Coupling Algorithms for Partitioned Multi-Physics Simulations

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Abstract: Partitioned coupling approaches are an important tool in order to achieve a decent time-to-solution for multi-physics problems with more than two physical fields or changing combinations of fields. We study different approaches to deduce coupling schemes for partitioned multi-physics scenarios, by means of a simple, but yet challenging fluid-structure-fluid model problem. To our knowledge, this is the first time that a fully implicit black-box coupling scheme for partitioned multi-physics scenarios is described. This allows the simulation of a new range of applications in a partitioned way.

1 Introduction

The simulation of the interaction between three or more physical fields, so-called multi-physics scenarios, is a crucial part of many engineering or biomedical applications. This includes, for example, fluid-structure-acoustic interaction (e.g. [LKBD09]), where the acoustic field can be seperated in a complex near field and a simplified far field, or simulations of the heart, where electro-magnetism, fluid flow, and structure equations are coupled ([LAVH12]). Also scenarios with multiple flow fields such as partially filled tanks on container ships ([GM08]) belong to this class. If the coupling is restricted to a lower-dimensional manifold in the computational-domain, which is the case in the mentioned examples, we speak of surface-coupled problems. Such applications benefit clearly from a partitioned approach where the physical domain is split into single-physics fields, all fields are simulated with their own solver, and these solvers are coupled via common interfaces. Since the coupling is restricted to a surface (instead of the whole volume of the domain), the amount of data to be transferred between the solvers is comparable to the amount of data transferred in a distributed memory parallelization of the single fields. The splitting into single fields gives us the possibility to reuse existing software which is

crucial to maintain a decent overall time-to-solution. Furthermore, multi-physics scenarios often adhere different time and spatial scales, making different spatial and temporal discretizations of each physical field necessary. However, the splitting into single fields introduces possible stability issues that need to be tackled by sophisticated coupling algorithms. The counterpart of partitioned methods, the monolithic approach tries to set up one large system of equations for all fields, which is solved within one program. This approach implies an unfeasible complexity, if multiple physical fields are considered that all have to be reimplemented for each new combination of fields.

Depending on the type of interaction between the single physical fields, the task of finding a stable and accurate coupling method can be easy or rather difficult. We speak of a uni-directional interaction if the mutual influence between two physical fields P_i and P_j is only one-sided, e.g., P_i influences P_j but not vice versa. Such problems can be handled easily and efficiently even with a file-based one-time data transfer between the single field solvers and is, therefore, not considered in this paper. We focus on bi-directional interactions where both fields have an impact on each other. Depending on the strength of the bi-directional interaction, more or less sophisticated methods are required for the numerical coupling of the fields.

Whereas many coupling algorithms for systems involving two surface-coupled physical fields have been developed in the last decade, there are hardly any general and robust methods available for the coupling of multiple fields. For the two-field coupling which we refer to as bi-coupling schemes in the following, a variety of methods from explicit schemes, executing only a fixed number of single field solves per multi-field time step [FL00], to stable implicit schemes, iterating between the single field solvers until the time step equation converges to the monolithic solution, are known from literature. Explicit coupling schemes are known to yield stable time-stepping only in cases with a relatively weak bi-directional coupling. Implicit schemes are particularly necessary for fluid-structure interaction (FSI) with incompressible fluids, a typical case for a strong bi-directional interaction due to the so-called added mass effect (e.g. [CGN05, Bru09]). Established methods are Aitken underrelaxation ([KW08]), or quasi-Newton methods ([DBV09, VLDV07, Mic04, MY10]). Usually, these methods execute the single field solvers in a staggered way, i.e., one after the other which limits the scalability of such a simulation on massively parallel systems. As massively parallel computations are necessary for multi-physics scenarios, since only a high resolution of all fields allows to take advantage of the more complex modeling of multi-physics scenarios compared to single-physics scenarios, we developed implicit coupling algorithms executing the involved single field solvers in parallel to each other ([UBGM13, MUB$^+$13]).

On the other hand, not many multi-coupling algorithms have been developed. In [SBE$^+$14], a multi-coupling algorithm that needs full Jacobian information (non-black-box) is described and tested for lower dimensional problems. The necessity of full Jacobian information makes it very cumbersome to integrate this algorithm in 2D or 3D scenarios already in the case of non-black-box solvers.

In this work, we develop and discuss multi-coupling algorithms that are derived from either a simple composition of bi-coupling schemes or a generalization of the underlying ideas to a true multi-coupling. To our knowledge, this is the first time, that a fully-implicit black-

box multi-coupling algorithm is described that allows to simulate scenarios in a partitioned way. The results in Section 5 show that the easiest approach, i.e., the composition of bi-coupling schemes, is not sufficient for already moderately difficult problems.

All coupling methods used in Section 5 are implemented in `preCICE`, a coupling library ([Gat14]) developed in our group, which offers, besides different coupling algorithms, also methods for mapping data between non-matching meshes and communication routines. The application interface of `preCICE` is formulated on a very high level, allowing for a minimally invasive inclusion in any single-physics code. Once a solver is adapted to `preCICE`, different other solvers can be coupled in a nearly plug-and-play manner. Different solvers including commercial tools such as `Fluent` or `COMSOL`, open source tools such as `OpenFOAM`, and in-house codes (e.g. [UCG$^+$14]), have been coupled successfully to `preCICE`. `preCICE` itself is open-source software[1]. For sake of brevity, implementation details of `preCICE` are left out in this work. The interested reader is referred to [Gat14].

The remainder of this work is organized as follows: Section 2 introduces a model problem containing three different physical fields. We use this model problem throughout this work to study multi-coupling algorithms. Section 3 discusses multi-coupling algorithms derived from a composition of bi-coupling schemes, whereas Section 4 studies multi-coupling methods based on a generalization of the underlying ideas of selected bi-coupling methods. Section 5 presents numerical results for our model problem and the derived algorithms. Section 6 concludes this work with some preliminary guidelines how to choose the right multi-coupling scheme for a particular scenario.

2 A Fluid-Structure-Fluid Model Problem

To include also commercial tools with a closed application interface, i.e., black-box solvers, we describe a solver P_i simply by an input-output relation

$$P_i : x_{\text{in}} \mapsto x_{\text{out}} .$$

Here, $x_{\text{in}} \in \mathbb{R}^N$ denotes a vector of nodal input values at the discretized coupling surface between P_i and other physical fields. P_i maps this input to output values $x_{\text{out}} \in \mathbb{R}^N$ by calculating one timestep or one iteration. A fluid solver in a so-called Dirichlet-Neumann coupling, for example, gets displacement values from (possible multiple) structure solvers at a common surface, calculates new pressure and velocity values, and finally gives back force values to the structure solvers at the same coupling surface. Note that black-box solvers, in particular, do not offer information concerning the Jacobian of P_i.

In general, we can model surfaced-coupled multi-physics scenarios as directed graphs where each vertex corresponds to a physical solver P_i and each edge from solver P_i to P_j means that an output variable x_k of P_i is an input variable of P_j. For the sake of clarity, we only consider a model problem comprising three solvers, two fluid solvers F_1 and F_2 and one structural solver S solving a scenario with two (possibly different) fluids separated by

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[1] `http://www5.in.tum.de/wiki/index.php/PreCICE_Webpage`

an elastic structure. As the two fluid solvers do not have a direct interaction with each other, this model scenario corresponds to the graph shown in Figure 1.
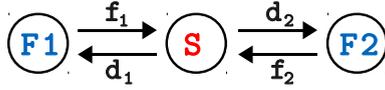

Figure 1: Multi-physics model problem, represented in a dependency graph. Two fluid solvers F_1 and F_2 compute force values f_1 and f_2, acting as input values for the structural solver S, who itself gives back displacement values d_1 and d_2 to the fluid solvers.

This model problem contains already many basic issues of general multi-physics scenarios. Therefore, we use it to study various multi-coupling approaches without the formalism needed to describe general multi-physics problems. We want to stress that all techniques developed in this work can be generalized in a straight-forward way to problems comprising more than three physical solvers. Also the implementation in `preCICE` allows for more complicated scenarios.

The densities ρ_{F1}, ρ_{F2}, and ρ_S in all three physical domains and the geometrical shape of the structural domain determine the strength of the three-field interaction, and, thus, have a crucial impact on the suitability and performance of coupling schemes. We use this fact to study different setups with strong or weak interaction between all three fields as well as strong interaction between two fields and weak interaction with the third field in Section 5. We introduce a variety of coupling approaches and discuss their theoretical applicability in the following two sections.

3   Composition of Bi-Coupling Schemes

This section discusses methods for multi-coupling that can be easily derived (and implemented) as a combination of existing bi-coupling methods. There are basically two ways to combine bi-couplings: concatenation (Section 3.1) and inclusion (Section 3.2).

3.1   Concatenation of Bi-Coupling Schemes

The straightforward idea to couple multiple physical solvers P_i is to use a bi-coupling scheme for each edge in the multi-physics graph as displayed in Figure 2 for our model problem from Section 2. In this setting, each bi-coupling scheme can be adjusted to meet particular needs. In particular, we can choose whether an explicit or an implicit scheme is required for each bi-coupling separately. Figure 3 shows that different choices lead to different execution orders of the physical solvers. All combinations sketched in Figure 3 are supported in `preCICE` (cf. [Gat14]). If two implicit schemes are used, they do, in general, not convergence at the same time. In this case, the early converged coupling scheme waits until convergence of all schemes.

Sophisticated implicit coupling schemes such as Aitken underrelaxation or quasi-Newton methods reuse information from previous iterations to estimate the response of the cou-
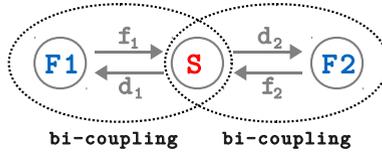
Figure 2: Schematic view of the three-field coupling for the model problem from Section 2 using a concatenation of two bi-coupling methods, one for each edge in the interaction graph.
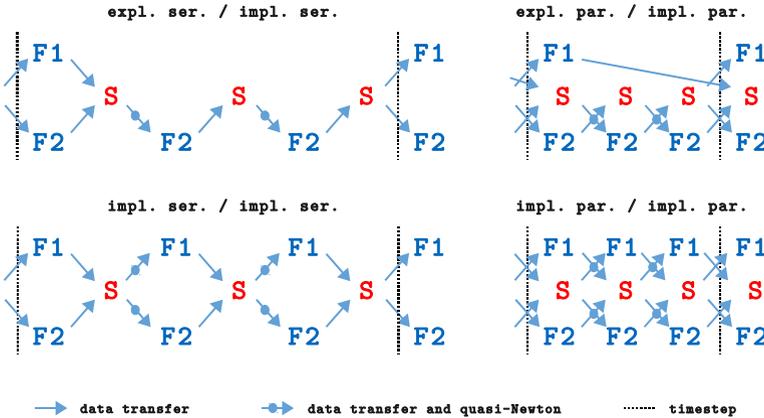

Figure 3: Execution orders for concatenation of different bi-coupling scheme. Whether the coupling scheme is explicit or implicit, serial or parallel, the execution order differs. The selection of sketches is restricted to reasonable and non-trivial combinations. For sake of simplicity, implicit coupling schemes are assumed to converge in 3 iterations.

pling partner, to choose the best coupling parameters. If, in our model problem, both coupling schemes are implicit, they hamper each other, since their adaptive choice of coupling parameters changes the response mechanism of the two coupled solvers seen from the third solver. In mathematical terms, this means that the Jacobian of both two-field subsystems in the three-field system changes substantially throughout the iterations which makes it almost impossible for the third solver to get any useful information on the response of such a subsystem. This is especially a problem if both fluid solvers F_1 and F_2 are indirectly strongly coupled which is the case for a very thin, elastic or lightweight structure. The impact of the described problem, depends, thus, on the geometrical shape and the stiffness of S. Numerical experiments in Section 5 confirm this conclusion. Obviously, this problem reappears in general multi-physics problems, if multiple implicit bi-coupling schemes influence each other. A simple concatenation of bi-coupling schemes does, in general, not result in a stable overall coupling. In Sections 3.2 and 4, we present possible remedies.

3.2   Inclusion of Bi-Coupling Schemes

The alternative to a simple concatenation of bi-coupling schemes is the inclusion of bi-coupling schemes. Here, two physical solvers coupled by a bi-coupling scheme are re-

garded as an entity from the outside and are coupled with the third solver using a bi-coupling approach again. Figure 4 shows a variant of this approach for our model problem: F_1 and S are bi-coupled and the converged entity $(F_1 S)$ is bi-coupled with F_2. Different from the methods in Section 3.1, this implies, that we have induced a nesting of iterations: In the inner iteration, we iterate between F_1 and S until convergence to the two-field monolithic solution. The result is then transferred to F_2 which returns its result to S. This outer coupling is then repeated until convergence to the three-field system. If we assume that M iterations are required for both the inner and the outer iteration, a total of M^2 solves is required for F_1 and S, whereas M solves are needed for F_2. Only if we apply an explicit scheme to the outside coupling, the overall scheme does not differ from a concatenation scheme.
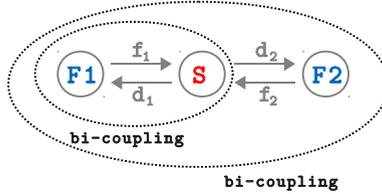

Figure 4: Schematic view of the three-field coupling for the model problem from Section 2 using an inclusion of two bi-coupling methods. Two coupled physical solvers are regarded as an entity from the outside and are coupled with the third solver. If two implicit schemes are applied, this results in two nested iteration loops.

We suppose that the inclusion of two implicit schemes always results in an overall stable coupling if we choose appropriate methods for each of the bi-couplings. The disadvantage of a multiple inclusion are its computational costs: the number of solver executions grows exponentially with the number of physical solvers if we assume that each coupling schemes needs a constant amount of iterations. This is the reason, why we did not implement such schemes in `preCICE`. In the next section, we develop a far more efficient and stable multi-coupling scheme.

4 Multi-Coupling Schemes

In this section, we generalize bi-coupling schemes to proper multi-coupling schemes by directly looking at the coupling of multiple solvers as depicted in Figure 5.

The basis of our coupling approach is the following observation: Each multi-physics problem can be reformulated as a fixed-point equation (FPE). This is widely used in classical partitioned FSI problems, where many coupling algorithms are derived from the FPE

$$(S \circ F)(d) = d . \tag{1}$$

S and F denote the structure and the flow solver, respectively, d is the displacement (or the velocity) of the structure surface. Below, f is the force exerted on the structure surface
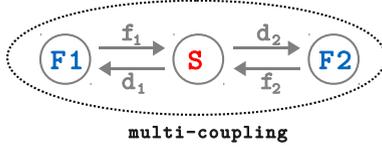
Figure 5: Schematic view of the three-field coupling for the model problem from Section 2 using an multi-coupling approach, i.e. considering all interactions in an overall coupling system.

by the fluid. We showed in [UBGM13, MUB$^+$13], that the same solution is achieved from the alternative FPE in matrix-like notation

$$\begin{pmatrix} 0 & F \\ S & 0 \end{pmatrix} \begin{pmatrix} f \\ d \end{pmatrix} = \begin{pmatrix} f \\ d \end{pmatrix} \ , \tag{2}$$

which we refer to as the vectorial system. This has the advantage that the fluid and structure solver can be executed in parallel to each other leading to a better parallel efficiency. To generalize this idea, we use the more compact notation

$$(S, F)\,(f, d) = (f, d) \ .$$

This notation stresses the fact, that we consider all input variables, and also all output variables as a closed entity. However, the notation does not specify any longer, which physical solver uses which variables as input and output. We can easily generalize this idea to more complex multi-physics problems. Basically, we calculate one iteration of the complete multi-physics dependency graph, taking all coupling variables as input values, computing all physical solvers P_i once, and considering all new calculated coupling variables as output values. For our model problem introduced in Section 2, the associated FPE reads

$$(F1, S, F2)\,(f_1, d_1, f_2, d_2) = (f_1, d_1, f_2, d_2) \ .$$

This FPE can now be solved with any FPE solver. We showed in [MUB$^+$13] that simple FPE solvers such as a fixed-point iteration or an Aitken underrelaxation for classical FSI problems lead to a two times slower convergence when using (2) compared to (1). Sophisticated quasi-Newton FPE solvers, based on the solution of a least-squares system in every iteration, however, show only a slight degradation of the convergence rate. Such a quasi-Newton least-squares (QNLS) solver (cf. [HDHV09]) was applied for (1) in [DBV09] based on similar ideas in [Mic04, VLDV07] and for (2) in [UBGM13, MUB$^+$13]. Algorithm 1 describes the QNLS technique for a general fixed-point equation $H(x) = x$. In a transient setting, the reuse of iteration values from previous time steps can lead to a far better efficiency (cf. [DBV09, MUB$^+$13]). We write QNLS(n) for a quasi-Newton solver reusing information from n time steps. If this algorithm is applied for the complete multi-physics problem, we refer to the coupling scheme as the "multi interface quasi-Newton" scheme (MIQN(n)).

The fixed-point x that Algorithm 1 seeks for consists, in our model problem, of four different subvectors: $x = (f_1, d_1, f_2, d_2)$. The solution of the least-squares system in QNLS

Algorithm 1 Quasi-Newton least squares method in pseudocode (cf. [DBV09, HDHV09])

initial value x^0
$\tilde{x}^0 = H(x^0)$ and $R^0 = \tilde{x}^0 - x^0$
$x^1 = x^0 + 0.1 \cdot R^0$
for $k = 1 \ldots$ **do**
    $\tilde{x}^k = H(x^k)$ and $R^k = \tilde{x}^k - x^k$
    if $\|R^k\|/\|x^k\| < \epsilon :$ **break**
    $V^k = [\Delta R_0^k, \ldots, \Delta R_{k-1}^k]$ with $\Delta R_i^k = R^i - R^k$
    $W_k = [\Delta \tilde{x}_0^k, \ldots, \Delta \tilde{x}_{k-1}^k]$ with $\Delta \tilde{x}_i^k = \tilde{x}^i - \tilde{x}^k$
    decompose $V^k = Q^k U^k$
    solve the first k lines of $U^k \alpha = -Q^{k^T} R^k$
    $\Delta \tilde{x} = W \alpha$
    $x^{k+1} = \tilde{x}^k + \Delta \tilde{x}^k$
end for

(Algorithm 1 uses a Q-R decomposition), depends highly on the mutual scaling of those subvectors. In classical FSI applications, force vectors often have substantially larger values than the displacements, in particular if displacement increments are used instead of absolute displacements relative to the initial state. In order to balance the influence of all subvectors we, thus, have to apply a normalization. In our studies, we found that a static scaling of the force vectors results in a sufficiently good balance. The potential of further improvements if using a dynamical scaling is subject of further research.
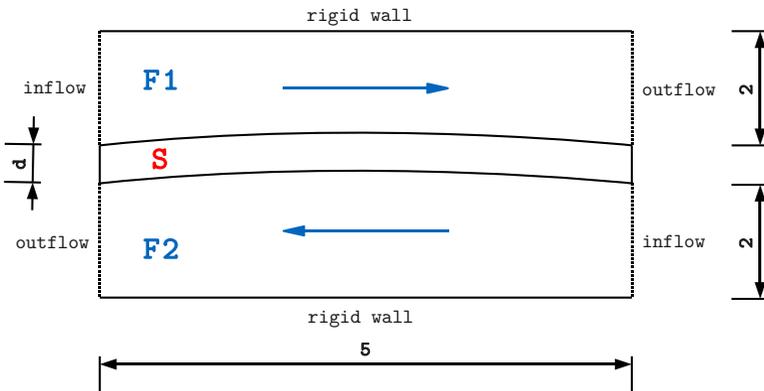
5 Numerical Results

5.1 Experiment settings

Figure 6: Scenario sketch: 2 fluid domains F_1 and F_2 are separated by an elastic beam S, which is fixed on both ends. Two variants of this scenario are studied, which differ in the vertical extent of the beam, $d_1 = 0.2$ or $d_2 = 1$ m.

Figure 6 depicts the geometry of the scenario that we study numerically. Two fluid domains F_1 and F_2 are separated by an elastic beam S, which is fixed on both ends. We distinguish two variants, which differ in the vertical extent of the beam: one thin variant S_1, $d_1 = 0.2$, and one thick variant S_2, $d_2 = 1.0$. The thin variant induces a stronger indirect interaction between the two fluid fields than the thick variant. The structure is modeled by a St. Venant-Kirchoff cantilever with Young's modulus $E = 5.6 \cdot 10^6$ kg/ms^2 and Poisson-ratio 0.4 . The density of the structure varies from 10^2 to 10^3 kg/m^3. Both fluids are modeled by the incompressible Navier-Stokes equations, with dynamic viscosity $\mu = 10^2$ kg/ms. The inflows show a parabolic velocity profile with mean velocity $U = 1.0$ m/s. The densities of both fluids, ρ_{F1} and ρ_{F2}, vary separately in the range from 10^0 to 10^3 kg/m^3, leading to either weak or strong interactions between both fluids and the structure.

We use `Alya`[2] as the physical solver for all three domains (cf. [UCG$^+$14, HVAC09]). There is, however, no particular need for this choice. Solvers can be exchanged in a nearly plug-and-play manner to meet particular needs. We showed, for example, in [MUB$^+$13] the bi-coupling of `COMSOL`, `Fluent`, or `OpenFOAM`. Both fluid domains are discretized with approximately 4000 elements each, whereas the structure consists of approximately 2250 (S_1), respectively 600 (S_2) elements. We use a matching grid at both interfaces. `Alya` uses finite elements for both, structure und fluids. The moving structure is resolved in the fluid by an arbitrary Lagrangian-Eulerian perspective, involving the movement of the underlying grid. For further numerical details of `Alya`, we want to refer to [UCG$^+$14]. The timestep size is set to $dt = 10^{-3}$ s. To decide on the overall convergence, we use relative convergence criteria of 10^{-3} for all coupling variables, displacement increments and forces on both interfaces. Figure 7 shows the results for the thin structure at $t = 10$ s
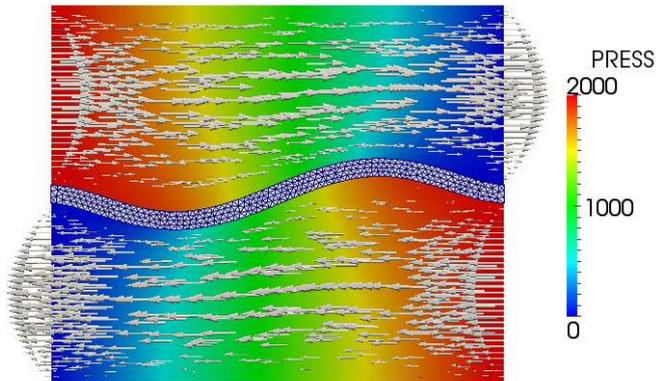

Figure 7: Pressure values (N/m^2), velocity vectors and grid deformation for the fluid-structure-fluid model problem at $t = 1.0$. This run uses the thin structure S_1, the densities $\rho_{F1} = \rho_{F2} = 10^3$ kg/m^3, $\rho_S = 10^1$ kg/m^3, and the MIQN(8) coupling scheme. For sake of clarity, the structural displacements are scaled by a factor of 5.

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[2]`Alya` is developed at the Barcelona Supercomputing Center, cf. http://www.bsc.es/computer-applications/alya-system.

5.2 Discussion

This subsection presents and discusses the results for the test case, described above. We vary the density of all three physical fields and the thickness of the structure. From literature (e.g. [CGN05, Bru09]), we know that the interaction between fluid and structure gets stronger, and thus the coupling gets more elaborate, if the density ratio ρ_F/ρ_S raises. Furthermore, by lowering the thickness of the structure, we can raise the indirect interaction between both fluid fields. Table 1 shows the average iteration numbers per timestep, listed for various density ratios, for both structures, and for the different coupling schemes, we introduced in Sections 3 and 4. By "IQN(5) / IQN(5)", we denote a concatenation of two interface quasi-Newton schemes (developed in [DBV09]), with re-used information from 5 timesteps, whereas CSS refers to the classical serial staggered scheme (the trivial explicit scheme, discussed in [FL00]). MIQN is the multi quasi-Newton coupling scheme, introduced in Section 4. Our experiences in [MUB+13] determines the choice of reused timesteps. For the MIQN scheme, we scale all force vectors by a factor of 10^{-7} (compare the discussion in Section 4).

ρ_{F1} - ρ_S - ρ_{F2}	Coupling	S_1	S_2
10^0 - 10^3 - 10^3	CSS / IQN(5)	4.62	4.62
10^0 - 10^3 - 10^3	CSS / IQN(0)	9.04	9.24
10^1 - 10^3 - 10^3	CSS / IQN(5)	crash	crash
10^1 - 10^3 - 10^3	IQN(5) / IQN(5)	crash	3.08 / 4.56
10^2 - 10^3 - 10^3	IQN(5) / IQN(5)	crash	crash
10^3 - 10^3 - 10^3	IQN(5) / IQN(5)	crash	5.10 / 5.18
10^3 - 10^2 - 10^3	IQN(5) / IQN(5)	crash	crash
10^3 - 10^1 - 10^3	IQN(5) / IQN(5)	crash	crash
10^1 - 10^3 - 10^3	MIQN(8)	5.64	5.62
10^2 - 10^3 - 10^3	MIQN(8)	5.88	5.76
10^3 - 10^3 - 10^3	MIQN(8)	7.40	7.00
10^3 - 10^2 - 10^3	MIQN(8)	11.78	11.72
10^3 - 10^1 - 10^3	MIQN(8)	20.90	17.20

Table 1: Average iteration numbers needed for convergence for various coupling configurations and different density ratios (all values in kg/m^3). S_1 and S_2 refer to the thin respectively thick structure. The values are averaged over the first 50 timesteps. The upper part shows results for the concatenation of an explicit and an implicit scheme, the middle part for the concatenation of two implicit schemes (cf. Section 3.1), and the lower part for the newly developed multi-coupling scheme (cf. Section 4).

In Table 1, we can observe several aspects. The upper part of the table shows results for a weak $F_1 - S$ interaction and a strong $F_2 - S$ interaction. For this case, we concatenate an explicit with an implicit scheme. We can argue from the results that, in this case, the strong interaction dominates the overall system. This means that the implicit coupling is not hampered by the explicit scheme. We can observe this in the better convergence of CSS / IQN(5) compared to CSS / IQN(0). If the density of F_1 is raised to 10^1 kg/m^3, the interaction between F_1 and S also becomes a strong one, leading to divergence if only an

explicit scheme is used. In general, the behavior of the overall system, in this case, is very similar to the bi-coupling of F_2 and S itself. The thickness of the structure, in particular, has no important influence here.

The middle block of Table 1 shows results for two strong interactions, demanding an implicit scheme on both interfaces. The concatenation of two implicit schemes, however, is not successful, since both interface systems hamper each other (compare the explanation in Section 3.1). It is striking that this problem, especially influences the test case with the thin structure S_1, since, here, both fluid fields possess a strong indirect interaction. In this case, all density settings lead to divergent runs. If the structure gets very thick, both interface systems get quasi independent from each other, and a concatenation of two implicit schemes can be successful. If we then lower the density of S, both interactions become stronger, and both interfaces influence each other again.

The lower block of Table 1 shows results for the same density settings, but now for the multi-coupling scheme MIQN. We observe a stable coupling for all settings. The number of iterations gets bigger for a stronger interaction between both fluid fields and the structure fields, but also for a stronger indirect interaction between both fluid fields.

6 Conclusions

In this work, we developed multi-coupling schemes based on either a composition of bi-coupling schemes or a true generalization of the underlying ideas of bi-coupling schemes. We tested those schemes by means of a simple, but yet challenging fluid-structure-fluid model problem. If one fluid-structure interface posseses a weak and the other one a strong interaction, the strong interaction dominates the overall stability and a concatenation of an explicit with an implicit scheme (cf. Section 3.1) leads to a stable multi-coupling scheme. If both fluid-structure interfaces show a strong interaction, a concatenation of two implicit coupling schemes does not result in an overall stable scheme, since both interface system hamper each other. For this case, we developed a true multi-coupling scheme, named "multi interface quasi-Newton" (MIQN) scheme by generalizing the ideas that we used in earlier work to deduce parallel FSI coupling schemes ([UBGM13, MUB$^+$13]. To our knowledge, this is the first time, that a fully implicit black-box multi-coupling scheme is described. This allows to simulate a new range of applications in a partitioned way.

We can easily generalize those ideas to multi-physics problems consisting of more than three physical fields. If one group of interface interactions is weak, and the other one strong, it should always be possible to couple all strong interactions with the MIQN scheme and then concatenate MIQN with explicit schemes for each weak interaction.

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