

Combined Optimization of Aircraft Maneuvers and RF Measurements for Passive Air-Air Ranging

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Abstract: When a tactical aircraft sets out on a mission in dense airborne and surface RF (radio frequency) emitter environment, the pilot desires to minimize the use of the onboard fire control radar to remain “silent.” Consequently, the pilot heavily relies on the onboard passive sensors and offboard reports received from wingmen and other sources. The onboard sensors may include a RWR (radar warning receiver) and an IRST (infrared search and track) sensor. The RWR detects RF (radio frequency) emissions from airborne target fire control radars and land/sea radars that are a part of complex weapon systems — such as SAM (surface-to-air missiles) and AAA (anti-aircraft-artillery) guns. The IRST detects hot emissions from airborne targets — such as target aircraft exhaust and thermal airframe heating. The critical need for the pilot is to have timely “situation awareness” of the targets (friendly or unfriendly) within a certain airspace (upper and lower hemispherical coverage) around the aircraft. Key target parameters to aid situation awareness include ID (identification) features, slant range, intent, mode (if the target has RF transmitters), and an assessment of the target’s near-term and far-term lethality (its weapon’s envelope). Timely, accurate range estimates provide information to the onboard flight path management system to compute route plans to avoid targets and vital information to the fire control computer when onboard standoff weapons are to be deployed. To estimate range passively for air-to-air target engagements is one of the most challenging research areas for the military tactical fighter aircraft industry today, and is the focus of this paper. Research has shown that 10 percent range estimates in 10 seconds are achievable under certain RF measurement and scenario (initial conditions and sensing aircraft maneuvers) conditions. The presentation reviews the mathematical models used, the Cramer-Rao lower bound formulation and range estimate errors for a typical air-air scenario. We conclude with a review of the complex relationship between the sensing aircraft maneuvers, required RF measurements, and initial uncertainties in emitter aircraft range, speed and heading angle; and the high-level requirements for fielding a real-time passive ranging system.

1 Introduction

The fighter aircraft today require stealth technology to maximize their effectiveness. As such, the pilot relies on the passive sensors to supply situation awareness. This situation awareness includes knowing what airborne and surface emitters are in the environment, their identification, their intent, lethality, and range. The electronic warfare (EW) system detects, identifies and has to assist in passive geolocation to the radio frequency (RF) emitters. Related ideas are captured in the open literature – such as work by Klaus Becker in Germany [1,2]. In this paper, we cover the following topics: Problem, Solution Approach, Math Model, Sample Scenario, Simulation Results, and Summary.

[1] “Passive Localization of Frequency-Agile Radars from Angle and Frequency Measurements,” Klaus Becker, IEEE Transactions on Aerospace and Electronic Systems, volume 35, No.4, October 1999, pp. 1129-1144.

[2] “An Efficient Method of Passive Emitter Location,” Klaus Becker, IEEE Transactions on Aerospace and Electronic Systems, Volume 28, No.2, October 1992, pp. 1091-1104.

PROBLEM - Passive Air-to-Air Ranging - Passively estimate the range of a target aircraft from an airborne platform at long range quickly and accurately.

SOLUTION APPROACH - Develop a theoretical range estimate performance model using the Cramer-Rao (CR) lower bound formulation in MATLAB to analyze the problem. Develop math models to characterize the passive RF measurements: angle and RF Doppler. Define, and incorporate, unknown initial conditions with regard to target aircraft (i.e., target aircraft range and speed, and heading with respect to the sensor aircraft). Assess ranging performance across large parametric conditions.

POTENTIAL APPROACHES –

IR Sensors

- ▶ Angle-only measurements
- ▶ Slow convergence time due to large sensor aircraft flight excursions
- ▶ Difficult to correlate measurements

RF Sensors

- ▶ Angle and frequency measurements
- ▶ Good detection range
- ▶ Potentially fast range convergence
- ▶ RF parameters useful to correlate reports

The **CR (Cramer-Rao) Lower Bound** provides the theoretical performance limits based on measurement parameter sets – since the measurement errors are jointly Gaussian and the estimation error distribution can be approximated by a joint Gaussian function, then the Cramer-Rao (CR) lower bound can be determined by as $C = (H^T R^{-1} H)^{-1}$, where H is the matrix of partial derivatives of the measurement equations with respect to the states, R is the measurement covariance matrix and the measurement errors are uncorrelated, jointly Gaussian. If the measurement errors are uncorrelated, jointly Gaussian, and have the same variance, then $C=R(H^T H)^{-1}$. The H matrix based on RF measurements alone consists of three columns containing the derivatives of the i th measured RF with respect to r_{e0} , v , and f_T . To combine these derivatives with the H matrix for the interferometer data, we add one column of zeros to the latter (no dependence of $\cos\beta$ on f_T) and then concatenate the two matrices. In the equation for C in the case of uncorrelated errors with equal variances, R is a scalar equal to the common variance. The Cramer-Rao bound gives the minimum variance estimate even when the relation between measurements and state variables is non-linear. If the measurement errors are unbiased and jointly Gaussian, the minimum variance estimates lie along the main diagonal of your matrix C. The ellipsoid defined by C, however, is no longer a constant probability surface as it is in the case of a linear relationship. If H is full rank and R is invertible, then C can be calculated. One can do a “rank test” or *singular value decomposition* to show that H is full rank. In all our work, H this was always full rank.

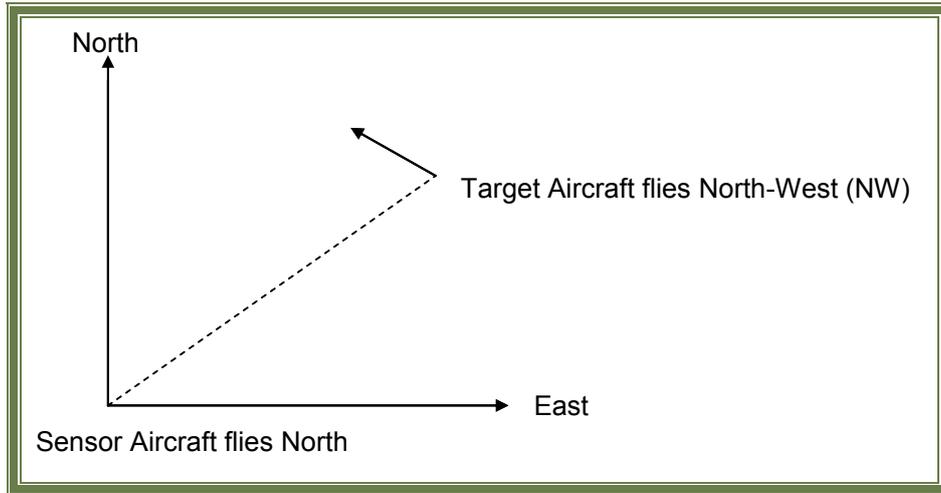


Figure 1 - North-by-North-West (NNW) Scenario

Angle measurements are cone angles from interferometer sensing devices.

$$\cos \beta_i = \frac{\mathbf{d}^T (\mathbf{r}_e - \mathbf{r}_{pi})}{|\mathbf{r}_e - \mathbf{r}_{pi}|} \quad (1)$$

where β_i is the cone angle about the interferometer axis at the i^{th} update ($i = 1,2,3,\dots,N$)
 \mathbf{r}_e is the vector to the emitter from the origin of coordinates

\mathbf{r}_{pi} is the vector from the origin to sensor aircraft position at the i^{th} update
 \mathbf{d}_i is a *unit* vector in the direction of the interferometer axis at the i^{th} update
(For straight and level flight, \mathbf{d}_i is a constant vector)

The superscript T denotes the transpose operation and $|x|$ denotes magnitude of x . The phase across the interferometer, which is assumed to be resolved, i.e., it contains the proper multiple of 2π , is equal to $(2\pi D/\lambda)\cos\beta_i$, where D is the length of the interferometer aperture and λ is the emitter signal wavelength (in the same units as D). If we assume that the phase measurement error is independent of the emitter angle of arrival, then the error in $\cos\beta_i$ is independent of β_i . The standard deviation $\sigma_{\cos\beta}$ of the error in $\cos\beta_i$ is given by

$$\sigma_{\cos\beta} = \frac{\lambda \sigma_{\psi}}{2\pi D} \quad (2)$$

where σ_{ψ} is the standard deviation of the phase error in radians. Note that this expression is also the first-order approximation to the interferometer boresight angle-of-arrival error, expressed in radians.

If the emitter is non-stationary, \mathbf{r}_e is no longer constant. In this case, we have to assume a model for the motion of the emitter platform. Suppose we assume a constant velocity model:

$$\mathbf{r}_e = \mathbf{r}_{e0} + \mathbf{v}_e t_i \quad (3)$$

where \mathbf{r}_{e0} is the position vector of the emitter platform at time $t = 0$, \mathbf{v}_e is the assumed constant velocity vector, and the subscript i again denotes update time. Since we now have to estimate both the initial position and the velocity of the emitter platform, we modify the expressions for $\cos\beta_i$ and $\cos\theta_i$ by substituting $\mathbf{r}_{e0} + \mathbf{v}_e t_i$ for \mathbf{r}_e .

Suppose the ownship velocity vector is also a constant, which we'll denote by \mathbf{v}_p . Then the expression for $\cos\beta_i$ becomes

$$\cos\beta_i = \frac{\mathbf{d}_i^T (\mathbf{r}_{e0} + (\mathbf{v}_e - \mathbf{v}_p)t_i)}{|\mathbf{r}_{e0} + (\mathbf{v}_e - \mathbf{v}_p)t_i|} \quad (4)$$

If we replace \mathbf{r}_{e0} by $K\mathbf{r}_{e0}$ and \mathbf{v}_e by $K\mathbf{v}_e + \mathbf{v}_p(1-K)$, where K is an arbitrary constant scalar, we find that the expression for $\cos\beta_i$ remains exactly the same, regardless of i . A similar remark applies to $\cos\theta_i$. This means that without additional information, our range and velocity estimates are ambiguous. For triangulation on an emitter platform moving with constant velocity, the ownship must therefore do a maneuver to eliminate ambiguities.

For the RF Doppler measurements, the measurement variable ψ is now the received RF, which we denote by f_R and which is related to the state variables by

$$f_R = f_T (1 - \mathbf{v}_R^T \mathbf{r}/cr) \quad (5)$$

where f_T is the frequency of the signal at the emitter, V_R is the emitter velocity minus the sensor aircraft velocity, and c is the speed of light. The vector r is the range vector from the sensing aircraft to the target aircraft emitter, and r has magnitude r .

For the angle measurements, we have

- measured variable is unambiguous cone angle
- since only a single-axis interferometer is used, and up-down ambiguity exists
- all phase measurement errors are independent of each other and are zero-mean Gaussian
- mechanical boresight errors are neglected
- cosine of cone angle is provided by unambiguous interferometer
- coordinate system origin is at initial position of sensor aircraft
- initial headings of sensor aircraft and emitter platform are measured from positive x-axis
- initial azimuth of emitter is defined relative to x-axis
- port and starboard antennas are available to minimize dropout from field of view
- the program allows for initial estimates of emitter platform speed, initial range, and heading
- the estimates are assumed to be normally distributed about the true values
- variables to be estimated are horizontal range, azimuth angle

To estimate these variables, we must also include emitter velocity vector, which is expressed as a magnitude (speed) and heading relative to the x-axis.

Range performance is a function of scenario parameters:

- ▶ sensor aircraft speed
- ▶ target aircraft speed
- ▶ initial sensor aircraft heading
- ▶ initial target aircraft range
- ▶ initial target aircraft heading
- ▶ initial target aircraft azimuth
- ▶ angle measurement accuracy
- ▶ angle measurement sample interval
- ▶ initial target aircraft range error assumed
- ▶ initial target aircraft speed error assumed
- ▶ initial target aircraft heading error assumed
- ▶ sensor aircraft maneuver (flight path [2-turn, sinusoid, etc.], time initiated, G's ["sign" & magnitude], straight-leg lengths [e.g., sinusoidal maneuver], total maneuver duration; total cross-range allowed with respect to initial AZ angle)

- ▶ sample range estimate requirements (e.g., TBD1% by 15 seconds; TBD2% by 40 seconds)

Air-to-Air passive ranging performance is a multivariable problem due to many critical “variations” for the same scenario due to the “knob adjustments” – as summarized in Table 1. Up to 16 filters are needed with this approach. In a later study, we replaced the approach using a 2-stage interacting multiple model (IMM) EKF approach – where an initial bank of 12 EKFs are run that monitor the residuals and then quickly reduce to a single 2-EKF IMM algorithm. Results for the IMM algorithm approach the CR bound. We applied the approach against the Raytheon engineer David V. Stallard scenario and performed slightly better than Dave. See the 1987 AIAA Guidance, Navigation and Control Conference, Monterey, CA, Aug 17-19, 1987 paper “An Angle-Only Tracking Filter in Modified Spherical Coordinates,” pages 542-550. Obviously augmenting the Doppler approach to this (later in our paper) vastly accelerates tracker convergence.

Table 1. Air-to-Air Passive Ranging Scenario Parameter Knob Settings
(errors are initial values-knobs 1-3)

KNOB	PARAMETER	TYPICAL SETTING
1	1-sigma emitter heading error in degrees	10, 30, 50
2	1-sigma emitter range error (percent)	30, 40, 50
3	1-sigma emitter speed error (percent)	10, 20, 30
4	1-sigma angle error at boresight (degrees)	0.1, 0.25, 0.5
5	sensor aircraft acceleration (Gs)	1, 2, 3
6	measurement sample interval (seconds)	0.5, 1, 2
in this table, there are $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$ ‘variations’		

Table 2. Sample Scenario

TOTAL FLIGHT TIME: 60 SEC UPDATE INTERVAL: 1 SEC SENSOR AIRCRAFT SPEED: 585 KNOTS TARGET SPEED: 585 KNOTS SENSOR AIRCRAFT HEADING FROM NORTH: 0 DEG TARGET HEADING FROM NORTH: -45 DEG INITIAL RANGE FROM SENSOR AIRCRAFT TO TARGET: 50 NMILES INITIAL TARGET AZIMUTH FROM NORTH: 45 DEG SENSOR AIRCRAFT ACCELERATION: 1.2 G MANEUVER START TIME: 0.05 SEC STANDARD DEVIATION OF ANGLE ERROR AT BORESIGHT: 0.5 DEG INTERFEROMETER AZIMUTH BORESIGHT: 34 DEG MAXIMUM ERROR OF INITIAL TARGET RANGE ESTIMATE (%): VARIABLE (BELOW) STANDARD DEVIATION OF INITIAL TARGET SPEED ESTIMATE (%): VARIABLE (BELOW) STANDARD DEVIATION OF INITIAL TARGET HEADING ESTIMATE (DEG): VARIABLE (BELOW) SENSOR AIRCRAFT MANEUVER FLIGHT PATH: 2-TURN (90 DEGREE TURN WITH 1.2 G, FOLLOWED BY 90 DEGREE TURN WITH -1.2 G)						
INITIAL ERRORS				PERCENT RANGE ERROR		
CASE	RANGE (%)	SPEED (%)	HEADING (deg)	15 SECONDS	40 SECONDS	15 SECONDS
1	100	0.5	0.5	20.39	3.02	20.39
2	100	10	0.5	24.74	8.21	24.74
3	100	20	0.5	32.02	11.36	32.02
4	100	30	0.5	37.59	12.59	37.59
5	100	40	0.5	41.21	13.13	41.21
6	100	0.5	10	20.39	3.02	20.39
7	100	0.5	20	20.39	3.02	20.39
8	100	0.5	30	20.39	3.03	20.39
9	100	0.5	40	20.4	3.03	20.4
10	100	10	10	24.74	8.22	24.74
11	100	20	20	32.03	11.54	32.03
12	100	30	30	37.61	13.14	37.61
13	100	40	40	41.26	14.18	41.26

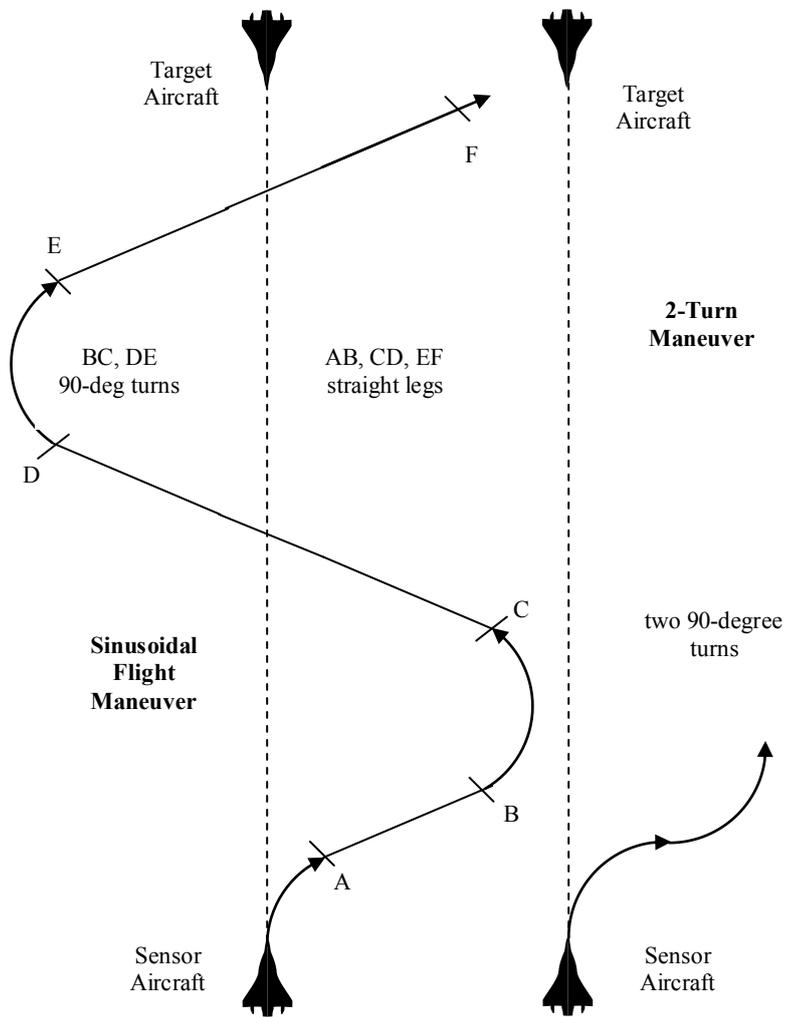


Figure 2 – Sensor Aircraft Maneuvers

Some Observations in Air-Air Ranging Performance North-by-Northwest (NNW) Scenario

The ranging performance has 3 basic regions: transient, transitory steady-state, and longer-term steady-state — corresponding to 15, 40 and 60 second time points, respectively

Head-on engagements are best addressed with sensor aircraft maneuvers to right or left

Near-perpendicular engagements are best addressed with sensor aircraft maneuvering in the counter (opposite) direction to the target aircraft.

The desired range error desired at the 3 time points (15, 40, and 60 seconds) for any scenario can be traded by looking at the percent range error curves for the sensor aircraft maneuver/acceleration levels selected.

Delaying the time to initiate the sensor aircraft maneuver can improve ranging performance for both “opening” and “closing” engagements for either sensor aircraft maneuver because the transient and steady-state responses are improved.

Given the 45-degree initial veer off the initial sensor aircraft heading: time to initiate the maneuver (particular payoff for “opening” engagements) balance between developing the baseleg [bearing spread] vs. getting in the sensor aircraft turn [angle acceleration “sign” and magnitude change] for steady-state and transient performance, respectively the magnitude and “sign” of the sensor aircraft acceleration deployed (the greater the sensor aircraft acceleration magnitude, the less time spent doing the sensor aircraft turns, thus the more time that can be devoted to the straight leg portion of the maneuver — to thus “extend” the leg length – and increase the baseleg to improve steady-state ranging performance.

Requirements have to be Driven by Pilot Tactical Maneuvers

- Ranging performance components — transient (15 sec) and near-steady-state (40 sec)
- Ranging performance contingent upon the sensor aircraft maneuver with respect to the target aircraft heading and sensor aircraft speed
- Sensor aircraft maneuver regulates the “observability” needed to estimate range
- Sensor aircraft maneuver flight path may be sinusoidal, 2-turn, or other defined flight paths
- Specific tactical sensor maneuver(s) available will be pilot- and scenario-dependent
- Ranging performance is contingent upon having the target aircraft in the sensor FOV (field-of-view)

- Need to know the total cross-range excursion permissible by the pilot for different scenarios
- Need to know the amount of time available to perform the sensor aircraft maneuver
- The level of a priori information handed-off by onboard Mission Systems directly drives the initialization uncertainty used in any proposed Kalman filter-based tracking algorithm (i.e., initial target aircraft range and speed, and heading with respect to sensor aircraft)

Simulation Results

Figures 3 and 4 depict sample simulation results. Conclusions we see:

- 10 percent range error is achievable quickly when the sensor aircraft executes higher G maneuvers
- To minimize range error at a given time, different sensor aircraft accelerations are required

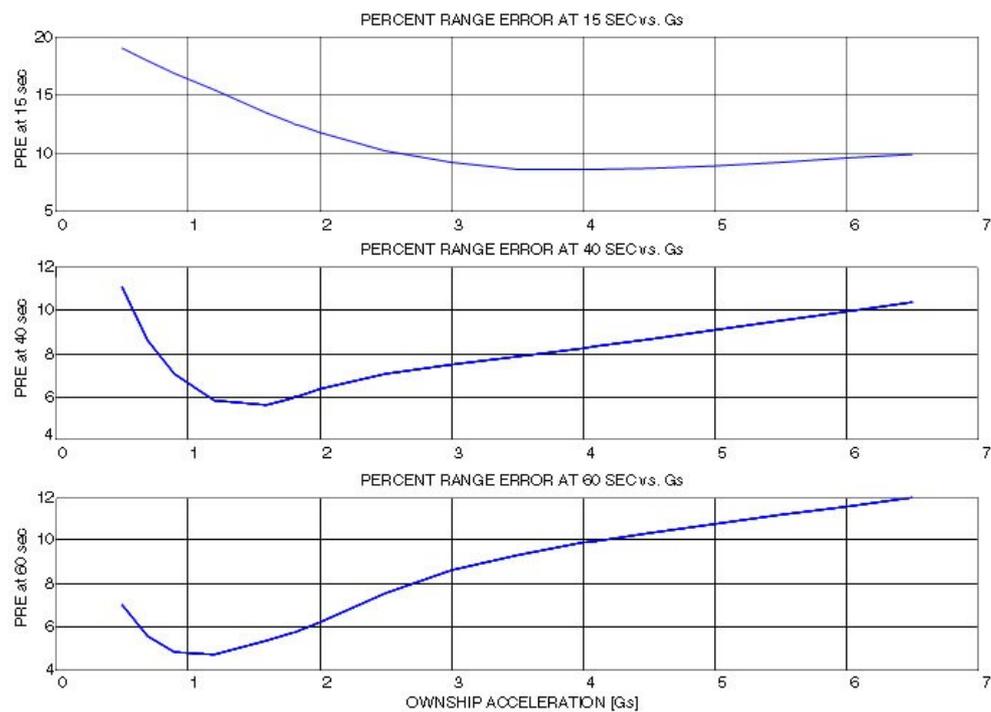


Figure 3 – Percent Range Error at 15 Seconds vs. Aircraft G's Pulled

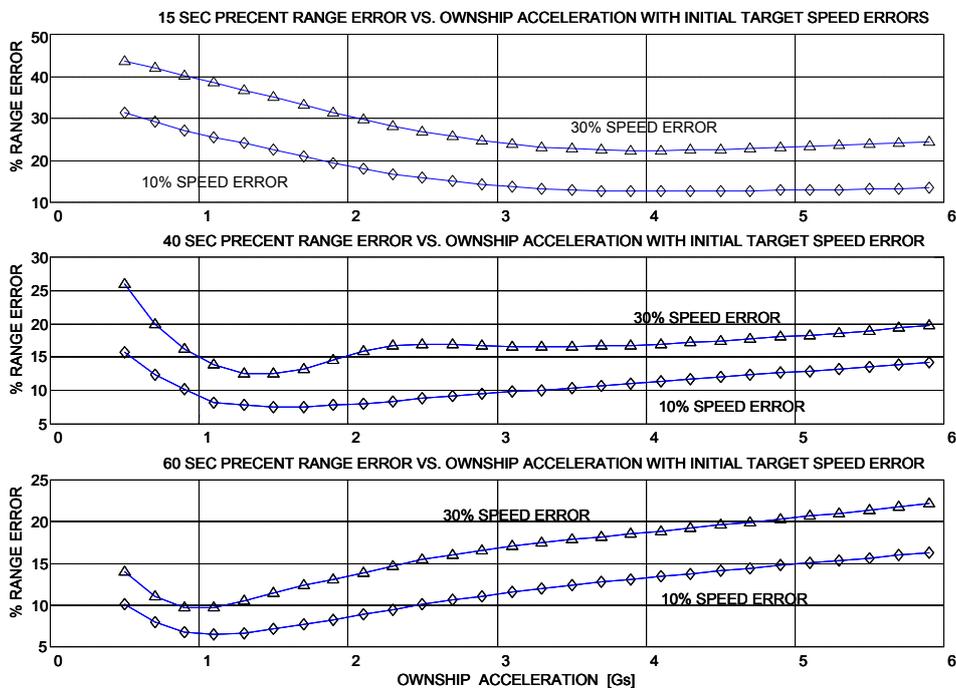


Figure 4 – Percent Range Error at 15 Seconds vs. Aircraft G's Pulled for 10 & 30% Speed Error

Real-Time Multiple-Hypothesis Passive Ranging Algorithm

Initial Conditions

- Accommodates uncertainties in initial target aircraft range, speed, and heading with respect to the sensor aircraft
- Several Kalman filter models are seeded with a spread of initial conditions

Target Aircraft Dynamics Models

Two models used

- constant velocity, constant heading
- acceleration (to accommodate heading changes)

Maneuver detection logic “built-in” to the IMM (interacting multiple model) formulation

Mission Avionics has to “Balance” the Dynamic Requirements of all the Subsystems to Meet Overall Sensor Aircraft Objectives

- pilot tactical maneuvers

- survivability maneuvers
- target geolocation maneuvers
- weapons targeting
- mission systems
- ingress and egress time constraints

Summary

- There is no closed-form RF-based passive ranging performance expression that captures all the cited parameters
- The optimal passive ranging solution is highly parametric and requires accurate received emitter angle and frequency measurements, coupled with the appropriate sensor aircraft acceleration
- With small (~2 Gs) sensor aircraft sinusoidal or 2-turn maneuvers, 10 percent range accuracy is achievable using angle and RF Doppler measurements
- Real-time, multiple model Kalman filter-based solutions will adequately perform the passive range calculations by incorporating initial condition uncertainties (i.e., target aircraft speed and range, and heading with respect to the sensor aircraft)
- The multiple model approach would be augmented by models to accommodate different target aircraft dynamics variations (i.e., to account for target aircraft heading changes with or without concurrent sensor aircraft heading changes)