Cognitive level in problem segments and theory segments

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Abstract: *Problems play an important role in mathematics instruction and are therefore frequently seen as central points of application for measures of instructional development.*
The research project "Quality of instruction and mathematical understanding in different cultures" examines the cognitive level of practice problems and theory problems in a three-lesson unit on the Introduction to Pythagorean theorem[1]*: Analogously to the TIMSS 1999 video study, a differentiation was made between the cognitive level of problem statement and the cognitive level of problem implementation. Additionally, the lesson time was also divided into practice and theory segments. The results show that teachers with a high proportion of connection activities in practice segments do not necessarily also spend a greater proportion of time on an analogous level for theory.*

ZDM-Classification: C33, D43

1. Theoretical Background

The presentation of theory is structured in two sections: First, problems will be described in general terms as central components of mathematics instruction (1.1). In a second part, problems will be analysed with regard to the thinking activities required (1.2).

1.1 Activities in mathematics instruction

With recourse to the TIMSS 1999 video study (Hiebert et al., 2003; Jacobs et al., 2003) and to the works of Aebli (1994), three didactic functions can be distinguished that mark the different segments in mathematics instruction: The solving of practice problems, the development of theory with problems, and the development of theory without problems. In the current work, these three functions will be used to divide the instruction time into "theory segment" and "practice segment".
A typical feature of mathematics instruction is working on problems. In different international studies, it became clear that most of the instruction time is spent working on problems: According to the TIMSS 1999 video study (Hiebert et al., 2003), this aspect amounts to more than 80% of the instruction time.
But what actually are "*problems*"? The definition used in the current study traces back to that of the TIMSS 1999 video study. "Problems comprise an explicit or implicit problem statement, which contains an unknown aspect that has to be determined by using mathematical operations or mathematical thinking" (Hugener, Pauli & Reusser, 2006, p. 67). It is therefore important to take into account that in this article, "problems" cannot be equated with problematic, cognitively demanding and complex tasks. Simple tasks that can be solved through routine operations are also deemed as problems. The term "problems" used here should therefore be distinguished from the term "Problems" used in current German-language mathematical didactics.
In the framework of entire teaching and learning processes (Aebli, 1994), working on problems serves purposes both of building up new knowledge, and working through, practising and applying previously held knowledge.
Accordingly, two types of problems can be distinguished in the sense of the definition presented above, which are called in the following "theory problems" and "practice problems":

1. "*Theory problems*": Problems for constructing new knowledge (cf. problem-based construction of cognitive structures by Aebli, 1994). With the help of these problems, an unknown concept is first developed or discovered. This concept can be developed both in classwork segments and discovered independently by the students.

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2. "*Practice problems*": Problems for practising (in the sense of making automatic, ability to carry out quickly), consolidating, and applying and transferring *previously held knowledge*.

In the TIMSS 1999 video study (Jacobs et al., 2003), the complete instruction time was divided into "problem" and "non-problem" segments. Based on the aforementioned theoretical considerations, in the current study, a three-segment classification of the complete instruction time was carried out (cf. Hugener et al., 2006). The mathematical work was divided into the following activity facets (In detail, the classification is more complex than described here. For reasons of readability and comprehensibility, the complete coding method is not presented here. The exact method can be found in Hugener et al. (2006)):

1. Practice problem: In the aforementioned sense, typical practice problems are solved.
2. Theory: New terms and rules are introduced or repeated without problem statements. This facet also includes the formulation of a mathematical theorem in various forms of representation, the proving of Pythagorean theorem and the discussion of historical aspects.
3. Theory problems: With the help of these problems, the unknown Pythagorean theorem is developed or discovered for the first time. This can be developed both in classwork and occur independently through the students. Theory problems take on an intermediate position between practice problems and theory.

In this article, the facets of theory and theory problem will be combined. The complete instruction time will be divided into the following two types of segment:

"*Practice segments*" are segments in which practice problems are worked on.

"*Theory segments*" are instruction segments in which either theory problems are worked on or else the theory is developed without problems. The theory segments are consequently defined differently than the non-problem segments in the TIMSS 1999 video study.

The following table 1 gives an overview of the relevant terms used in this article.

It is important to keep in mind that during the teaching of a mathematical theme, theory segments and practice segments can alternate repeatedly.

Problems are solved	yes			no
Activities are related to	practice problem		theory problem	theory
This part of lesson segment is called	practice segment		theory segment	
Cognitive level coded for	each *problem statement* and the relating *segment*	each *problem implementation* segment or sub-segment	the whole *theory implementation* segment (no coding of problem statement)	

Table 1: Overview of terms

1.2 Problems and the required cognitive activities

On a theoretical basis, it is assumed that different types of problems can trigger different types of thinking processes and consequently influence the learning of the students (see also Hiebert & Wearne, 1993). It is assumed that students think more deeply about mathematics if they are working on challenging problems and when a demanding subject-based discussion occurs in class (e.g. De Corte, Greer & Verschaffel, 1996; Klieme & Reusser, 2003). It is also assumed that problems with higher cognitive demands are not only of importance for acquiring competences in themselves, but also comprise a considerable potential for achieving overall and general learning goals (e.g. Schoenfeld, 1985; Winter, 1995; Wittmann, 1981) The problem-based mathematical didactical research was given an essential impulse by the TIMSS 1995 and 1999 video studies.

In the TIMSS 1999 video study (Hiebert et al., 2003; Jacobs et al., 2003), a distinction was made between different types of problem statements:

1. Using procedures: A problem statement that implies the problem would be solved by applying one or more procedures.

2. Stating concepts: Problem statements that suggest that the problem was typically solved by remembering mathematical properties or definitions or an example of a mathematical concept.
3. Making connections: A problem statement that called for construction relationships among mathematical ideas, procedures and facts. Mathematical reasoning is often observable. These problems cannot be solved only by applying a known procedure or a concept.

Problem statements in mathematics instruction form the framework for the mathematical activities in the instruction: They initiate learning activities and create opportunities for using those thinking processes that are ideally necessary and are achieved for the implementation of the problem.

However, the problem statements do not determine the observable activities in the instruction or the cognitive level actually achieved in their implementation to a sufficient extent. As the TIMSS 1999 video study (Hiebert et al., 2003) showed, and also Stein and Lane (1996) as well as Stein, Grover and Henningsen (1996), the potential of a problem statement is not necessarily also exhausted in the following implementation segment. For example, a teacher can transform a problem statement that would typically require problem-solving skills into a purely routine problem in which he or she reveals to the learners the core of the solution at the beginning of the solving process, meaning that the learners only then need to carry out routine procedures.

For this reason, the distinction between "*problem statement*" as an actual problem and "*problem implementation*" as its realisation in the instruction is important.

The two TIMSS video studies revealed that the participating countries differ considerably both in terms of problem statement and in terms of problem implementation (Hiebert et al., 2003). A palpable difference concerned, for example, the implementation of problems at the making-connections level. For instance, it was apparent that in the USA, problems at the making-connections level were implemented much more often at a procedure level than in the highly achieving countries. At the same time, the proportion of making-connections problems, which were also implemented at a connection level, was practically 0% in the USA.
In Germany, too, in the course of the international school achievement study TIMSS, the quality, the level and the complexity of problems were queried and critically examined (cf. Blum & Neubrand, 1998; Klieme, Schümer & Knoll, 2001; Knoll, 2003; Neubrand, 2002). This critical reflection was given added fuel by the results of the PISA study, according to which from an international comparison, German students find it particularly difficult to solve difficult and complex problems correctly, while they produced comparative achievements in problems that required stronger procedural and technical skills (Knoche et al., 2002).

In summary: From a theoretical perspective, problems that stimulate problem-solving and connective thinking seem to be particularly relevant for the development of mathematical and general competences, while problems that focus on remembering and applying procedures and concepts appear to be of rather minor importance in terms of demanding mathematical learning processes. In this respect, it can be suggested that in different types of problems, different "cognitive levels" of problems can be found.

2. The project "Quality of instruction and mathematical understanding in different cultures"

This study is embedded in the project "Quality of instruction and mathematical understanding in different cultures", which is being conducted by the German Institute for Educational Research in Frankfurt/Germany and by the Institute of Education of the University of Zurich/Switzerland (e.g. Klieme & Reusser, 2003).

The project group has addressed the main question of how and which instructional features impact the development of achievement and motivation of secondary school students.

In order to answer this question, we combined a micro-genetic research approach with a longitudinal one. The micro-genetic research approach examined the impact of instructional quality in two micro-modules. One module focused on the "Introduction to Pythagorean theorem", while the other related to word problems. The data presented here relate to the

"Introduction to Pythagorean theorem". In 19 classes of the 9th grade (Germany) and 20 classes of the 8th grade (Switzerland), three lessons were videotaped. The videotaped teachers took part in the study voluntarily, meaning that on the whole, it should be assumed that they were a particularly motivated group of teachers. Accordingly, the concern is not with a representative sample of teachers.

The recordings were made with two cameras: One was moveable and was focused on the teachers, while the other was fixed on the class. The video codings presented below all refer to the recordings from the teacher camera. In addition to the video recordings, further data were collected with various different instruments (interview, questionnaire, achievement test etc.). However, these are not relevant to the analyses in this article.

A main emphasis in the video study is given to a more subject-related analysis of the videotaped lessons. On the one hand, we examined the level of problem statement and problem implementation, as reported here; on the other hand, we analysed the quality of theory segments of mathematical instruction focusing on relevant concepts of Pythagorean content, on quality of proof phases, on coherence of mathematical content and adaptivity of teachers' actions.

3. Research questions

Based on the theoretical reflections presented above, this article addressed the following questions:

- For how long were problems of different cognitive levels worked on in the framework of the lesson unit on Pythagorean theorem? On what cognitive level do problem implementation and theory implementation actually occur? (cf. 5.1)
- Are there any relationships between the cognitive level of problem statements and the cognitive level of problem implementation in practice segments on the one hand and the cognitive level of theory segments on the other hand? (cf. 5.2)

4. Method

In this section, the methodological procedure will be elucidated in terms of its main features: The database (4.1) and the video analysis (4.2).

4.1 Database

In order to answer the research questions, it was possible to include 37 of the 39 classes. Of the 37 classes, 18 come from Germany (9 Gymnasium [higher-track secondary school]; 9 Realschule [medium-track secondary school]) and 19 from Switzerland (3 Gymnasium [higher-track secondary school]; 16 Sekundarschule [medium-track secondary school]).

For several reasons, possible country differences are not examined here. Firstly, the concern is with a small, non-representative sample of classes. Secondly, the classes are not equally distributed across the two school types included, meaning that it cannot be ruled out that putative country differences are attributable to differences between the school types.

4.2 Video analysis

In this section, the method of the video analyses will be presented. The terms presented in the following such as "theory segments" and "problem segments" as well as "problem statement" and "problem implementation" were introduced in section 1.1 (see also table 1).

The section is structured as follows: First of all, it is briefly shown how, in the available video data, on the one hand the practice segment and theory segment and on the other hand the problem statement and problem implementation segments were identified. Because the cognitive level within the practice segments and theory segments is captured differently, the description of this coding step is provided in the following in two separate sections.

As the method is fairly complex, table 2 shows, as an advanced organizer, an overview of the associations.

Differentiation of practice segments and theory segments

On the basis of available codings, the practice and theory segments could be formed as in section 1.1 (cf. Hugener et al, 2006). For the current evaluations, a small number of segments

were excluded in which the Pythagorean theorem was not worked on.
Of importance is the above-mentioned central difference to the TIMSS 1999 video study: problem statements with the help of which Pythagorean theorem was discovered or derived belong to the theory segment.

	Types of cognitive level	Cognitive level is coded for
Practice segment (Low-inference)	Problem statement • Connection • Concept • Procedure • (Rest)	Each single problem statement segment
	Problem implementation • Connection • Concept • Procedure • Results only • (Rest)	Each identifiable sub-segment of a single problem implementation segment
		Cognitive level is rated for
Theory segment (High-inference)	Theory implementation • Connection and concept general • Connection and concept with student participation	The whole theory segment: The rating scale captures the relative proportion of time in terms of the total theory time.

Table 2: Survey of the coding system of cognitive level

Delimitation of problem statement and problem implementation in the practice segments

As the cognitive level[2] of the problem statement does not have to correspond with the cognitive level of the respective implementation, we undertook the following distinction: "*problem statement*" means the actual stating of the problem, while "*problem implementation*" refers to the following work on the problem (cf. section 1). The terms "problem statement segment" and "problem implementation statement" both describe the time span in which a certain problem statement or concurrent problem statement is worked on. The difference lies in the fact that with the problem statement segment, only the cognitive level of the problem statement was coded, while with the problem implementation segment, the actual implementation was recorded.
The coding of problem statement and problem implementation in the practice segment requires information about the time points at which problems were solved. Corresponding codings were carried out in an earlier stage of analysis (cf. Hugener et al., 2006). At the same time, analogously to the TIMSS 1999 video study, it was also recorded whether the concern was with a) several concurrent problem statements consisting of several practice problems, which the learners are given during a problem implementation segment simultaneously to the implementation, or with b) independent problem statements. In terms of the implementation of concurrent problems, in the current study it was only possible to determine the beginning of the implementation of the first problems of the block, but not the time point at which the students finished the respective problems of the block. For independent problems, by contrast, the beginning and end of the problem implementation was the same for all students and could accordingly be clearly determined. This results in the fact that with concurrent problems, it was not possible to determine the corresponding implementation code for each concurrent problem statement individually, but only generally for all corresponding problem statements (cf. below).
In addition, it was possible to draw on the social form (Hugener et al., 2006) both for concurrent and for independent problems, meaning that a distinction can be made regarding how long the concurrent or independent problems were worked in classwork or in seatwork.

Coding of the cognitive level of the practice segment

Problems can, as illustrated in section 1, be set and implemented on different levels. Here, only

[2] Please note that the cognitive level of problem statement and problem implementation reported here are not the same variables as the "cognitive activation of students" reported in other publications of the project (e.g. Hugener et al., 2006).

the cognitive level is of interest. The general degree of difficulty, the linguistic level and further conceivable distinctions were not considered.

The following rules formed the basis of the coding:

1. Each problem statement that was implemented or at least mentioned in the teaching of the three Pythagoras lessons was assessed with regard to its cognitive level. Furthermore, for each individual problem statement, it was determined for how long it was implemented. Analogously to this, if several problems were given to the students set at the same time working on all these so-called concurrent problems.
2. Generally speaking, every single problem statement or every concurrent problem statement is followed by a corresponding problem implementation segment. The cognitive level of this problem implementation is also coded. It is important that this does not have to correspond with the level of the problem statement. If the cognitive level of the implementation in the problem implementation segments does not change, only one code is allocated for the whole implementation segment. If the cognitive level on which the problem is implemented changes during the implementation, the time point of the change is determined. Each sub-segment contains a code.

The method we applied is oriented towards a system of analysis that was developed in the framework of the TIMSS 1999 video study (cf. Hiebert et al., 2003, p. 97 ff; Jacobs et al., 2003). However, our method also goes beyond the TIMSS system of analysis on several points: In contrast to the TIMSS system, it plays no role here whether a public discussion of the problem took place in the instruction or not. Furthermore, for Jacobs et al. (2003), the whole problem implementation segment formed the coding unit, i.e. only one cognitive level was allocated per problem implementation, while we divided the problem implementation segment into individual sub-segments, provided observable changes in the cognitive level occurred.
Therefore, both the problem statements and the resulting problem implementation segments were assessed in terms of their cognitive level. In the following, the two codings will be presented in greater detail separately.

Coding of problem statement in practice segments

In the TIMSS 1999 video study (cf. Hiebert et al, 2003; Jacobs et al., 2003), the problem statements were divided into the categories “making connections”, “stating concepts” and “using procedures” (cf. section 1.2). We adopt these categories in terms of their fundamental ideas, but change the coding on some essential points, as is shown below. For purposes of delimitation and simplification, we therefore use the following categories:

- *Connection* instead of “making connections”,
- *Concept* instead of “stating concepts”,
- *Procedure* instead of “using procedures”.

The categories were supplemented with a rest category, which contains further codes on non-mathematical work or work that cannot be coded.

In the following, examples for the three categories of problem statements are presented:

Connection: Calculate the height of an equilateral triangle with a side length of 12 cm.
Reason: Provided that the formula is not known, in order to solve this problem, a suitable right-angled triangle within the equilateral triangle has to be recognised as a central element. This is a connection activity.

Concept: Formulate the Pythagorean theorem in words.
Reason: Provided that the linguistic formulation of the theorem is known, the theorem has to be recalled and reproduced here. No connection is necessary.

Procedure: Calculate in the triangle ABC with Gamma = 90° the hypotenuse c from the legs a = 6.2 cm and b = 8.4 cm.
Reason: If the formula is known, the numerical values can be directly put in and calculated.

Coding was carried out with the help of the available problem sheets. Each problem statement segment is defined as the phase in the instruction in which the corresponding independent problem is implemented or the corresponding concurrent problem statements are

implemented. Each of these segments contains a code.

Segments in which the students are working simultaneously on several concurrent problems received, in a multi-step procedure, the highest occurring code of all individual problem statements in the following order: 1. Connection, 2. Concept, 3. Procedure. If the teacher therefore sets several problems at the same time, which have both a connection level and a concept level, then the category of connection is allocated. The precise procedure and the differences to the coding of the TIMSS 1999 video study (cf. Jacobs et al., 2003) can be found in Hugener et al. (2006).

All identical problem statements receive the same code. The order was therefore not taken into account. The inter-rater reliability of the raters, who were working independently of one another, amounted to 100%. For the evaluations, the number of problems was not included in the calculations, as was the case with Jacobs et al. (2003), but rather the time in which the corresponding problem statement was worked on, as we assume that the time duration of the problem implementation plays a greater role for the students' learning than the number of problems implemented.

Coding of problem implementation in practice segments

Each problem implementation segment is defined as the temporal phase in the instruction in which the corresponding independent problem statement is implemented or the corresponding concurrent problem statements are implemented. (It should be noted that although for each individual problem statement, the corresponding problem statement segment and the corresponding problem implementation segment are of the same length, their coding is described somewhat differently: For the cognitive level of problem statement segment, the potential of the problem statement is coded, while for the problem implementation segment, by contrast, its actual implementation in the lesson is coded.)
The coding of the problem implementation was based on the same categories as the coding of the problem statements: connection, concept and procedure:

Connection: Explicit mathematical links between concepts, examples or principles are produced. Justification, generalisation, and technical argumentation are carried out. Concepts and procedures are used here as tools for argumentation. They are therefore not merely defined or carried out in a recipe-like fashion. Examples: The development of the geometric importance of Pythagorean theorem, the justification of a proof step, the construction of an equation, the recognition of right-angled triangles in complex figures.

Concept: Mathematical terms or properties that are already known are recalled. Examples: Providing the formula for Pythagorean theorem or the definition of a square.

Procedure: Known routine operations are carried out. No links are made. Examples: Solving of equations, calculations etc.

In addition, analogously to the TIMSS 1999 video study, the category "Results only" was added, which records as a special case of problem implementation the mere naming of the solution.
The actual coding of the problem implementation segments was more complex and is described in Hugener et al. (2006).
With the help of video recordings, the time segments were coded in which work was carried out on a particular level.
In public class instruction, the time in which one problem (or several problems in a concurrent problem implementation segment) was worked on could be divided into several different problem implementation sub-segments, as long as the cognitive level changed visibly. This was the case, for example, if on the connection level first of all an equation was constructed, which was then solved with the help of routine procedures.
In the student work segments, only one implementation code was allocated, as changes in the cognitive level could not be clearly determined.
In contrast to the problem statements, the problem implementation segments were coded according to the method of consensus. The exact method is described in Hugener et al. (2006).

Examples of a problem statement with three different corresponding problem implementations

A fictitious but realistic example aims to show how differently the same problem can be imple-

mented (fig. 1). On the vertical axis, the time duration is given, and on the horizontal axis, three teachers are represented who have implemented the same problem statement:

- Teacher A begins with a short connection segment, followed by a procedure segment.
- Teacher B uses, in total, more time for implementing the same problem. The implementation ensues exclusively on the connection level.
- Teacher C uses the most amount of time for the problem implementation. She switches repeatedly between the procedure and concept levels.

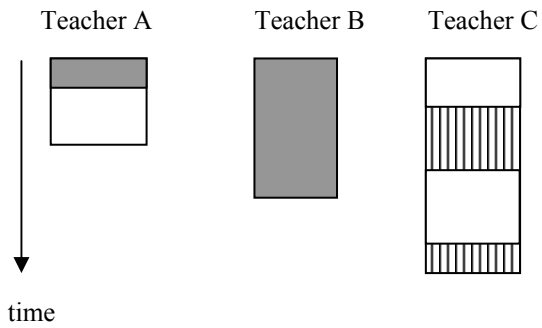

Figure 1: Different teachers' work with students on the same problem statement (Legend: dark: connection; hatched: concept; white: procedure)

Rating of the cognitive level of the theory segment

In this article, the cognitive level of the problems and their implementation is examined not only in the practice segment, but also in the theory segment. The cognitive level of the theory segment was measured fundamentally differently to that of the practice segment: In the theory segments, problems are worked on only in part (cf. table 1). Conversely, exactly like in the practice segments, mathematical work takes place during the whole of the theory segment. For this reason, in the theory segments, only the implementation of the theory was coded, and not the problem statement segments (cf. table 2). In addition, the coding method differed: In contrast to the problem segments, the coding ensued more in a high-inference way (on the terms high and low inference, cf. Hugener, Rakoczy, Pauli & Reusser, 2006). In concrete terms, first the whole theory segment was observed on the video, and then one rating was given for the whole segment. More precisely, the following was rated:

1. The proportion of the total theory time in which work was carried out on the connection and concept level;
2. The proportion of the total theory time in which students participated in activities on the connection and concept level.

The proportion of connection and concept was, in this regard, assessed together, as the levels could not be separated. The lack of discriminatory power of the two segments can be most likely explained through the fact that in the introduction of concepts, connection activities are always required also.

Ratings were fixed on a five-point scale from (1) 0%, (2) "less than 25%", (3) "less than 50%" to (4) "less than 75%" and (5) "less than 100%". Intermediate stages were also permitted. For both assessments, 100% represents the total theory time. The theory segments in the 37 classes were rated in summary independently by two raters. The generalization coefficient for "the proportion of theory time in which the students participated in connection and concept activities" amounted to .79, and for the variable "proportion of theory time in which work was carried out on the connection and concept level", it lay at .63.[3] Building on the individual ratings of the two raters, a method of consensus rating was additionally carried out. The exact method can be found in Hugener et al. (2006).

5. Results

The results are presented in two sections: In the first section (5.1), the results of descriptive analyses on the proportion of theory and practice time, on the cognitive level of practice segments and on the cognitive level of theory segments are presented. In the second section (5.2), the associations between the cognitive level in the theory segments and the practice segments are described.

5.1 Descriptive analysis

Proportion of practice and theory segments

[3] The generalization coefficient is interpreted analogously to the kappa coefficient. From .65, a satisfactory inter-rater reliability can be assumed.

The following table (table 3) displays the proportion of time in the three lessons in which practice problems on Pythagoras were worked on and theory was introduced. Table 3 illustrates that on average, only slightly over one third of the complete instruction time was used for implementing practice problems, while 56.3 % of the time was used for theory.

	Mean	Standard deviation	Extremes	
Practice segment	37.4	21.3	5.0	81.6
Theory segment	56.3	21.5	15.2	90.7

Table 3: Proportion of practice and theory segments (in percentage of total lesson time, *N = 37 classes*)

The proportion of time in which practice was carried out is therefore clearly lower than in the TIMSS 1999 video study (Hiebert et al., 2003). This lies above all in the definition of "problem segment", which is more broadly conceived in the TIMSS 1999 video study than the definition of "practice segment" in the current analyses. In addition, the concern with the lessons examined here is with introductory lessons in which a new concept is developed.

All non-mathematical segments as well as all segments in which mathematics was carried out but Pythagorean theorem was not worked on are combined into so-called rest time (not listed in table 3). This rest time makes up, on average, approximately 7% of the instruction time. The sum of rest time, theory time and practice time makes up the complete lesson time.

Problem statements and problem implementation in the practice segment

Both the number of problem statement segments and the duration of their implementation vary considerably between teachers (table 4): In total, in the three lessons of the 37 teachers, there were 229 problem statements (Each concurrent problem statement segment is counted in this process as only one segment. Segments that extend beyond the end of the lesson into the next lesson are counted twice.). On average, each teacher accounts for 6.2 problem statement segments. However, there is at least one teacher in each case in whose three lessons 1 and 27 problem statement segments occur, respectively.

The problem implementation segments lasted for an average of approximately 8 minutes (463.6 seconds); the extreme cases lie far apart from one another at 14 seconds and approximately 43 minutes, respectively.

	Mean	Standard deviation	Extremes	
Number of problem statement segments	6.2	4.9	1	27
Duration of one problem statement segment (in seconds)	463.6	481.9	14	2'592

Table 4: Number and duration of problem statement segments

Cognitive level of the practice segment

Table 5 provides an overview of the time proportion from the complete instruction time in which problem statements of the corresponding level were worked on. This does not yet reveal the level on which the problems were actually implemented. As table 5 illustrates, in an average of 25.7% of the complete instruction time, connection problem statements were worked on. Clearly less time was spent working on procedure problem statements (10.5%), and concept problem statement segments are very rare (1%).

	Mean	Standard deviation	Extremes	
Procedure	10.5	10.4	0	32.7
Concept	1.0	3.5	0	21.0
Connection	25.7	19.3	0	76.3
Rest	0.2	0.4	0	2.0

Table 5: Cognitive level of the problem statement segments in the practice segment (Proportion in percentage of total lesson time, *N = 37 classes*)

If one then adds all three time proportions and takes into account the 0.2% of problem statements for which no clear code could be allocated (category rest), one arrives at the 37.4% of the instruction time during which practice was implemented (cf. table 3). The

presentation of the extreme cases shows how large the differences between the teachers already are if one only considers the time in which problems of the same level were implemented.

Table 6 shows the cognitive level that was actually worked on independently of the respective problem statement. To understand the meaning of these values in table 6, it is important to keep in mind the coding method: Each problem implementation segment was divided during the coding into one or several sub-segments of differing cognitive levels. Since teachers work mostly on more than one problem during the three lessons, there are mostly several short or longer problem implementation sub-segments of each cognitive level per teacher. All of these small sub-segments were added up for each teacher and expressed in percentage of total lesson time. Following this, the mean and standard deviation across all teachers and the extremes of the teachers were determined.

	Mean	Standard deviation	Extremes	
Procedure	6.8	7.3	0	32.1
Concept	9.4	6.9	0	24.0
Connection	14.2	15.5	0	67.8
Result only	1.1	1.5	0	5.9
Rest	5.8	5.9	0	21.0

Table 6: Cognitive level of the problem implementation segments (Proportion in percentage of total lesson time, *N = 37 classes*)

With the exception of rounding errors, all of the proportions indicated add up again to a proportion of 37.4 %, which was used on average for implementing practice problems (cf. table 3). It can be recognised that the majority of the time of problem implementation is carried out on the connection level (14.2 %). The concept level comprises 9.4 %. The least work is carried out on the procedure level (6.8 %). The category "results only" occurs so rarely that it was left out of the further analyses. Approximately 5.8 % of the total instruction time was used in practice problems either for non-mathematical work or for non-codable activities. This value was also no longer considered in the further evaluations.

It should be noted that also in terms of the level of problem implementation, the extreme cases lie far apart, particularly in the connection implementation.

Cognitive level of theory segments

Also within the theory segments, activities take place on a different cognitive level. Table 7 shows the mean, standard deviation and extremes of the rating of the cognitive level of the theory segment.

	Mean	Standard deviation	Extremes	
Connection and concept general	3.1	0.7	2	5
Connection and concept with student participation	2.6	0.7	1	4

Table 7: Rating of the cognitive level of theory segment, *N = 37 classes*

The interpretation is rather difficult with the current response format: (1) 0%, (2) "less than 25%", (3) "less than 50%", (4) "less than 75%" and (5) less than 100%", in each case in percentage of the theory time. On average, approximately half of the theory time (3.1) was spent working on the connection and concept level. However, the students only spent approximately one third of the total theory time (2.6) participating in connection and concept activities. As is the case with the practice segments, the extreme cases show a broad range. In particular, there is at least one class in which in the theory segment, the students were not involved at all in concept or connection.

5.2 Correlations between the cognitive level of the theory segment and the level of the practice segments

Does the cognitive level in the theory segments correspond with the cognitive level in the practice segments? Are the classes that work on the connection and concept levels comparatively frequently in the theory segments the same classes that work on the connection and concept levels in the practice segments? In order to address this question, in the following, correlations will be determined between the relative time proportions of the cognitive level of theory

segments and practice segments. For this purpose, two points should be taken into account:

1. The cognitive level of the practice segments (problem statement and problem implementation) and the cognitive level of the theory segments were measured differently: The practice segments were measured with a low-inference method, and the theory segments more with a high-inference method (for the terms high and low inference, cf. Hugener, Rakoczy et al., 2006). In particular, for the practice segments, the codings are more refined in terms of time. This complicates the calculation of associations. The results of the following analyses therefore need to be interpreted with caution and should be replicated with further data.
2. In the calculation of associations between the cognitive level in theory segments and in practice segments, it further needs to be taken into account that the proportion of time of these two segments adds up, apart from the small amount of rest time, to the total instruction time (cf. section 5.1). A high percentage value in terms of the total instruction time in the theory segment therefore rules out a high percentage value in the practice segment.

For this reason, for the calculation of associations, the percentage of the individual cognitive levels was formed not in terms of the total instruction time, but rather only in terms of the total practice time or theory time. In other words: The cognitive level of the problem statements and problem implementation was related to the total practice time. The method was similar for the theory segment: The cognitive level of the theory segment was related to the complete theory time, which precisely corresponds to the available rating values. Thus, the above-mentioned mutual dependency is broken and it is possible for high percentage values to occur both in the theory and the practice segments. As not all variables are normally distributed, for reasons of simplicity, all correlations were calculated with the Spearman rank-order correlation. Tables 8 and 9 show that the cognitive level of the theory segments is not significantly associated either with the level of the problem statement or that of the problem implementation.

Practice segment / Theory segment	Problem statement		
	Connection	Concept	Procedure
Connection and concept general	-.151	.092	.124
Connection and concept with student participation	-.275^{+}	.195	.222

Table 8: Correlation between the cognitive level of theory segment and the cognitive level of problem statement segment (*N= 37; ** p < 0.01,* p < 0.05, $^{+}$ p < 0.1, Spearman rank-order correlation, two-tailed)*

Practice segment / Theory segment	Problem implementation		
	Connection	Concept	Procedure
Connection and concept general	.095	.086	.089
Connection and concept with student participation	-.136	.232	.172

Table 9: Correlation between the cognitive level of theory segment and the cognitive level of problem implementation segments (*N= 37; ** p < 0.01,* p < 0.05, + p < 0.1, Spearman rank-order correlation, two-tailed)*

6. Discussion

In the discussion of the results, the following points should be kept in mind: Our codings and analyses differ in fundamental points from those of the TIMSS 1999 video study (Hiebert et al., 2003):

1. The cognitive level was also analysed here for segments in which no problems were solved (in this article, these segments are called theory segments).
2. Calculations are carried out with the time duration in which a certain cognitive level

was worked on, and not with the number of problems of the respective level.

3. A problem implementation segment does not (always) only contain one implementation code, but rather different sub-segments with different cognitive levels are coded if these are observable. Therefore, it is possible for there to be a repeated change in the cognitive level within one implementation of a single problem statement.

In the classes examined, the greatest proportion of time in the practice segments was used for implementing connection problem statements: A quarter of the total instruction time or approximately two thirds of the practice time is, according to the present coding, spent working on connection problems. Procedure problem statements are worked on in 10 percent of the lesson time or approximately a quarter of the practice time. Concept problem statements occur very rarely. The problem implementation also takes place predominantly on the connection level: Approximately 14 % of the total lesson time (this corresponds to somewhat less than 40 % of the practice time) is spent on problem implementation on the connection level, around 10 % on the concept level and approximately 7 % on the procedure level.

The comparison of problem statement and problem implementation shows the following: It is apparent that the proportion of connection problem statement segments of the total lesson time is substantially larger than the proportion of connection problem implementation segments (25.7 % and 14.2 %). The proportion of procedure problem statement segments is also larger than that of the procedure problem implementation segments (10.5 % and 6.8 %). Conversely, the proportion of concept problem statement segments is smaller than that of the concept problem implementation segments (1.0 % and 9.4 %). It is already evident from these data that the interplay between the cognitive level of the problem statement and the cognitive level of the problem implementation segments is very complex. In the analyses presented in this article, the cognitive level of the problem statement and the problem implementation were analysed separately. The article does not yet make any comment about whether the cognitive level of the problem statement corresponds with the cognitive level of the problem implementation: Are connection problems also indeed, at least in part, implemented on the connection level? We will address this question in a later analysis.

In the introduction of new theory, approximately half of the time is spent working on the connection and concept level. The proportion of time in which the learners are participating in connection and concept activities lies at approximately one third of the total theory time.
Interestingly, across all teachers, no significant correlations were found between the cognitive level of problem statements and problem implementation in practice segments on the one hand and the cognitive level in theory segments on the other hand. With due caution, the results from tables 7 and 8 can be provisionally understood as indications that teachers who use a greater proportion of time for introducing contents on the connection and concept levels do not necessarily also leave a greater proportion of time on an analogous level for practice. The cognitive level of the theory segments appears, accordingly, to be largely independent of the level in the practice segment. According to the tendency, it even appears that a high proportion of time spent on connection problem implementation is accompanied with a low proportion of time spent on connection and concept in the theory segments. This is an indicator that there could be something like “compensation effects” between theory and practice segments. In any case, the results reveal that it appears worthwhile to analyse practice segments and theory segments separately.

In the interpretation of the current results, it should be taken into account that the cognitive level of theory and practice segments was determined differently. It should be examined whether a high-inference rating of the practice segments leads to similar results. Fundamentally, it should be taken into account that the sample used was a non-representative sample and that with the three videotaped lessons, we only examined a small and with Pythagorean theorem also a special section of the curriculum. Possibly, the associations for other, more algebraic themes would turn out differently.

Our further analyses will deal with this association between types or levels of problem statements and students’ achievement gains, thereby controlling important influencing variables.

References

Aebli, H. (1994). *Zwölf Grundformen des Lehrens* (8. Aufl.). Stuttgart: Klett-Cotta.

Blum, W. & Neubrand, M. (1998). *TIMSS und der Mathematikunterricht. Informationen, Analysen und Konsequenzen.* Hannover: Schroedel.

De Corte, E., Greer, B. & Verschaffel, L. (1996). Mathematics teaching and learning. In D.C. Berliner & R.C. Calfee (Eds.), *Handbook of educational psychology* (pp. 491-549). New York: Macmillan.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K.B., Hollingsworth, H., Jacobs, J.K., Wearne, D., Smith, M., Kersting, N. & Stigler, J. (2003). *Teaching Mathematics in seven countries: Results from the TIMSS 1999 Video Study.* Washington, DC: National Center for Educational Statistics, U.S. Department of Education.

Hiebert, J. & Wearne, D. (1993). Instructional tasks, classroom discourse and students' learning in second-grade arithmetic. *American Educational Research Journal, 30* (2), 393-425.

Hugener, I., Pauli, C. & Reusser, K. (2006). Videoanalysen. In E. Klieme, C. Pauli & K. Reusser (Hrsg.), *Dokumentation der Erhebungs- und Auswertungsinstrumente zur schweizerisch-deutschen Videostudie "Unterrichtsqualität, Lernverhalten und mathematisches Verständnis". Teil 3. Materialien zur Bildungsforschung, Band 15*. Frankfurt am Main: GFPF.

Hugener, I., Rakoczy, K., Pauli, C. & Reusser, K. (2006). Videobasierte Unterrichtsforschung: Integration verschiedener Methoden der Videoanalysen für eine differenzierte Sicht auf Lehr-Lernprozesse. In S. Rahm, I. Mammes & M. Schratz (Hrsg.), *Schulpädagogische Forschung. Unterrichtsforschung. Perspektiven innovativer Ansätze* (S. 41-53). Innsbruck: Studienverlag.

Jacobs, J., Garnier, H., Gallimore, R., Hollingsworth, H., Givvin, K.B., Rust, K., Kawanaka, T., Smith, M., Wearne, D., Manaster, A., Etterbeek, W., Hiebert, J., & Stigler, J.W. (2003). *TIMSS 1999 Video Study Technical Report: Volume 1: Mathematics Study.* Washington DC: National Centre for Education Statistics, U.S. Department of Education.

Klieme, E. & Reusser, K. (2003). Unterrichtsqualität und mathematisches Verständnis. *Unterrichtswissenschaft, 31* (3), 194-205.

Klieme, E., Schümer, G. & Knoll, S. (2001). Mathematikunterricht in der Sekundarstufe I: Aufgabenkultur und Unterrichtsgestaltung. In: Bundesministerium für Bildung und Forschung (BMBF) (Hrsg.), *TIMSS - Impulse für Schule und Unterricht. Forschungsbefunde, Reforminitiativen, Praxisberichte und Video-Dokumente.* Bonn: Bundesministerium für Bildung und Forschung.

Knoche, N., Lind, D., Blum, W., Cohors-Fresenborg, E., Flade, L., Loeding, W., Moeller, G., Neubrand, M., Wynands, A. (2002). Die Pisa-2000-Studie. Einige Ergebnisse und Analysen. *Journal für Mathematik-Didaktik, 23* (3-4), 159-202.

Knoll, S. (2003*). Verwendung von Aufgaben in Einführungsphasen des Mathematikunterrichtes.* Edition Wissenschaftliche Reihe Pädagogik, Band 66. Marburg: Tectum. (Dissertation: Freie Universität Berlin)

Neubrand, J. (2002). *Eine Klassifikation mathematischer Aufgaben zur Analyse von Unterrichtssituationen: Selbsttätiges Arbeiten in Schülerarbeitsphasen in den Stunden der TIMSS-Video-Studie.* Hildesheim: Franzbecker.

Schoenfeld, A.H. (1985). *Mathematical Problem Solving.* San Diego, CA: Academic Press.

Stein, M.K., Grover, B.W. & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33,* 455-488.

Stein, M.K., & Lane, S. (1996). Instructional task and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2* (1), 50-80.

Winter, H. (1995). Mathematikunterricht und Allgemeinbildung. *Mitteilungen der Gesellschaft für Didaktik der Mathematik, 61,* 37-46.

Wittmann, E.C. (1981). *Grundfragen des Mathematikunterrichts.* Braunschweig: Vieweg.

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