Evacuation from a Disk for Robots with Asymmetric Communication

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Abstract

We consider evacuation of two robots from an Exit placed at an unknown location on the perimeter of a unit (radius) disk. The robots can move with max speed 1 and start at the center of the disk at the same time. We consider a new communication model, known as the SR model, in which the robots have communication faults as follows: one of the robots is a Sender and can only send wirelessly at any distance, while the other is a Receiver in that it can only receive wirelessly from any distance. The communication status of each robot is known to the other robot. In addition, both robots can exchange messages when they are co-located, which is known as Face-to-Face (F2F) model.

There have been several studies in the literature concerning the evacuation time when both robots may employ either F2F or Wireless (WiFi) communication. The SR communication model diverges from these two in that the two robots themselves have differing communication capabilities. We study the evacuation time, namely the time it takes until the last robot reaches the Exit, and show that the evacuation time in the SR model is strictly between the F2F and the WiFi models. The main part of our technical contribution is also an evacuation algorithm in which two cooperating robots accomplish the task in worst-case time at most $\pi + 2$. Interesting features of the proposed algorithm are the asymmetry inherent in the resulting trajectories, as well as that the robots do not move at full speed for the entire duration of their trajectories.

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1 Introduction

Evacuating a group of autonomous mobile agents (or robots) operating over a specified planar domain (e.g., circle, square, convex set, etc.) from an unknown (to the robots) Exit is an important paradigm in group search for understanding the tradeoffs of communication and mobility in distributed computing. The communication capabilities of the robots that have been considered in the literature so far are limited mainly to Wireless (WiFi) and Face-to-Face (F2F), see [2]. An important aspect in this type of group search is the ability of the robots to evacuate despite the presence of communication faults. Two types of faults have been considered in the literature: crash (passive and non-malicious) and Byzantine (active and malicious). Note that in our model only the robots' communication capabilities are affected and both robots are honest (i.e., they send correct messages and the communication is always available). Despite its importance, fault tolerant group search has been considered less extensively mainly due to the complexity of the resulting optimization problem for this task (see [3]).

In this paper we explore two robot evacuation in the plane under a different type of faulty behaviour whereby the robots have different communication capabilities, namely one robot is a Sender in that it can only send messages wirelessly and the other is a Receiver in that it can only receive messages wirelessly. For this setting we obtain the following contributions. First we show that any algorithm is bound to have running time strictly more than the provably optimal evacuation time of two Wireless robots. Second, we provide a novel evacuation algorithm whose running time is better than the best lower bound known for two Face-to-Face robots, separating this way the two well-studied models.

1.1 Preliminaries and notation

Two robots are initially collocated in the plane and can travel with speed at most 1. An Exit is placed at known distance 1 from the robots at an unknown location (therefore defining a disk with center the original position of the robots). The robots can always communicate F2F (when co-located). In addition, one of the robots is a Sender (i.e., can send messages wirelessly, but cannot receive messages from distance) and the other is a Receiver (i.e., can receive wirelessly, but cannot send messages from distance). The communication capabilities of the robots are known to themselves and to each other, namely the robots know who is the Sender and who is the Receiver and moreover the Sender knows it can send and the Receiver it can receive messages wirelessly, the messages are transferred instantly. The goal is to design an evacuation algorithm which specifies trajectories which enable the robots to reach the unknown Exit. During their trajectories the robots can take shortcuts in the interior of the disk and can recognize its perimeter. The quality of the algorithm is measured by the time it takes the last robot to reach the Exit, which is defined as the evacuation time (of the algorithm).

Next we define more precisely the concept of evacuation time restricted to our search domain, namely the unit disk, which will be used in the rest of the paper.

▶ **Definition 1.** *The evacuation time $\mathcal{E}_A(p)$ of an algorithm A for an Exit placed at an unknown (to the robots) location p on the perimeter of the unit disk is the time it takes for the two robots starting from the centre of the unit disk and following algorithm A to evacuate from the Exit placed at p. The worst case evacuation time of an algorithm A is defined as the* $\sup_p \mathcal{E}_A(p)$*, where p may be any point on the perimeter of the unit circle.*

Consider a class $\mathcal{A}$ of all possible evacuation algorithms on the unit disk arising from the chosen communication model (e.g., F2F, WiFi, etc.).

▶ **Definition 2.** *The evacuation time for a class $\mathcal{A}$ of algorithms is defined as* $\inf_{A\in\mathcal{A}} \mathcal{E}_A$.

For the purposes of the current work, a robot may be in one of four possible communication states, namely F2F (Face-to-Face), WiFi (Wireless, both send and receive), S (only send wirelessly), and R (only receive wirelessly). All previous studies considered evacuation in which the robots have identical communication capabilities (e.g., both F2F, or both WiFi). Our focus in this paper is to consider robots with different communication capabilities. Before proceeding any further it will be useful to make an observation which clarifies the communication potential of an ensemble of two evacuating robots assuming that they both maintain their F2F (both send and receive only when co-located) communication capability.

▶ **Observation 3.** *There are only three possibilities for the communication capabilities of an ensemble of two robots which are (at least) able to communicate F2F: either both are WiFi, or both are only F2F, or one is a Sender and the other a Receiver.*

To see why this is true, recall that by assumption, at a minimum the robots are assumed to be able to communicate F2F. Moreover, one can think of a robot with WiFi communication potential as a robot which is both Sender and Receiver. If one of the robots can only communicate F2F then regardless of the communication capabilities the other robot has, the ensemble of two robots behaves as if both robots have only F2F communication. Similarly, if both robots are Senders or both are Receivers; the robots can communicate only F2F with each other. If one of the robots has WiFi communication but the other is only either a Sender or Receiver then the WiFi cannot use its full communication potential and therefore can only use its Sender status with a Receiver and its Receiver status with a Sender. Thus the validity of the observation follows by combining all three previous assertions.

Let $\mathcal{E}_{F2F}$, resp. $\mathcal{E}_{WiFi}$, be the evacuation times when both robots use F2F, resp. WiFi, communication. Further, let SR denote the mixed Sender and Receiver model described above. From previous studies we know that $\mathcal{E}_{WiFi} < \mathcal{E}_{F2F}$. The main questions of interest are the following:

1. Can we find an optimal evacuation algorithm in the Sender/Receiver model?
2. It is easy to see that the evacuation time in these models satisfies $\mathcal{E}_{WiFi} \leq \mathcal{E}_{SR} \leq \mathcal{E}_{F2F}$. Can we differentiate the three models and show that the inequalities are strict?

It turns out that this communication asymmetry between Sender and Receiver gives rise to interesting trade-offs which are unique to the SR model. Regarding the proposed questions, we answer them as follows.

1. We provide a technical algorithm with evacuation time strictly less than $\mathcal{E}_{F2F}$ and that we conjecture is optimal.
2. We show that any algorithm is bound to have evacuation time strictly more than $\mathcal{E}_{WiFi}$.

1.2 Mobility Model

We use the standard evacuation model for search on a unit disk where the Exit is placed on the perimeter and the robots may take short-cuts in its interior. The robots are autonomous and can exchange messages instantly using the SR communication model. The robots run synchronized clocks. They know the unit disk (its center and unit radius) and can recognize its perimeter. At any time they can stop and start, change direction and speed but their speed can never exceed 1. An evacuation algorithm is defined by the trajectories of the two robots. A robot trajectory is a continuous function $f : [0, T] \to \mathbb{R}^2$ over time, such that $f(t)$ is the location of the robot at time t and T is the duration of a robot's trajectory. Moreover, the robot's speed can never exceed 1, meaning that $\|f(t) - f(t')\|_2 \leq |t - t'|$, for all $0 \leq t, t' \leq T$, where $\| \cdot \|_2$ denotes the Euclidean norm in the plane $\mathbb{R}^2$.

1.3 Related work

In all the results below we consider worst-case evacuation time and limit our discussion mostly to the case of two robots. The general evacuation model on a disk discussed in our paper was first introduced in [2]. In this paper, among other results, the optimal value for the WiFi model was determined, namely $\mathcal{E}_{WiFi} = 1 + \frac{2\pi}{3} + \sqrt{3} \approx 4.826$, while for the value of the F2F model it was shown that $5.199 \approx 3 + \frac{\pi}{4} + \sqrt{2} \leq \mathcal{E}_{F2F} \leq 5.74$. The lower bound was later improved to $3 + \frac{\pi}{6} + \sqrt{3} \approx 5.255 \leq \mathcal{E}_{F2F}$ in [6]. Most recently, [12] improving on results of [1], has improved the upper bound from 5.74 (which was first proved in [2]) to $\mathcal{E}_{F2F} \leq 5.6234$. However, it is worth noting that, in general, worst-case tight bounds for evacuation of two robots in the F2F model remain elusive to this day.

Search and evacuation with multiple faulty robots on an infinite line was initiated in the work of [5] and [11]. For three robots one of which is Byzantine, [16] shows that the proportional schedule presented in [11] can be analyzed to achieve an upper bound of 8.653055. More importantly, [10] gives a new class of algorithms for n robots when the number of Byzantine among them is near majority, which in turn implies the best known upper bound of 7.437011 on an infinite line for three robots one of which is Byzantine.

There is also a limited number of studies for the unit disk. In [3] the authors consider evacuation of three robots from a disk one of which may be a crash or Byzantine faulty robot. For the case of crash faults the lower bound achieved is ≈ 5.188 and the upper bound ≈ 6.309, while for Byzantine faults the lower bound is ≈ 5.948 and the upper bound ≈ 6.921. In [13] and [15] the authors consider search on a disk for n robots at most f of which may be faulty, i.e., crash or Byzantine (it turns out that search is simpler to analyze than evacuation since success is achieved when the first non-faulty robot finds the Exit.) In addition, [8] studies search for n robots in the plane with faulty robots; in this model the robots have arbitrary (not necessarily identical) max speeds and visibility ranges. Moreover, in [14] the authors study the problem on the disk with two robots one of which can have speed more than 1, while the robots can communicate wirelessly. Finally, [7] presents a survey of group search which could be useful to the reader.

The SR mixed communication model studied in our current paper was considered as a way to model group search in the presence of robots with faulty communication capabilities and was first introduced for an infinite line in [9], but otherwise has never been studied in any other domain, such as the unit disk.

1.4 Results of the paper

We show that evacuation in which one robot is a Sender and the other is a Receiver is more powerful than evacuation when both robots use F2F communication and less powerful when both robots use WiFi communication. We give upper and lower bounds for the evacuation time of two robots in the SR communication model. For the upper bound, see Section 2, we design a new evacuation algorithm which leads to evacuation time at most $\pi + 2$. The novelty of our upper bound pertains also to the asymmetry of the proposed algorithm, as well as that the robots follow trajectories in which their speed is not always 1. A similar feature was deployed previously only in the F2F model for searching on the line [4], but with an objective different than minimizing the evacuation time. For the lower bound, see Section 3, we show that there is no evacuation algorithm in the SR model whose evacuation time is equal to the optimal evacuation time in the WiFi model.

2 The Upper Bound of $2 + \pi$

In the present section we give an evacuation algorithm in the SR model and analyze its competitive ratio. The main result is the following.

▶ **Theorem 4.** *Evacuation of two robots from a disk in the Sender Receiver communication model can be accomplished in time* $2 + \pi$.

For the proof of Theorem 4 we design trajectories for the robots and show that for every placement of the Exit on the unit circle, the evacuation time is at most $2+\pi$. For convenience we think of the search domain, the unit circle, in a Cartesian system, i.e. $\mathcal{C} = \{\textsc{cycle}\,(t)\,, t \in [0, 2\pi]\}$, where $\textsc{cycle}\,(t) = (\cos t, \sin t)$ is a parametric description of the unit circle. Note that both robots start from the center of circle $\mathcal{C}$.

We give trajectories for the robots in two steps, parameterized by the same function $\alpha\colon [0,\pi] \mapsto \mathbb{R}$ (to be fixed later). For notational convenience, we occasionally write α_t instead of $\alpha(t)$. The trajectories depend on when (and if) robots find the Exit at some point $\textsc{cycle}\,(t)$. The description below uses the assumption that should the Sender locate the Exit in $\textsc{cycle}\,(t)$ then the information will be transmitted to the Receiver instantaneously and the Receiver will abandon her trajectory in order to reach $\textsc{cycle}\,(t)$ in the fastest possible way (along a line segment). The time that the Receiver will reach $\textsc{cycle}\,(t)$ will be the total evacuation time for that placement of the Exit. Therefore, the Sender's trajectory needs to be described only under the assumption that no Exit has been found by her. The trajectory of the Sender will be fully determined once we fix the aforementioned function $\alpha(\cdot)$. It is worthwhile mentioning in advance that the Sender's trajectory will not be unit speed at all times during the execution of the algorithm.

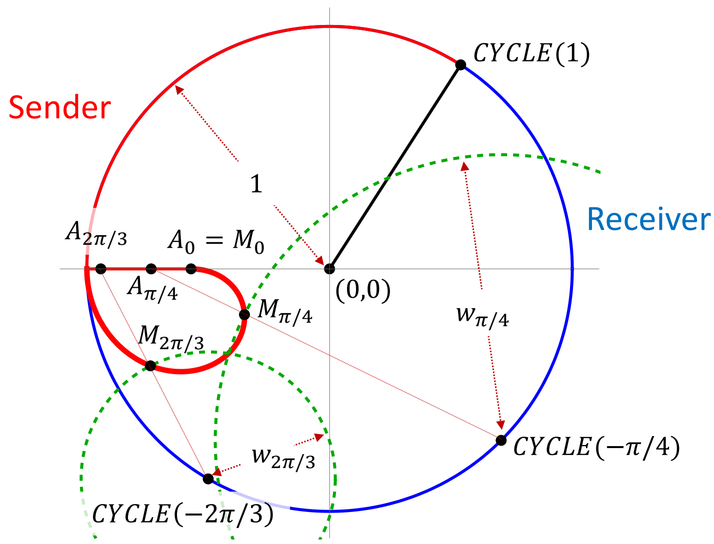


Figure 1 Robots' trajectories, for the optimal choice of parameter function $\alpha(\cdot)$. Red and blue are the Sender's and Receiver's trajectories respectively (assuming no Exit is found). The thick red curve corresponds to the Sender's partial trajectory M_t, $t = 0, \ldots, \pi$. Note that when the Sender reaches point $(-1, 0)$ she starts moving towards the center $(0, 0)$. Figure also depicts two examples of Exit placements, $\textsc{cycle}\,(-\pi/4)\,, \textsc{cycle}\,(-2\pi/3)$, along with the corresponding circles with these centers and radii $w_{\pi/4}, w_{-2\pi/3}$, along with the directive points $A_{\pi/4}, A_{-2\pi/3}$ (indicating the direction the Receiver attempts to meet the Sender), along with the intended meeting points $M_{\pi/4}, M_{-2\pi/3}$, respectively.

In contrast, the Receiver's trajectory will depend on whether she has found the Exit or not. If the Receiver finds the Exit at $\textsc{cycle}\,(t)$, then starting from $\textsc{cycle}\,(t)$ she will follow a trajectory in order to meet the Sender in her trajectory so that together they return to the Exit in time at most $2+\pi$ in total. The Receiver's trajectory will be determined uniquely once we fix function $\alpha(\cdot)$, but the Receiver will maintain a unit speed at all times (except from some placements of the Exit, in which case the Receiver will wait idle in some point in order to meet the Sender and return together to the Exit). The description of our algorithm is coupled with the illustration of Figure 1 that we explain as we present the technicalities of the algorithm.

2.1 Robots' Trajectories

We now give more formally the robots' trajectories and the main algorithm of the paper.

Algorithm 1 Sender-Receiver Search Algorithm.

1: **Sender's Trajectory (input: speed compliant SPT pair $(\mathcal{S}, \tau)$)**
2: Phase S1: At unit speed go to CYCLE (1).
3: Phase S2: At unit speed, search $\mathcal{C}$ counter-clockwise up to point CYCLE (π), or until exit is found. If exit is found at CYCLE (x) then send the Receiver the position for her to come to the exit and wait.
4: Phase S3: At unit speed go to point $(1-\pi/2, 0)$.
5: Phase S4: Reset the clock and traverse curve $\mathcal{S}$ with timing rule τ.

1: **Receiver's Trajectory (input: reserved function $\alpha(\cdot)$)**
2: Phase R1: At unit speed go to CYCLE (1).
3: Phase R2: Search $\mathcal{C}$ clockwise towards point CYCLE (π). If Exit is found at CYCLE (t), $t \in [0, 1]$, start R3. If Exit found at CYCLE ($-t$), $t \in (0, \pi]$, start R4.
4: Phase R3: Go to $(1-\pi/2, 0)$ and wait for Sender, then return to CYCLE (t).
5: Phase R4: Move toward $A_t := (\alpha(t), 0)$ for time $w_t := (\pi - t)/2$ (to meet the Sender at point M_t) and return to CYCLE ($-t$).

2.1.1 Sender's Trajectory

Let $f, g : [0, \pi] \mapsto \mathbb{R}$ be continuous and differentiable real functions defining a closed curve $\mathcal{S} := \{(f(s), g(s)), s \in [0, \pi]\}$. Consider also continuous and differentiable $\tau : [0, \pi/2] \mapsto \mathcal{S}$ (that we will call the *timing-rule* for $\mathcal{S}$). The pair $(\mathcal{S}, \tau)$ will be called SPT (Sender's Partial Trajectory) if the following conditions are satisfied:

$$\lim_{t\to 0} f(t) = 1 - \pi/2, \lim_{t\to\pi} f(t) = -1, \lim_{t\to 0} g(t) = \lim_{t\to\pi} g(t) = 0 \quad (1)$$

$$\tau(0) = \lim_{t\to 0}(f(t), g(t)), \tau(\pi/2) = \lim_{t\to\pi}(f(t), g(t)) \quad (2)$$

We think of the timing rule τ as specifying the position $\tau(t) \in \mathcal{S}$ of the Sender at time t, which is used only during the traversal of curve $\mathcal{S}$. Hence, the pair $(\mathcal{S}, \tau)$ determines uniquely the speed $\|\tau'(t)\|$ of the Sender while traversing curve $\mathcal{S}$, that takes a total time of $\pi/2$. The pair $(\mathcal{S}, \tau)$ will be called *speed compliant*, if the movement along $\mathcal{S}$ with the timing rule τ induces speed which is at most 1.

The Sender's trajectory is parameterized by a speed compliant SPT pair $(\mathcal{S}, \tau)$. Both $\mathcal{S}, \tau$ will be determined later as a function of some $\alpha\colon [0, \pi] \mapsto \mathbb{R}$. It is worthwhile mentioning that, strictly speaking, curve $\mathcal{S}$ is induced by timing rule τ, so one would only need to specify τ. Nevertheless, it is a technicality of our argument that requires first to determine $\mathcal{S}$ (as a function of $\alpha(\cdot)$), and then the timing rule $\tau(\cdot)$ (implicitly, as a function of $\alpha(\cdot)$).

The formal description of the Sender's trajectory can be found in Algorithm 1, and is also depicted in Figure 1 as the red trajectory (for a specific choice of a SPT pair $(\mathcal{S}, \tau)$). Note that the SPT pair $(\mathcal{S}, \tau)$ guarantees that point CYCLE (π) $= (-1, 0)$ is visited twice by the Sender, and what $(\mathcal{S}, \tau)$ determines is the nature of the curved trajectory after the Sender visits point $(1 - \pi/2, 0)$, denoted as A_0 in Figure 1.

Note that since $(\mathcal{S}, \tau)$ is SPT, the above trajectory is indeed feasible (it defines a continuous curve on the plane; see transition between phases S3, S4). Moreover, if $(\mathcal{S}, \tau)$ is speed compliant, then the Sender's trajectory has speed at most 1 during the execution of the move. The Sender, during phase S1, will move from the center of the circle to CYCLE (1), which takes 1 time unit. During phase S2 the sender moves counter-clockwise on the circle from CYCLE (1) towards the point $(-1,0)$, and this phase takes time $\pi - 1$. In phase S3 the sender moves from point $(-1,0)$ to the point $(1-\pi/2,0)$ which is the point A_0 as shown in Figure 1, this phase takes time $2-\pi/2$. Finally, for phase S4 the sender moves in a trajectory from point $(1-\pi/2,0)$ to reach point $(-1,0)$ for the second time. Phase S4 takes time $\pi/2$, which gives us the total time of $2+\pi$.

2.1.2 Receiver's Trajectory

A continuous and differentiable $\alpha\colon [0,\pi] \mapsto [-1,1]$ will be called *reserved* if $\alpha_0 = 1-\pi/2$, and the limit $\lim_{t\to\pi} \frac{\alpha_t - \cos t}{\sin t}$ exists and is not equal to ± 1 (the latter technical condition will be used in the Proof of Claim 6 in order to show that $\alpha(\cdot)$ induces SPT pair $(\mathcal{S},\tau)$ for the Sender). Note also that the latter condition also implies that $\lim_{t\to\pi} \alpha_t = -1$. The trajectory of the Receiver is parameterized by some reserved $\alpha(\cdot)$. Intuitively, α_t will determine its movement if the Exit is found at point CYCLE $(-t)$, when $t \in [0,\pi]$.

The formal description of the Receiver's trajectory can be found in Algorithm 1, and is also depicted in Figure 1 as the blue trajectory (for a specific choice of a reserved $\alpha(\cdot)$). According to the description of the trajectory, there are two cases in which the Receiver attempts to visit the Sender, see phases R3 and R4. Correctness of phase R3 will be shown later in Claim 8, and is independent of function $\alpha(\cdot)$. The correctness of phase R4 will be shown later in Claim 9, and will follow once we fix $\alpha(\cdot)$ that will also determine the Sender's trajectory (hence pair $(\mathcal{S},\tau)$). Also this step will be correct independently of whether $(\mathcal{S},\tau)$ is speed complaint.

Note that the total duration of the trajectory of the Receiver, under the assumption that no Exit is found, is $2+\pi$, when she meets with the Sender at point $(-1,0)$, and they arrive at that point simultaneously.

2.2 Determining a Partial Trajectory and a Timing-Rule for the Sender

In this section we show how a choice of a function $\alpha(\cdot)$, as the one used to determine the Receiver's trajectory, can also determine the Sender's trajectory and in particular phase S4 of her trajectory. More specifically, given function $\alpha(\cdot)$ we determine a pair $(\mathcal{S},\tau)$ that we also call α-induced.

Intuitively, for every $0 < t \le \pi$ we require that the Sender is at point M_t at the same time the Receiver would arrive there if the Exit was found at CYCLE $(-t)$. First we determine points M_t. As per the description of the Receiver's trajectory, if the Exit is found at some CYCLE $(-t)$, for $t \in (0,\pi]$, then the Receiver, starting from CYCLE $(-t)$, moves with unit speed for time[1] $w_t = (\pi - t)/2$ towards point A_t. Therefore point $M_t = (f(t), g(t))$ can be found as the intersection of the line passing through points CYCLE $(-t) = (\cos t, -\sin t)$ and $A_t = (\alpha_t, 0)$, and the circle of center CYCLE $(-t)$ and radius w_t, lying within the unit circle. Point M_t can be therefore determined as one of the roots of the following quadratic system with $x = f(t)$ and $y = g(t)$.

[1] This is the maximum time that the receiver can move away from the exit in order to be able to return back to it for a total time of $2+\pi$. We just need to ensure that the Sender reaches M_t at the same time the Receiver does.

$$(y + \sin t)(\alpha_t - \cos t) = (x - \cos t)\sin t$$
$$(x - \cos t)^2 + (y + \sin t)^2 = w_t^2$$

Solving the system gives one of the roots to be

$$f(t) = \cos t + \frac{(\alpha_t - \cos t)\frac{\pi - t}{2}}{\sqrt{\sin^2 t + (\alpha_t - \cos t)^2}} \tag{3}$$

$$g(t) = -\sin t + \frac{\sin t \frac{\pi - t}{2}}{\sqrt{\sin^2 t + (\alpha_t - \cos t)^2}} \tag{4}$$

The above pair is the point on the line segment with endpoints CYCLE $(-t)$, A_t, and from CYCLE $(-t)$ to the direction of A_t (the other root is its antipodal on the circle with center CYCLE $(-t)$ and radius w_t).

We now define the α-*induced* pair $(\mathcal{S}, \tau)$. We set $\mathcal{S} := \{M_t : t \in [0, \pi]\}$. Also, assuming that the Sender reaches point $(1 - \pi/2, 0)$ (i.e. no Exit is found earlier by the Sender), reset the clock for the Sender and require that for every $t \in [0, \pi]$, the Sender lies at M_t exactly at time $t/2$. Note that this defines indeed a timing-rule $\tau : [0, \pi/2] \mapsto \mathcal{S}$. Later we show that pair $(\mathcal{S}, \tau)$ is indeed SPT (i.e., Conditions 1 and 2 are satisfied) for a proper choice of $\alpha(\cdot)$.

2.3 Correctness and Performance Analysis

In this section we show that starting with a reserved $\alpha(\cdot)$, the α-induced pair $(\mathcal{S}, \tau)$ is indeed SPT, and that this guarantees that robots meet as per the description of their trajectories, as well as that the running time is at most $2 + \pi$. The results are summarized in Lemma 5.

We need to emphasize that results of this section do not touch on the Sender's speed induced by the choice of $\alpha(\cdot)$. Finding a proper $\alpha(\cdot)$ that induces a speed-compliant pair $(\mathcal{S}, \tau)$ (i.e., a pair that keeps the Sender's speed to at most 1) is the topic of the next section.

▶ **Lemma 5.** *If $\alpha(\cdot)$ is reserved, then Algorithm 1 solves the sender-receiver problem with evacuation time at most $2 + \pi$. In particular, the Receiver's move is of unit speed (while it is moving), while the Sender's move has speed (possibly more than 1) that depends on the choice of $\alpha(\cdot)$.*

The proof of Lemma 5 is given by the following claims. First we show that the α-induced pair $(\mathcal{S}, \tau)$ of Section 2.2 is SPT (and therefore α determines indeed continuous trajectories).

▷ Claim 6. If $\alpha(\cdot)$ is reserved, then the α-induced pair $(\mathcal{S}, \tau)$ is SPT.

Proof. Since α is reserved we have that $\alpha(0) = 1 - \pi/2$ and $\alpha(\pi) = -1$. We use the formulas for $M_t = (f(t), g(t))$ of Section 2.2 as a function of $\alpha(\cdot)$. A direct substitution gives

$$f(0) = 1 + \frac{(\alpha_0 - 1)\frac{\pi}{2}}{\sqrt{0 + (\alpha_0 - 1)^2}} = 1 - \pi/2,$$

since $\alpha_0 - 1 < 0$. The value $f(\pi)$ is indefinite, so we compute

$$\lim_{t \to \pi} f(t) = -1 + \left(\lim_{t \to \pi} \frac{\alpha_t - \cos t}{\sqrt{\sin^2 t + (\alpha_t - \cos t)^2}} \right) \left(\lim_{t \to \pi} \frac{\pi - t}{2} \right)$$
$$= -1 + \left(\lim_{t \to \pi} \frac{\alpha_t + 1}{\sqrt{(\alpha_t + 1)^2}} \right) \left(\lim_{t \to \pi} \frac{\pi - t}{2} \right) = -1.$$

Again, a direct substitution gives $g(0) = 0$. The next step is the only one that uses that $\lim_{t\to\pi} \frac{\alpha_t - \cos t}{\sin t}$ exists and is not equal to ± 1 (since $\alpha(\cdot)$ is reserved). We have

$$\lim_{t\to\pi} g(t) = \lim_{t\to\pi} \frac{\sin t \frac{\pi - t}{2}}{\sqrt{\sin^2 t + (\alpha_t - \cos t)^2}} = \left(\lim_{t\to\pi} \frac{\sin t}{\sqrt{\sin^2 t + (\alpha_t - \cos t)^2}}\right)\left(\lim_{t\to\pi} \frac{\pi - t}{2}\right)$$
$$= \left(\lim_{t\to\pi} \frac{1}{\sqrt{1 + \left(\frac{\alpha_t - \cos t}{\sin t}\right)^2}}\right)\left(\lim_{t\to\pi} \frac{\pi - t}{2}\right) = 0,$$

where the second to last limit exists and is some constant, due to the fact that $\alpha(\cdot)$ is reserved.

Next we study the timing-rule $\tau(\cdot)$, and we verify that $\tau(0) = (f(0), g(0)), \tau(\pi/2) = (f(\pi), g(\pi))$. Indeed, reset the clock to 0 at the time the Sender reaches point $(1 - \pi/2, 0)$. Recall that for every $t \in [0, \pi]$, τ was defined so that at time $t/2$ the Sender is at point M_t. For $t \to 0$, our previous calculations for f, g show indeed that $\lim_{t\to 0}(f(t), g(t)) = (1 - \pi/2, 0)$, which is exactly where the Sender is at time 0. Finally, for $t \to \pi$, we know that $\lim_{t\to\pi}(f(t), g(t)) = (-1, 0)$, while M_π was indeed defined as point $(-1, 0)$. This is because M_π is the attempted meeting point if the Exit is found at point CYCLE (π), in which case, by definition, M_π is $w_\pi = (\pi - \pi)/2 = 0$ away from CYCLE (π). ◁

Next note that, given the described trajectories (and independently of $\alpha(\cdot)$), the Sender searches all points CYCLE (t) with $1 \le t \le \pi$, while the Receiver searches all points CYCLE (t) with $-\pi \le t \le 1$. Now assume that the Exit is indeed in some CYCLE (t), where $-\pi \le t \le \pi$ (i.e., anywhere on the unit circle). Claim 7 below covers the case that $1 \le t \le \pi$, hence the Exit is found by the Sender. Claim 8 below covers the case $0 \le t \le 1$, hence the Exit is found by the Receiver. Lastly, Claim 9 below covers the case $-\pi \le t \le 0$, hence the Exit is found by the Receiver, not before she has spent time 1 searching on the circle. All these claims assume that the provided $\alpha(\cdot)$, determining robots' trajectories, is reserved.

In the first case, the evacuation cost is strictly less than $2 + \pi$.

▷ Claim 7. If the Exit is found by the Sender in CYCLE (t), with $1 \le t \le \pi$, then the evacuation time is less than $2 + \pi$.

Proof. Consider the Exit at point CYCLE $(1 + x)$ for some $x \in [0, \pi - 1]$. The Exit is discovered by the Sender at time $1 + x$, after spending time x moving counter-clockwise, starting from CYCLE (1). Until the Exit is found, the Receiver moves in the opposite direction, starting again from CYCLE (1). Therefore, when the Receiver's receives the message from the Sender, the two are at arc-distance $2x$, or equivalently at Euclidean distance $2\sin x$. It follows that the worst case evacuation in this case equals

$$\max_{0\le x\le \pi-1} \{1 + x + 2\sin x\} \le \pi + 2\sin 1 < \pi + 2.$$ ◁

In the second case, the evacuation cost is at most $2 + \pi$, and equality holds for selected placements of the Exit.

▷ Claim 8 (Correctness of phase R3 of Receiver's Trajectory). If the Exit is found by the Receiver in CYCLE $(1 - t)$, with $0 \le t \le 1$, then the evacuation time is at most $2 + \pi$.

Proof. As per the Receiver's trajectory, if the Exit is found in $T := \{\text{CYCLE}\,(1 - t) : t \in [0, 1]\}$, the Receiver attempts to meet the Sender in point $P = (1 - \pi/2, 0)$. Note that for the duration that the Receiver is exploring T the corresponding part of the circle, the Receiver's distance to P is increasing. Hence, the latest time that the Receiver reaches point P, over all placements of the exits that induce this move, is when the Exit is placed at $\textsc{cycle}(0)$. It is also important to note that the move is the same limiting move that the Receiver makes in order to meet the Sender if the Exit is at $\textsc{cycle}(t)$, and t tends to 0, either from the left or the right. In this case it is easy to see that the Receiver reaches point P at time $2+\pi/2$. Hence, for any placement of the Exit in T, the Receiver reaches P at time at most $2+\pi/2$, and waits for the Sender who arrives at P at time exactly $2+\pi/2$ (1 to reach the perimeter, $\pi-1$ to explore her part of the circle, and $2-\pi/2$ to reach P from $\textsc{cycle}(\pi)$). Noting that P is $\pi/2$ away from $\textsc{cycle}(0)$, and in fact at most $\pi/2$ away from any point in T, concludes the proof. ◁

In the third case, we show that the evacuation cost stays invariant (for infinite and uncountable many placements of the Exit).

▷ Claim 9 (Correctness of phase R4 of Receiver's Trajectory). If the Exit is found by the Receiver in $\textsc{cycle}(-t)$, with $0 \le t \le \pi$, then the evacuation time equals $2+\pi$.

Proof. Consider the Exit at point $\textsc{cycle}(-t)$. The Receiver arrives at the point in time $2+t$ (1 to reach the perimeter, extra time 1 to reach $\textsc{cycle}(0)$, and another t to reach $\textsc{cycle}(-t)$). Then, the Receiver spends time w_t toward the (intended) meeting point M_t, which is reached at time $2+t+w_t = 2+t+(\pi-t)/2 = 2+\pi/2+t/2$.

Now we show that the Sender arrives at M_t at the exact same time (and note that the Sender follows her trajectory without knowing the findings of the Receiver). Observe first that the Sender reaches point $M_0 = (1-\pi/2, 0)$ in time $2+\pi/2$ (1 to reach the perimeter, extra time $\pi-1$ to reach $\textsc{cycle}(\pi)$, and extra time $2-\pi/2$ to reach M_0). Since the Sender's trajectory uses the α-induced pair $(\mathcal{S},\tau)$ thereafter (see definition in Section 2.2), the Sender reaches M_t in additional time $t/2$ for a total of $2+\pi/2+t/2$, which is simultaneously with the Receiver.

Since the two robots do indeed meet at point M_t in time $2+\pi/2+t/2$, they return together to $\textsc{cycle}(-t)$ in extra time w_t, for a total of $2+\pi/2+t/2+w_t = 2+\pi$. ◁

This concludes the proof of Lemma 5.

2.4 Speed Compliant Partial Trajectory (Proof of Theorem 4)

In this section we provide the missing component toward the proof of Theorem 4, a reserved function $\alpha(\cdot)$. To this end we define $\alpha\colon [0,\pi] \mapsto [-1,1]$ as

$$a(t) := 1-\pi/2+(-2+\pi/2)\sin(t/2)$$

which also gives rise to the α-induced pair $(\mathcal{S},\tau)$ as per Section 2.2.

▶ **Lemma 10.** *Function $\alpha(\cdot)$ is reserved.*

Proof. Direct substitution shows that $\alpha(0) = 1-\pi/2, \alpha(\pi) = -1$. Moreover, using l'Hopital's rule

$$\begin{aligned}\lim_{t\to\pi} \frac{\alpha_t - \cos t}{\sin t} &= \lim_{t\to\pi} \frac{1-\pi/2+(-2+\pi/2)\sin(t/2)-\cos t}{\sin t} \\ &= \lim_{t\to\pi} \frac{\frac{1}{2}(-2+\pi/2)\cos(t/2)+\sin t}{\cos t} = 0,\end{aligned}$$

which concludes that $\alpha(\cdot)$ is indeed reserved. ◀

Since by Lemma 10 the function $\alpha(\cdot)$ is reserved, Lemma 5 applies, according to which the robots' trajectories of Algorithm 1 solve the sender-receiver problem with evacuation time at most $2+\pi$. The Receiver maintains speed 1 (except from when she is idle), while the Sender's speed is induced by $\alpha(\cdot)$. Therefore, Theorem 4 follows once we show that the α-induced pair $(\mathcal{S}, \tau)$ is speed compliant. This is done in Lemma 11 below.

We emphasize that our proof is computer-assisted in the following sense (but still rigorous). First we rely on numerical evaluations of formulae that admit closed (symbolic) forms. Specifically for our goal to show that a certain function (the one describing the Sender's speed) takes values at most 1 over a domain, we discretize the domain and we identify a "safe" subdomain where the function is bounded away from 1. Given that we show that the function is smooth enough, the provided discretization shows that the calculations are robust against (possible) numerical inaccuracies and hence the function is at most 1 in the safe subdomain. In the complementary "critical" subdomain where the function admits values close to 1, we study the monotonicity of our function. To that end, we identify points in the critical subdomain in which the function provably evaluates to 1, and then we show that these points correspond to local strict maximizers (where the locality of the argument covers the entire "critical" subdomain).

▶ **Lemma 11.** *The α-induced pair $(\mathcal{S}, \tau)$ is speed compliant.*

Proof. We show that the α-induced pair $(\mathcal{S}, \tau)$ gives rise to a movement of speed at most 1. Indeed, as per the definition of the trajectory, and for $t \in [0, \pi]$, the Sender is at point $M_t = (f(t), g(t))$ in time $t/2$. Therefore, for $t \in [0, \pi/2]$, the Sender's speed is given by

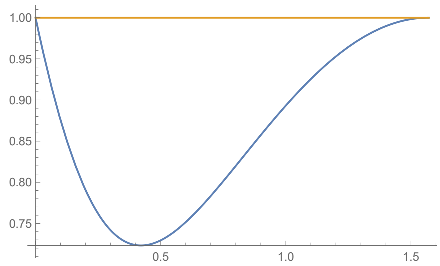
$$speed(t) := \left(\left(\frac{\mathrm{d}}{\mathrm{d}t} f(2t) \right)^2 + \left(\frac{\mathrm{d}}{\mathrm{d}t} g(2t) \right)^2 \right)^{1/2},$$

where functions f, g are given in (3), (4) (see Figure 2a for the plot of the function). We need to verify that $speed(t) \le 1$, for all $t \in [0, \pi/2]$. For this we rely on a computer assisted proof that relies on a refined enough discretization of domain $[0, \pi/2]$. By calculating $speed(t)$ exclusively on the discretized domain, and after we identify key monotinicity properties of the function, we show that indeed the speed is at most 1. Towards that direction we define functions $f_0(t) = speed^2(t), f_1(t) = f_0'(t), f_2(t) = f_1'(t)$; see Figures 2b, 2c and 2d for their plots, respectively. We will verify that $f_0(t) \le 1$, for all $t \in [0, \pi/2]$. The main idea behind our argument is that for values of t for which $f_0(t)$ is bounded away from 1, numerical (computer-assisted) evaluations of the function are robust enough to guarantee that the function is indeed at most 1. A special treatment is required for the neighborhood of values in which $f_0(t)$ attains provably value equal to 1.

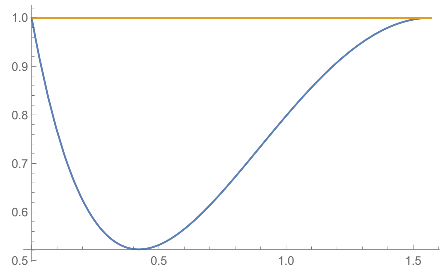
Note that all functions $f_i(t), i = 0, 1, 2$ admit closed formulae, so all arguments below that refer to Figures 2b, 2c, 2d for the sake of intuition, are actually supported by concrete symbolic evaluations of functions $f_i(t)$ in the discretized domain.

From the corresponding evaluation of $f_0(t)$ in the discretized space of $[0, \pi/2]$, we see that $f_0(t)$ is bounded away from 1 as long as t is bounded away from 0 and $\pi/2$. Indeed, by Figure 2b we have that $f_0(t) \le 0.97$ for all $t \in [0.1, 1]$. Hence, it remains to show that $f_0(t) \le 1$ also in the neighborhoods close to $t = 0$ and $t = \pi/2$, covering in particular the intervals $[0, 0.1]$ and $[1, \pi/2]$.

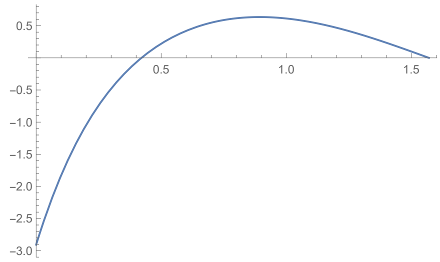
Starting with the neighborhood of $t = 0$, and with direct symbolic calculations, we see that $\lim_{t \to 0^+} f_0(t) = 1$. At the same time we see that $f_1(t) < -1$ for all $t \le 0.1$, see Figure 2c. It can be shown that the derivative of $f_1(t)$ is bounded and therefore a refined evaluation of $f_1(t)$ that shows it is negative for $t \le 0.1$ is enough to justify that $f_0(t)$ is at most 1. We



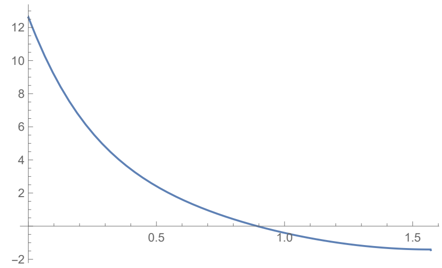
(a) The plot of function $speed(t), t \in [0, \pi/2]$.



(b) The plot of function $f_0(t), t \in [0, \pi/2]$.



(c) The plot of function $f_1(t) = f_0'(t), t \in [0, \pi/2]$.



(d) The plot of function $f_2(t) = f_0''(t), t \in [0, \pi/2]$.

Figure 2 Numerical evaluation of functions $speed(t), f_i(t), i = 0, 1, 2$ for $t \in [0, \pi/2]$.

conclude that even with the considered discretization, function $f_0(t)$ is strictly decreasing for $t \leq 0.1$. That accounts for any possible numerical inaccuracies when computing $f_0(t)$ numerically, and hence $f_0(t) \leq 1$ for all $t \leq 0.1$.

Next we study the neighborhood of $t = \pi/2$. Direct symbolic calculations show that $\lim_{t \to \pi/2} f_0(t) = 1$. However, $\lim_{t \to \pi/2} f_1(t) = 0$, so we need to follow a different argument than before. In that direction, we study $f_2(t)$, see Figure 2d. We observe that when $t \in [1, \pi/2]$, then $f_2(t) < -0.4$. Hence, we have numerical verification that $f_2(t)$ is strictly negative in $[1, \pi/2]$ (bounded away from 0), and hence $f_0(t)$ is strictly concave in the same interval. As a result, $t = \pi/2$ is a strict local maximum of $f_1(t)$, and therefore $f_0(t) \leq 1$ for all $t \in [1, \pi/2]$, concluding our argument. ◀

3 Lower Bound

We prove the following theorem which differentiates the WiFi and the SR models with respect to evacuation time.

▶ **Theorem 12.** *The evacuation time of any algorithm which accomplishes evacuation from an unknown Exit in a unit disk using the SR model is strictly bigger than* $1 + \frac{2\pi}{3} + \sqrt{3}$*, the optimal evacuation time for the WiFi model.*

Proof. The proof is based on the fact that if the evacuation time is exactly $1 + \frac{2\pi}{3} + \sqrt{3}$ then the robots must follow specific trajectories. The robots begin their search starting from the center of the unit disk. They need at least one time unit to reach the perimeter after which they begin their exploration of the perimeter in order to find the Exit. Consider the two robots at time exactly $1 + \frac{2\pi}{3}$. Together the robots can cover a length of at most $\frac{4\pi}{3}$ on the perimeter. We first prove a useful lemma which is similar to Lemma 3 in [2].

▶ **Lemma 13.** *If at time exactly* $1 + \frac{2\pi}{3}$ *for some* $x > 0$ *the total perimeter explored by the two robots is at most* $\frac{4\pi}{3} - x$ *then there is a chord on the circle of length* $> \sqrt{3}$ *none of whose endpoints has been visited by a robot.*

Proof. Assume on the contrary there is no such chord, i.e., for every chord of length $> \sqrt{3}$ at least one of its endpoints has been explored by a robot. Consider a point A on the perimeter which has not been explored by a robot. Consider two points A_1 and A_2 to the left and right of A such that we get two arcs, one starting from A_1 and moving clockwise towards A_2 without passing from A, and one starting from A_1 and moving counter-clockwise passing through A and then reaching A_2 as shown in Figure 3.

We choose points A_1 and A_2 such that every point in the arc $\widehat{A_1A_2}$ has been explored by a robot.

If there is an unexplored point B to the left of A then we can use the same reasoning as above to expand the explored arc below the arc $\widehat{A_1A_2}$. Moreover, the farther the point B from A the longer the arc. Take the point B farthest and to the left of A which is unexplored by a robot. Using the same reasoning take the point C farthest and to the right of A which is unexplored by a robot. Clearly, the arc $\widehat{BA_1A_2C}$ is fully explored by robots with the exception of the endpoints B and C. However, by the hypothesis of the lemma the arc $\widehat{BA_1A_2C}$ has length at most $\frac{4\pi}{3} - x$. It follows that $|BC| > \sqrt{3}$ but this is a contradiction since both endpoints B and C have not been explored by a robot. This proves Lemma 13. ◀

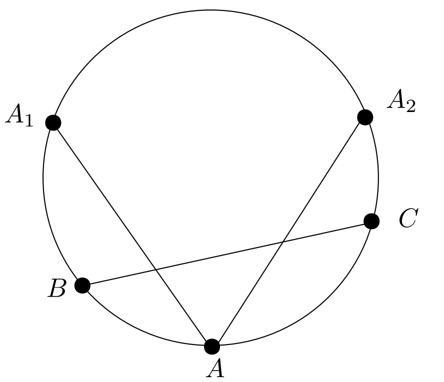


Figure 3 Robot configuration at time $1 + \frac{2\pi}{3}$. A is a point that has not been visted by any of the two robots. The arcs $\widehat{AA_1}, \widehat{AA_2}$ have length $\frac{2\pi}{3}$.

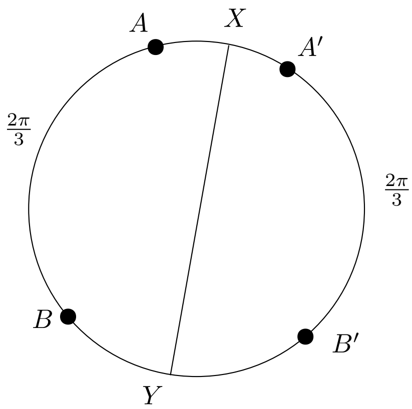


Figure 4 Two non-overlapping contiguous trajectories, forming respective arcs AB and $A'B'$, each of length $\frac{2\pi}{3}$ by the Sender and the Receiver, respectively, at time $1 + \frac{2\pi}{3}$.

Now we proceed to the main proof of Theorem 12 and to this end we consider three cases.

Case 1. The trajectories of the robots either have a nontrivial overlap or one of the trajectories is non-contiguous.
If the trajectories of the robots have an overlap of length ℓ, for some $\ell > 0$ then it is clear that in time $2\pi/3$ the robots cannot cover together more than a length of $\frac{4\pi}{3} - \ell$ of the perimeter. Therefore by Lemma 13 there is a chord of length $> \sqrt{3}$ none of whose endpoints has been explored by a robot. Similarly, if one of the robots' trajectories is not contiguous, i.e., one of the robots moves through the interior of the disk during its trajectory from one point on the perimeter to another, then it is clear that the robots cannot cover together more than a

length of $\frac{4\pi}{3} - \ell$ of the perimeter, where ℓ is the length of the arc a robot avoids by moving across the chord of this arc. Therefore again making use of Lemma 13 we see that there is a chord of length $> \sqrt{3}$ none of whose endpoints has been explored by a robot. In either case, depending on which endpoint of this chord is visited first by a robot the adversary places the Exit at the other endpoint and the additional time required will be bigger than $\sqrt{3}$. This proves the theorem in *Case 1*.

So without loss of generality from now on we may assume that both robot trajectories are contiguous (they do not skip any length on the circle) and have no non-trivial overlap. We consider two additional cases.

Case 2. If the perimeter explored by both robots by time $1 + \frac{2\pi}{3}$ is itself non-continguous. Note that each robot explores a contiguous arc of the perimeter of length $\frac{2\pi}{3}$. Moreover the two trajectories are non-overlapping as depicted in Figure 4. Hence there is a chord of length 2 none of whose endpoints has been explored by a robot. Arguing as before the resulting evacuation time will be at least $1 + \frac{2\pi}{3} + 2$. Hence the theorem is proved in *Case 2* as well.

Therefore it remains to consider the cases where the trajectories of the robots on the perimeter form a single contiguous curve of length $\frac{4\pi}{3}$. This gives rise to the following case.

Case 3. The perimeter explored by both robots by time $1 + \frac{2\pi}{3}$ is itself a contiguous arc of length $\frac{4\pi}{3}$.
There are four Subcases to consider depending on where the robots start and finish on the perimeter. If any of the following three Subcases depicted in Figure 5 occurs the evacuation time will be $1+\frac{2\pi}{3}+2$. Indeed, in each Subcase the trajectories are contiguous and overlapping

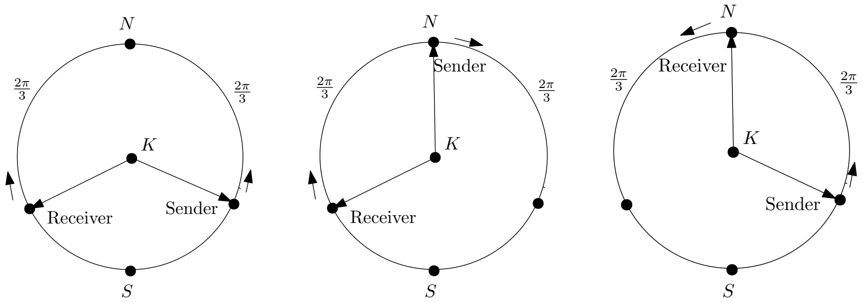


Figure 5 Three cases of possible trajectories of the two robots at time $1 + \frac{2\pi}{3}$ forming a single contiguous segment on the perimeter of total length $\frac{4\pi}{3}$.

at the single point N (at the north pole of the perimeter of the circle). Since all the points in the arc of length $\frac{2\pi}{3}$ at the bottom are unexplored we can find a point S at the bottom (namely the south pole) unexplored by any of the robots. If the adversary places the Exit at the south pole S (a point antipodal to N) either the Sender or the Receiver will have to spend an additional time 2 to evacuate. Arguing as before the resulting evacuation time will be at least $1 + \frac{2\pi}{3} + 2$. Hence the theorem is proved for these three Subcases of *Case 3* as well.

The last remaining Subcase is depicted in Figure 6 in which both robots go together to the north pole N and then traverse the perimeter for a time $\frac{2\pi}{3}$. In this final case we will make use of the fact that the Receiver cannot send messages wirelessly. Without loss of generality let the Receiver follow the left arc NX and the Sender the right arc NY. Consider the two robots at an additional time $\frac{\sqrt{3}}{2}$. On the one hand, if the Sender robot does not reach the midpoint M of the chord XY then the adversary can place the Exit at a suitable point close to X in the arc XSY so that evacuation takes additional time which is more

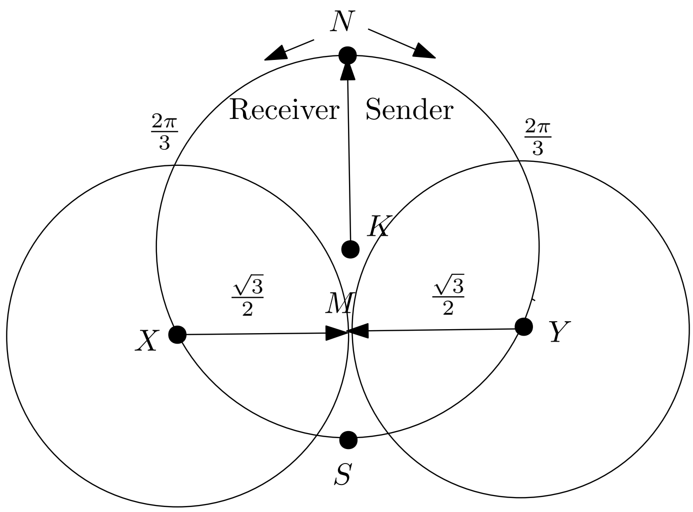


Figure 6 Robot configuration at time $1 + \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$. The robots meet at the midpoint M of the chord XY.

than $\frac{\sqrt{3}}{2}$ thus proving that the evacuation time exceeds $1 + \frac{2\pi}{3} + \sqrt{3}$. On the other hand, if the Sender reaches the point M it cannot have explored any point in the arc XSY other than the point Y. Therefore a similar reasoning applies if at the same time the Receiver does not reach the midpoint M of the chord XY: the adversary can place the Exit at a suitable point close to Y in the arc XSY so that evacuation takes additional time which is more than $\frac{\sqrt{3}}{2}$ thus proving again that the evacuation time exceeds $1 + \frac{2\pi}{3} + \sqrt{3}$.

So without loss of generality we may assume that in time exactly $1 + \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$ both robots meet at the point M without having explored any other part of the perimeter other than the arc XNY. Moreover, it is clear that none of the robots has explored any point on the arc XSY. Consider a point X' close to X on the arc XSY which is at distance ϵ from X for some $\epsilon > 0$ sufficiently small. The adversary places the Exit at one of the endpoints of the chord $X'S$ and the evacuation will require additional time $|MS| + |X'S| - \epsilon$ which is clearly at least $\frac{\sqrt{3}}{2} + \frac{1}{2} - \epsilon$. To see this, depending on which of the endpoints of the chord $X'S$ is visited first by a robot the adversary places the Exit at the other endpoint. Therefore in this case the evacuation time exceeds $1 + \frac{2\pi}{3} + \sqrt{3} + \frac{1}{2} - \epsilon$. This gives the proof in this case as well and the proof of Theorem 12 is now complete. ◀

4 Conclusion

In this paper we considered the evacuation problem for two robots in the SR model from an unknown Exit placed on the perimeter of the unit disk. We proved upper and lower bounds for evacuation and showed that the evacuation time of our algorithm is at most $\pi + 2$. Closing the gap between the lower and upper bounds remains a challenging open question. The mixed communication model considered is interesting for multiple robots. For example, suppose we have n robots all of which can communicate F2F while s among the n are senders (can only send wirelessly) and the remaining r are receivers (can only receive wirelessly) such that $n = s + r$. Nothing is known about the optimal evacuation time in this more general model. (Note that if a sender and a receiver join and move together as one robot then they behave as a fully wireless robot.) In our study, the communication status of the robots (i.e., either sender or receiver) is known in advance. An interesting extension worth considering is when the communication status of the robots may be set by an adversary.

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