefit from seeing the presentation of the evidence and arguments in support of them. The report is available as a real book and also on-line at the National Academy Press website for reading with convenient searching.

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