

# Constrained Production Schedule Optimization of Output-Normalized Expenditures under Uncertainty in Shift Duration and Energy Price Forecasts

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**Abstract:** Variability in day-ahead energy prices offers the potential for reductions in energy expenditures through flexible load planning in which production is shifted to periods of low predicted energy prices. In this paper we present a means of optimizing a production run for the following day and week given basic constraints on the start and end times of production shifts as well as the total production within a specified time frame (planning horizon). The general problem is a difficult, non-convex optimization problem over stochastic variables that can only be solved in its original form by global optimization and Monte Carlo simulation techniques. However, reasonable approximations to the original problems render it more tractable. By linearizing the energy price forecasts and the plant consumption profile, as well as discretizing the stochastic variables involved, we can reduce the planning problem to a mixed integer quadratic programming (MIQP) problem, with the customary linear bounds.

**Keywords:** Industrial load scheduling, variable energy pricing, day-ahead energy price forecasting.

## 1 Introduction

Certain energy intensive industrial processes offer the possibility of shifting production schedules (within set constraints) in order to minimize energy costs by giving precedence to low energy cost regimes. These are periods of low demand coupled with high renewable energy availability, mostly determined by weather conditions, whose forecast is uncertain. The scheduling of production volume for the upcoming day (or week, month, etc.) is further challenged by inherent variability in the actual duration and energy cost per unit produced, due to inhomogeneity in the quality of the raw material or the stochastic nature of basic sub-processes (labor, reaction dynamics, environmental conditions etc.) in a production pipeline. Many processes are discrete and serial in nature: the product is made in discrete batches/units run sequentially, meaning that variances in batch duration accumulate with time.

We present a means of optimizing a production run for the following day and week given basic constraints on the start and end times of production shifts as well as the total production within a specified time frame (horizon). The inputs to the optimization

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algorithm in our example are hourly spot market energy price data and a stochastic dynamic simulation of electric steel production. Based on the results of this analysis and taking into account the aforementioned constraints, daily and weekly batch schedules are generated, which minimize some critical aspect (e.g. expected value) of a probability distribution of the output normalized energy cost (e.g. in megawatt hours per ton of product).

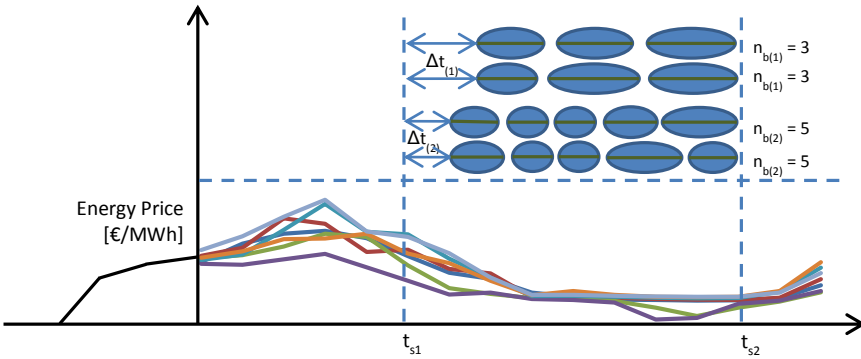


Fig. 1: Batch scheduling utilizing energy price forecast data

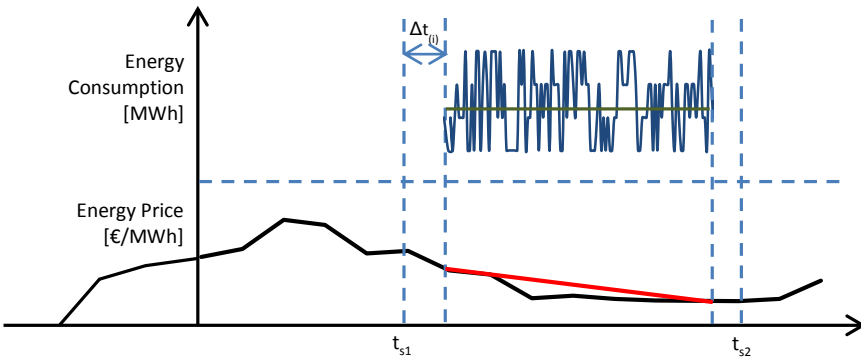


Fig. 2: Linear approximation of stochastic energy demand and spot prices

## 2 Approach and Constraints

The overall goal of the optimization is to minimize the sum of the cost of each shift  $i$  over a specified number of days  $j$  within the planning horizon such that the total amount of material produced within that horizon (e.g. the next week) is reached:

$$\min E \left[ \sum_{k=1}^M \frac{c_s(n_b(k), \Delta t_k)}{n_b(k)} \right] \text{ where } \sum_{k=1}^M n_b(k) = N \quad (1)$$

where  $c_s$  represents the total energy cost of the shift on day  $k$ , and  $n_b$  and  $\Delta t$  are the control parameters representing the number of batches scheduled for the particular shift and the delay before the start of the first batch in the daily production run, respectively. A production run consists of a daily serial run of batches, which are planned some hours in advance to allow for the logistics of personnel and material. To compute the cost of a shift, which is dependent on the number of batches planned during the shift and the time delay before the first batch with respect to the regularly scheduled shift start for that day, the integral of the energy cost components over the length of the shift is calculated:

$$c_s(n_b, \Delta t) \cong \int_{s_1 + \Delta t}^{s_1 + \Delta s(n_b, \Delta t)} (c_b(t) + \rho c_m(t)) dt \quad (2)$$

where  $\Delta s$  is the duration of the production shift<sup>3</sup>,  $c_b$  is the energy cost for one batch and  $c_m$  is the fixed cost of energy. The variables  $\Delta s$  and  $c_s$  are stochastic variables. For example, the cost at time  $t$  is equal to the base price plus paid by the consumer plus a fraction of the market price at that point in time (assuming a price structure that reflects a long-term stable price plus a current market reflecting component). Note that the fixed cost (as opposed to the variable cost) can be subtracted out for optimization purposes with no meaningful effect.

### 3 Optimization using Quadratic Mixed Integer Approximation

The problem described in the previous section is an optimization of scalars (the delays of the actual production runs each day) and integers (the number of batches per day) and is subject to constraints both in scalars (the end of the planned production run must arrive within the shift end) and in integers (the number of batches within the programming horizon must be preset and greater than or equal to one). We now present a means of simplifying these computations and minimizing the complexity of dynamic simulations while improving estimates of cost savings, and discuss how this basic framework is extensible to more complex situations (adaptive optimization, soft boundaries on total amount produced within the forecast horizon, etc.)

In reality, energy spot market prices vary nonlinearly with time and industrial processes may exhibit highly stochastic energy consumption. Through the linearization of energy price forecasts and the plant consumption profile, the integral of which is supra-quadratic

<sup>3</sup> Production shift length,  $\Delta s = n_b \cdot \Delta t_b$

within the integration limits of the optimization, as well as discretization of the stochastic variables involved, we reduce the planning problem to a MIQP approximation, bound by the duration of the production run. This approximation allows a more computationally efficient optimization of energy costs in comparison with non-linear methods. For a single forecast  $i$ :

$$c_{s_i}(n_b, \Delta t) \cong \int_{t_{s_1} + \Delta t}^{t_{s_1} + \Delta t + \Delta s(n_b, \Delta t)} (a_{(i)} + b_{(i)}t) dt \quad (3)$$

$$E(\Delta s) \cong \sum_j P_{(j)} \Delta s_{(j)} = \sum_j P_{(j)} \Delta t_{b(j)} n_b \quad (4)$$

$$c_{s_i}(n_b, \Delta t) \cong \mathbb{P}_2(a_{(i)}, b_{(i)}, \Delta t, \Delta s) \quad (5)$$

Note that  $\Delta t_b$  is a stochastic variable denoting the duration of one batch; whereas,  $\Delta t$  is a deterministic design variable signifying the delay of a batch run relative the start of a shift.  $P_{(j)}$  is a scalar which signifies the discrete probability of an event  $j$  whereas  $\mathbb{P}_2$  signifies a second order polynomial: the coefficients are not given here for compactness but are easily derived –  $n$ -degree polynomials are closed under summation. Utilizing linear cost approximations and discretizing  $\Delta t_b$ , the expected cost over all likely cost trajectories  $i$  and production run lengths  $j$  is calculated.

$$E(c_s) \cong \sum_i P_{(i)} c_{s_{(i)}} \quad (6)$$

$$E(c_s) \cong \sum_i \sum_j \mathbb{P}_{2(i,j)}(\Delta t_{(i)}, (n_b \cdot \Delta t_b)_{(j)}) = \mathbb{P}_2(\Delta t_{b(j)}, a_{(i)}, b_{(i)}, \Delta t, n_b) \quad (7)$$

The constraints, apart from Eqn. 1 (linear equality constraint) are linear inequality constraints, in that the numbers of batches per day are zero or positive integers and that the end of the expected planned production run arrives within the end of the shift minus a preset safety factor. The predicted expenditures are integrations of strictly positive pricing functions and are therefore always positive; however, the method presented here does not guarantee an absolute minimum solution. In this case other general methods for global optimization can be utilized to select the greatest probabilistic minimum expenditure for each shift.

## 4 Conclusion and Future Work

We have presented a method for approximating a difficult, NP-hard industrial planning problem – time constrained, sequential production batch planning under uncertain (stochastic) energy costs and batch duration, such that tractable, efficient and known classes of algorithms (constrained MIQP) can be applied to unit energy cost optimization. The trade-off between the computational efficiency afforded by this approximation and the quality of the optimized solution will be quantified in future work, in which we shall compare it with a Monte-Carlo based optimization over nonlinear forecasts and dynamic simulations of the industrial process (based on our previous work), and will present preliminary results in that regard.

If price forecasts are non-linear, it follows that their integral (the batch energy expenditure) will be supra-quadratic in terms of the integration limits, which are in effect the variables for which we optimize, namely the onset and duration of the production run. Although the expenditures remain positive definite (being an integrals of strictly positive functions, namely prices), there are no guarantees of a unique local minimum, and in such case only general methods for global optimization can be used as in the relevant literature (genetic algorithms, simulated annealing, particle swarm, etc.). Over Monte Carlo evaluated cost functions, this can potentially be quite expensive from a computational standpoint. Given that forecasts are themselves uncertain, the effective performance gain of computationally expensive methods may be modest. At any rate the golden standard, against which optimization quality and the methods employed should be judged, is the expenditure, assuming perfect fore-knowledge of energy prices.

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