

# Fast and Powerful Hashing Using Tabulation<sup>\*†</sup>

Mikkel Thorup

Department of Computer Science, University of Copenhagen, Copenhagen,  
Denmark

mikkel2thorup@gmail.com

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## Abstract

Randomized algorithms are often enjoyed for their simplicity, but the hash functions employed to yield the desired probabilistic guarantees are often too complicated to be practical. Here we survey recent results on how simple hashing schemes based on tabulation provide unexpectedly strong guarantees.

*Simple tabulation hashing* dates back to Zobrist [1970]. Keys are viewed as consisting of  $c$  characters and we have precomputed character tables  $h_1, \dots, h_q$  mapping characters to random hash values. A key  $x = (x_1, \dots, x_c)$  is hashed to  $h_1[x_1] \oplus h_2[x_2] \dots \oplus h_c[x_c]$ . This scheme is very fast with character tables in cache. While simple tabulation is not even 4-independent, it does provide many of the guarantees that are normally obtained via higher independence, e.g., linear probing and Cuckoo hashing.

Next we consider *twisted tabulation* where one character is “twisted” with some simple operations. The resulting hash function has powerful distributional properties: Chernoff-Hoeffding type tail bounds and a very small bias for min-wise hashing.

Finally, we consider *double tabulation* where we compose two simple tabulation functions, applying one to the output of the other, and show that this yields very high independence in the classic framework of Carter and Wegman [1977]. In fact, w.h.p., for a given set of size proportional to that of the space consumed, double tabulation gives fully-random hashing.

While these tabulation schemes are all easy to implement and use, their analysis is not.

This keynote talk surveys results from the papers in the reference list.

**1998 ACM Subject Classification** E.1 [Data Structures] Tables, E.2 [Data Storage Representations] Hash Table Representations, F.2.2 [Analysis of Algorithms and Problem Complexity] Nonnumerical Algorithms and Problems – Sorting and Searching, H.3 [Information Storage and Retrieval] Information Search and Retrieval – Search Process

**Keywords and phrases** Hashing, Randomized Algorithms

**Digital Object Identifier** 10.4230/LIPIcs.ICALP.2017.4

**Category** Invited Talk

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\* Research partly supported by Advanced Grant DFF-0602-02499B from the Danish Council for Independent Research.

† A similar talk abstract also appears in *Proceedings of the 36th IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS)*, pages 1:1–1:2, 2016 – <http://dx.doi.org/10.4230/LIPIcs.FSTTCS.2016.1> –, and in *Proceeding of the 14th ACM/SIGEVO Conference on Foundations of Genetic Algorithms (FOGA)*, page 1–1, 2017.



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44th International Colloquium on Automata, Languages, and Programming (ICALP 2017).

Editors: Ioannis Chatzigiannakis, Piotr Indyk, Fabian Kuhn, and Anca Muscholl;

Article No. 4; pp. 4:1–4:2



Leibniz International Proceedings in Informatics

LIPICs Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany



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