# Phase Transitions and Emergent Phenomena in Random Structures and Algorithms*

**Dana Randall**

**School of Computer Science, Georgia Institute of Technology, Atlanta, GA, USA**
randall@cc.gatech.edu

## Abstract

Markov chain Monte Carlo methods have become ubiquitous across science and engineering to model dynamics and explore large sets of configurations. The idea is to perform a random walk among the configurations so that even though only a very small part of the space is visited, samples will be drawn from a desirable distribution. Over the last 20 years there have been tremendous advances in the design and analysis of efficient sampling algorithms for this purpose, building on insights from statistical physics. One of the striking discoveries has been the realization that many natural Markov chains undergo phase transitions, whereby they change from being efficient to inefficient as some parameter of the system is modified, also revealing interesting properties of the underlying random structures.

We will explore how phase transitions can provide valuable insights in three settings. First, they allow us to understand the limitations of certain classes of sampling algorithms, potentially leading to faster alternative approaches. Second, they reveal statistical properties of stationary distributions, giving insight into various interacting models. Example include colloids, or binary mixtures of molecules, segregation models, where individuals are more likely move when they are unhappy with their local demographics, and interacting particle systems from statistical physics. Last, they predict emergent phenomena that can be harnessed for the design of distributed algorithms for certain asynchronous models of programmable active matter. We will see how these three research threads are closely interrelated and inform one another.

The talk will take a random walk through some of the results included in the references.

**1998 ACM Subject Classification** F.2 Analysis of Algorithms and Problem Complexity, G.2.1. Combinatorics/Counting Problems, G.3. Probability and Statistics – probabilistic algorithms (including Monte Carlo)

**Keywords and phrases** Markov chains, phase transitions, sampling, emergent phenomena, programmable matter

**Digital Object Identifier** 10.4230/LIPIcs.DISC.2017.3

**Category** Keynote talk

## References

1 M. Andres Arroyo, S. Cannon, J.J. Daymude, D. Randall, and A.W. Richa. A stochastic approach to shortcut bridging in programmable matter. In *Proc. 23rd International Conference on DNA Computing and Molecular Programming* (DNA), 2017.

2 P. Bhakta, S. Miracle, and D. Randall. Clustering and mixing times for segregation models on $\mathbb{Z}^2$. In *Proc. 25th ACM-SIAM Symposium on Discrete Algorithms* (SODA), 2014.

* This work was supported in part by NSF CCF-1526900 and CCF-1637031.


31st International Symposium on Distributed Computing (DISC 2017).
Editor: Andréa W. Richa; Article No. 3; pp. 3:1–3:2
Leibniz International Proceedings in Informatics
LIPICS Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

3. A. Blanca, Y. Chen, D. Galvin, D. Randall, and P. Tetali. Phase coexistence for the hard-core model on $\mathbb{Z}^2$. *Submitted*, 2017.
4. S. Cannon, J.J. Daymude, D. Randall, and A.W. Richa. A markov chain algorithm for compression in self-organizing particle systems. In *Proc. 2016 ACM Symposium on Principles of Distributed Computing* (PODC), pages 279–288, 2016.
5. E. Lubetzky and A. Sly. Critical ising on the square lattice mixes in polynomial time. *Communications in Mathematical Physics*, 313:815–836, 2012.
6. F. Martinelli. Lectures on glauber dynamics for discrete spin models, lectures on probability theory and statistics. Lecture Notes in Math., page 93–191, 1999.
7. S. Miracle, D. Randall, and A. Streib. Clustering in interfering binary mixtures. In *Proceedings of the 14th International Workshop and 15th International Conference on Approximation, Randomization, and Combinatorial Optimization: Algorithms and Techniques* (RANDOM), pages 652–663, 2011.
8. D. Randall. Rapidly mixing markov chains with applications in computer science and physics. *Computing in Science and Engineering*, 8:30–41, 2006.
9. R. Restrepo, J. Shin, P. Tetali, E. Vigoda, and L. Yang. Improving mixing conditions on the grid for counting and sampling independent sets. *Probability Theory and Related Fields*, 156:75–99, 2013.