First Order Multiple Hypothesis Testing for the Global Nearest Neighbor Data Correlation Approach

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Abstract: The growing necessity in multiple targets tracking (MTT) in surveillance systems, with the recent dramatic increase in computational capabilities, has lead to a major interest in improving the performance of classical methods, such as the Global Nearest Neighbor (GNN), to enhanced schemes of Data Correlation. Today, the Multiple Hypothesis Testing (MHT) is generally accepted as the preferred approach for MTT systems, as it demonstrates better results in more complicated and uncertain environments. However, embedding such a mechanism to a deployed GNN-based system requires an extensive software change, and may introduce a major engineering risk to the working environment. Moreover, in a system that is deployed at different sites, which addresses operational environments of different complexities, such a change may be too costly and even superfluous. In this paper we will present a method which will address the challenge of multiple targets tracking in changing environments through a First Order Multiple Hypothesis Testing for a Global Nearest Neighbor engine. We will start with presenting the basics of multiple targets tracking, followed by a review of the proposed solution and conclude with simulations to verify its performance in different scenarios.

1 Introduction

Estimating a single target's state from a noisy set of observations is one of the fundamental problems in estimation theory. Several solutions have been suggested to minimize different error measures. Focusing on the mean square error (MSE) measure, and under the constraint of a discrete time linear estimator, the Kalman filter provides an efficient, recursive, real-time, optimal solution, which minimizes the MSE and converges to the non-linear optimal solution if the noise is known to be normally distributed. Assuming a simplified dynamic true state model:

$$\underline{X}_k = F \cdot \underline{X}_{k-1} + B \cdot a_k$$

$$\underline{Y}_k = \underline{X}_k + \underline{V}_k$$

Where $\underline{X}_k$ is the target's state vector (position and velocity), F is the state transition model, B and a_k are the acceleration update model, and V_k is a normally distributed measurement noise:

$$\underline{X}_k = \begin{bmatrix} X_k \\ \dot{X}_k \end{bmatrix}, \mathrm{F}=\begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, \mathrm{B}=\begin{bmatrix} \frac{\Delta t^2}{2} \\ \Delta t \end{bmatrix}, a_k \sim N(0, \sigma_a^2), \underline{V}_k \sim N(0, R_k).$$

We define:

$$w_k = B \cdot a_k \sim N(0, Q_k)$$

$$Q_k \triangleq Var(B \cdot a_k) = \sigma_a^2 B B^T$$

$\hat{\underline{X}}_{k|k}$ - The a-posteriori state estimate at time *k* given observations up to and including time *k*.
$\mathrm{P}_{k|k}$ - The a-posteriori error covariance matrix at time k, given observations up to and including time *k*.

Then, the Kalman Filter solution consists of two phases, as follows:

Predict
Predicted (a priori) state:

$$\hat{\underline{X}}_{k|k-1} = F \cdot \hat{\underline{X}}_{k-1|k-1}$$

Predicted (a priori) estimate covariance:

$$\mathrm{P}_{k|k-1} = F \cdot \mathrm{P}_{k-1|k-1} \cdot F^T + Q_k$$

Update
Innovation or measurement residual:

$$\tilde{\underline{Y}}_k = \underline{Y}_k - \hat{\underline{X}}_{k|k-1}$$

Innovation (or residual) covariance:

$$\mathrm{S}_k = \mathrm{R}_k + \mathrm{P}_{k|k-1}$$

Optimal Kalman gain:

$$\mathrm{K}_k = \mathrm{P}_{k|k-1} \mathrm{S}_k^{-1}$$

Updated (a posteriori) state estimate:

$$\hat{\underline{X}}_{k|k} = \hat{\underline{X}}_{k|k-1} + \mathrm{K}_k \tilde{\underline{Y}}_k$$

Updated (a posteriori) estimate covariance:

$$\mathrm{P}_{k|k} = (\mathrm{I} - K_k) \mathrm{P}_{k|k-1}$$

However, in a surveillance system where multiple targets have to be tracked and monitored simultaneously, the problem becomes more complicated. Assuming we are given the association between each target and its observation, multiple targets tracking through independent Kalman filters will independently minimize each estimated target's MSE, thus minimize the overall MSE. Therefore, using the structure of the Kalman Filter, an optimal solution for the MTT problem can be described as in Figure 1.

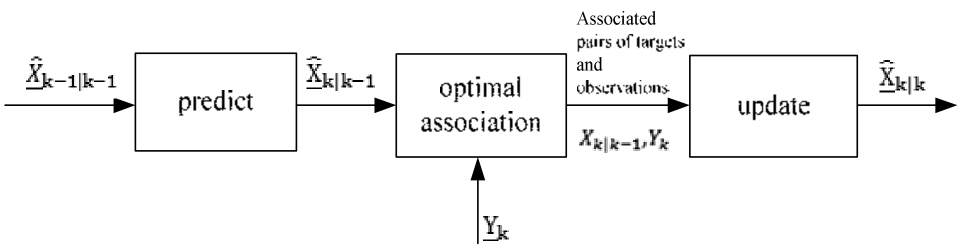

Figure 1: Block diagram for a multi-target estimation process

Based on this approach, the better the performance of the association phase is, the closer we get to the optimal solution.

Several methods were introduced to reflect the association's sub-optimality in the model and by that improve the estimation performance. In [Na69], [SS71] it was proposed to reflect the association uncertainty in the targets' estimate covariance matrix. A different method, the Joint Probabilistic Data Association (JDPA) [Fo80], suggests allowing weighted sum association of a single observation to multiple targets in uncertain association conditions (dense and cluttered environment, maneuvering targets). This means that a single observation may contribute to the update of several targets at a time, based on a probabilistic approach.

However, forcing an increased Kalman Filter estimate covariance matrix to reflect errors in different modules of the system may actually result with a reduced overall performance of the system [FV87], while associating a single observation to several targets may cause extremely similar targets in a dense environment, where several close observations contribute to several close targets.

In this paper we will tackle the sub-optimality of the association phase by taking into consideration a number of feasible associations and choosing the one which achieves the best association result, given a set of incoming observations. By that, we will address the possibility of false association by allowing an immediate recovery followed by corrected continuous tracking

2 Association Phase

As illustrated above, the association phase receives a predicted targets vector $\underline{\hat{X}}_{k|k-1}$ and an observations vector $\underline{Y}_k$, and generates pairs of predicted targets and corresponding observations. The association is calculated so the global price of association is minimized, where the price is defined as a distance measure between a predicted target and its associated observation. For example:

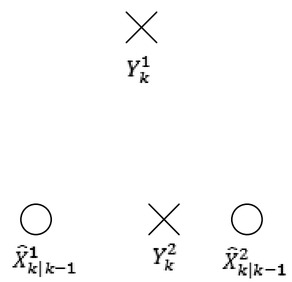

Figure 2: Observation-to-target association

Where $\hat{X}_{k|k-1}^1$, $\hat{X}_{k|k-1}^2$ are the positions of the predicted targets at time *k*, and Y_k^1, Y_k^2 are the positions of the observations at time *k*.

In this example, we chose the distance measure to be:

$$d^T S^{-1} d + \log(\det(S))$$

Where *d* is the geographical distance vector between the observation and the target and *S* is the observation's covariance matrix. The logarithm of the determinant of the covariance matrix is a value which corresponds to the volume of the covariance matrix, thus the uncertainty of the observation.

Therefore, the association of $\hat{X}_{k|k-1}^1$with Y_k^1 and $\hat{X}_{k|k-1}^2$with Y_k^2 will minimize the global statistical distance, assuming both observations‘ covariance matrices are equal.

The two-dimensional association problem is a well studied problem in the optimization field which can be solved using the generalized Auction algorithm [Be98] or a network flow algorithm [Go91] in (pseudo) polynomial time. Since these algorithms are well known they will not be described in this paper. The reader is referred to [Be98] and [Go91] for further details.

In order to reduce the computational load of the association phase, and define the minimal distance above which a target and an observation are not considered as candidates for association, we use the concept of Gating. In the pre-association gating phase the observations' and targets' positions are compared to each other to generate for each target its list of observations that are candidates for association. Graphically:

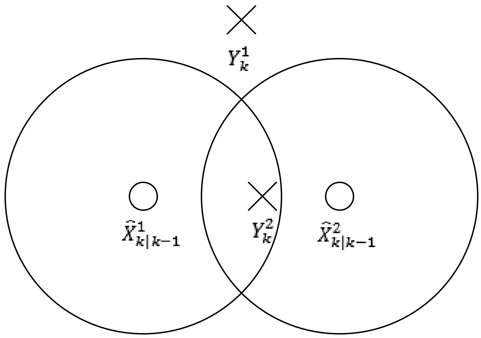

Figure 3: Gating the observation-to-track association

Where the circles represent the distance measure above which observations will not be considered as candidates for association for that target.

3 Dense Environment and Maneuvering Targets

In a well-spread target environment, with slow maneuvering targets, the number of candidate observations for each target is limited and result with a certain and undoubtedly preferred observation-to-target association. However, in a dense environment with fast maneuvering targets, the preferred association may only be slightly better than other feasible ones. Moreover, the specific environment which resulted several preferred associations may become clearer when future observation are available, thus making the decision easier. Therefore, by choosing only the best association at any given time we may even cause a propagation of error, when the environment is ambiguous.

For example:

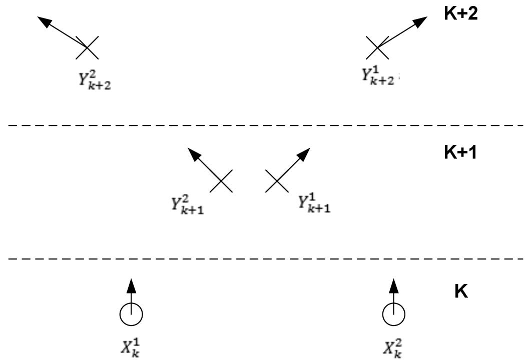

Figure 4: Crossing targets and corresponding observations

Where X_k^m and Y_k^m are target *m* and observation *m* at time *k*, and the arrows represent velocity vectors for each target and observation. In this example, the two targets cross each other between time stamps *k* and *k+1*. However, the best association at time *k+1* will be $X_{k+1|k}^1$, Y_{k+1}^2 and $X_{k+1|k}^2$, Y_{k+1}^1 which will result with an increasing MSE and falsely target tracking at time *k+2* and beyond. To overcome such hard decisions, we introduce the concept of First Order Multiple Hypothesis Testing for the Global Nearest Neighbor Data Correlation Approach.

4 Multiple Hypothesis Testing for the Global Nearest Neighbor Approach

As described above, making a hard decision at any time interval may lead to a propagation of error in targets tracking. Therefore, we shall regard not only the best association, but also store a configurable number of possible target sets that were generated from the N best (or better) feasible associations. We will then compare it against the new set of incoming observations. The target set which will result with the best association with the new observations will be updated with those observations and displayed to the user. The N-1 best (or better) associations shall be updated with the observations and stored for the next iteration.

In a pseudo code:

1) **First set of observations introduced to the system (initialization):**
   All observations generate new targets (no existing targets yet)

2) **Second set of observations:**
   a. Acquire sensor's observations.
   b. Predict targets (compute $\underline{\hat{X}}_{k|k-1}$)

c. Generate observations candidate list for association, for each target (Gating phase).
d. Find N best (better) observation-to-target associations.
e. Update targets ($\hat{\underline{X}}_{k|k}$) according to the N associations to generate N hypotheses (target sets), and store in memory.
f. Display the best hypothesis.

3) For the following sets of observations:
a. Acquire sensor's observations
b. Predict all stored target sets (compute $\hat{\underline{X}}_{k|k-1}$)
c. For each of the N stored hypotheses (targets sets):
   - Generate observations candidate list for association, for each target (Gating phase).
   - Find N best (better) observation-to-target associations.

d. Find the N best (better) observation-to-target associations from the associations calculated in the previous step.
e. Update targets ($\hat{\underline{X}}_{k|k}$) according to these N associations to generate N hypotheses, and store in memory.
f. Display the best hypothesis.

To guarantee that the system will not overwork in non-ambiguous scenarios, in which the observations are well-spread and easily associated, leading to a single association which achieves significantly better results, we introduce the Quality Factor. The Quality Factor is a threshold which defines a relative value of the best association below which an association will not be considered as possible hypothesis and will not be stored.

Therefore, by setting the Quality Factor to a high enough value, we make sure the system will only generate multiple hypotheses in an ambiguous environment, and converge to a standard Global Nearest Neighbor solution in spread and easily analyzed scenarios.

A great deal of work has been done in the area of calculating N-best associations [CM95], [DN93], [Mi95], [Mu68], in the context of solving an S-D assignment problem [DP92a], [De97]. These methods were used as guidelines in our algorithm's implementation, where their sub-optimality was handled by the Quality Factor threshold, as described above.

5 Simulations and Performances

In order to verify the performance of the First Order Hypothesis Testing for the Global Nearest Neighbor algorithm and quantify its behavior in different environments, we randomly generated targets in different scenarios and observed them through real deployed sensors models. We then injected the observations to the system and compared the estimated targets we got with their corresponding simulated ones, in terms of MSE. In all simulations the sensors models generated observations according to time-constant covariance matrices, providing a set of observations every two seconds. The area of experiment is 0.5 square kilometer.

- Simulation no. 1: Increasing Velocities

  Quality Factor: 0.7

  Number of simulated targets: 20

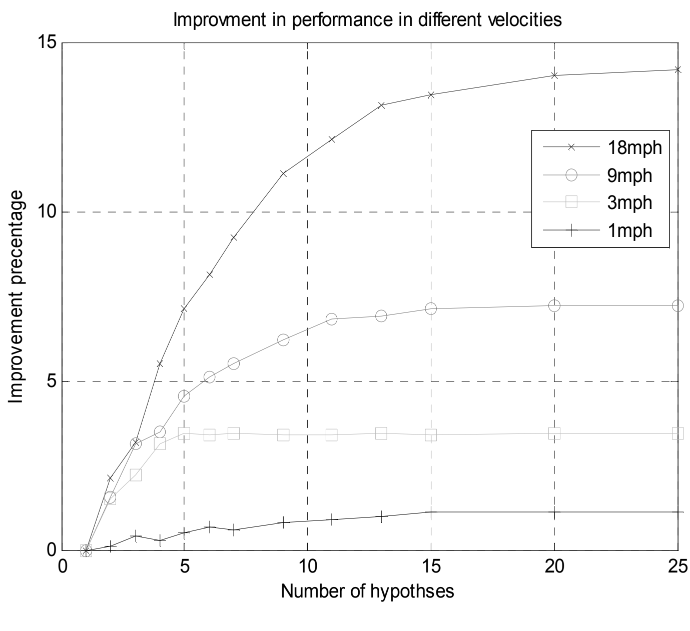

Figure 5: Improvement percentage as a function of maximal number of hypotheses

The following chart describes the relative improvement in the MSE of the simulated and the estimated targets at different average targets' velocities. The X axis describes the maximal number of hypotheses the system may generate and store, while the Y axis describes the percentile MSE improvement, compared to a single hypothesis testing (GNN).

We can first notice that the relative improvement increases, on average, as we allow more hypotheses testing to be made for a given scenario. This happens up to a saturation level above which increasing the number of hypotheses does not result with a significant improvement. The reason is that for a given scenario, the algorithm tends to converge to a preferred target tracking (in the described sense) for a certain number of hypotheses. From this point on, increasing the number of allowed hypotheses will not show a significant increase in the relative improvement.

Moreover, we can see the relative improvement in the MSE becomes more significant as the velocity of the simulated targets increases. The greater velocity makes the tracking of the simulated targets more complicated, which results with more uncertainty in the association phase. Therefore, in higher velocities, the Multiple Hypothesis Testing demonstrates greater improvement in the performance of the system.

- Simulation no. 2: increasing number of targets

  Quality Factor: 0.7

  Targets average velocity: 9 mph.

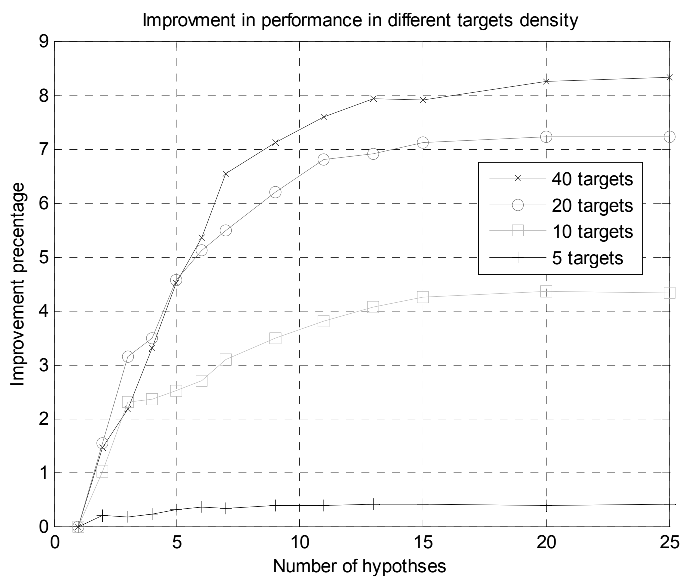

Figure 6: Improvement percentage as a function of maximal number of hypotheses

In this simulation we can see the effect of the targets density on the performance of the algorithm. Increasing the number of targets at a given area results in a more ambiguous association making, which is reflected in greater and closer association results. Therefore, allowing more hypotheses increase the performance in higher density environments.

- Simulation no. 3: decreasing Quality Factors

  Number of targets: 20

  Targets average velocity: 9 mph

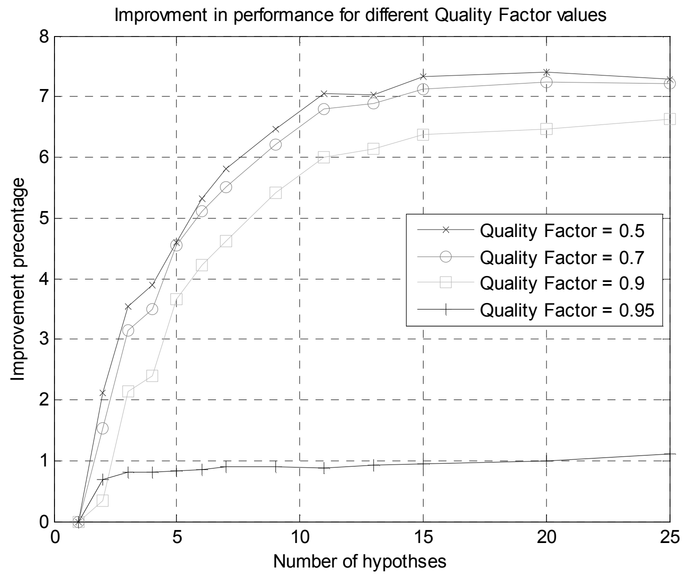

Figure 7: Improvement percentage as a function of Quality Factor values

Here we can see that in uncertain environments, decreasing the quality factor improves the performance of the system to a certain point above which it tends to saturate. The reason for this behavior is that decreasing the Quality Factor allows more, but less valuable, hypotheses testing. These hypotheses may improve the performance only in very ambiguous situations, in which the environment was so hard to track that a very unlikely association ended up being the better one to choose.

6 Conclusions

In this paper we introduced the concept of First Order Multiple Hypothesis Testing for a Global Nearest Neighbor Data Correlation Approach. Generating and storing multiple target hypotheses allows an immediate recovery in case of a false decision in uncertain association environments. In addition, it improves the system's ability to handle multiple target tracking, and creates a more accurate situational picture for the system's operator.

Introducing the Quality Factor, and a configurable number of maximum hypotheses testing, assures the system can be easily adjusted to different environments to meet the exact necessary tradeoff between its estimation accuracy and computational load.

The simulations we held demonstrated significant improvement comparing to the classical Global Nearest Neighbor Approach in more ambiguous scenarios, such as dense and cluttered environment or fast maneuvering targets.

Implementing a First Order Multiple Hypothesis testing for the Global Nearest Neighbor Approach does not necessitate an extensive software change; it is easily adjustable to meet different performance requirements and does not entail sophisticated or complex pruning mechanisms as other MHT-based methods. All these qualities make the upgrade from the classical GNN to an enhanced performance engine less complicated, in terms of code change and risk mitigation, for real-time deployed Command and Control surveillance systems.

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