

Social Reflection in Mathematics Classes: Cooperation or Denial

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Abstract: *Is it possible to reflect reasonably with pupils on social behaviour by means of mathematics? Which importance can this subject have for those who learn as well as for society? Within an educational project, and with the help of games like the Prisoners' Dilemma and texts, situations were discussed in which cooperation of all parties involved would show an optimal result but which also have a great appeal to denial for each party involved. Mathematics can help to distance oneself from consternation and also creates the possibility of abstraction and helps to precise possibilities of reflection. After the end of this project the first question can be answered with "yes".*

ZDM-Classification: D30, E20

1. Social Reflection

The centre of the following thoughts is the philosophy of mathematics education that focuses on the reflection on an extra-mathematical subject. Mathematics itself is not the subject of learning – it in fact is a tool to deal with the main idea.

The objective is to deliberate on social behaviour at a "higher" level with those who learn, which means reflecting on human decisions and acts regarding social interactions. At the same time it will be analysed whether or not this subject can be accepted by the pupils as a content of mathematics classes or even be judged as important. Own reflections of pupils shall be promoted on the one hand during decision taking and on the other hand at the analysis of observed behaviours. What is meant by "higher" level and what mathematics can contribute to reach this level, will be presented in detail.

I want to name deliberation on social¹ behaviour as "*Social Reflection*". The term "Social Reflection" shall in the following get a double meaning that for further thoughts has always to be taken into consideration:

¹ social: "in its consequences affecting (at least two) human beings"

- On the one hand Social Reflection means reflection on a concrete subject, namely deliberations on behaviours regarding social questions. The term "social question" can absolutely be replaced by "social circumstances", what results in an equivalent definition of Social Reflection as *deliberation on social circumstances*.
- On the other hand, Social Reflection means to *reflect together with others* – unlike individual deliberation. In the project this objective is met by working groups, lectures and collective questioning of the input. All that enables a more comprehensive view (The own opinion shall not be the sole centre of reflection.) of the question observed, emphasized by the term of "Social Reflection".

The concrete content that shall lead to Social Reflection is the reflected deliberation about the question "Cooperation or Denial". This reflected deliberation is the central educational aim of the developed educational project on Social Reflection. Why do I attribute importance to this aim regarding school – even regarding society?

1.1. Cooperation or Denial

Starting point and motivation for the treatment of the content "Cooperation or Denial" is the naivety of society regarding this subject. The fact that problems are identified as such, but that deliberations do not reach far enough, shall be illustrated by the example of "*The Tragedy of the Commons*":

Picture a pasture open to all. It is to be expected that each herdsman will try to keep as many cattle as possible on the commons. (...) Therein is the tragedy. Each man is locked into a system that compels him to increase his herd without limit -- in a world that is limited. Ruin is the destination toward which all men rush, each pursuing his own best interest in a society that believes in the freedom of the commons. (Harding 1968)

Situations like this are commonly known, (modern) examples are e.g. armament races: Two rivalling countries that dispose of similar financial means and that both feel forced to invest much money in armament, cannot ameliorate their positions by doing so. The opponents refuse cooperative behaviour, because denial

seems to be better. Each of them causes damage to society (and in the end to him). And that is the Tragedy of the Commons. Each herdsman can conclude that enlarging his herd is the best solution for himself – independent from the behaviour of all other herdsmen. If they keep only few cattle on the commons, it will be his advantage to use a greater part of the commons by keeping more cattle. If in contrast all herdsmen keep as many cattle as possible on the commons, he will be at a disadvantage by keeping just few cattle. He therefore will increase the size of his herd in any case. During a first phase, the individual can seemingly ameliorate his position, but if too many of the parties involved deny cooperation, all will lose. It can be observed that in many situations we all are unable to free ourselves from the straight jacket that is described by the example given above. Egoistic behaviour, that only consists of pursuing one's own interests, does not lead to optimal results.

One step toward liberation from situations like this can only come from more reflection on the subject "Cooperation or Denial". *Social Reflection can help to recognize the common structures behind the dilemmas of commons and armament.* When (new) dilemmas are encountered, this recognition of common points and the analysis of examples from a higher level help to better judge the situation and to identify the consequences of different options of action at an early stage. Social Reflection, in terms of reflecting together with others, also means to put oneself in the position of the other and helps to relativize one's own point of view. The individual has to see, when it has to defer its own interests in favour of the acceptance of the conditions of social coexistence.

Social Reflection in terms of deliberation on social behaviour can be a step toward a cooperative attitude in two ways:

- Social Reflection in terms of reflecting on social circumstances can contribute to underline the importance of cooperation for society.
- Social Reflection in terms of reflecting together with others can help to put one's own interests in the background and to discern greater contexts.

Therefore, Social Reflection on the basis of the subject "Cooperation or Denial" is of importance for society.

1.2. Demands on Education

On the basis of the social question outlined, what do I demand from my teaching?

Reflecting together with others on the social question "Cooperation or Denial" includes inevitably a discussion about ethics: "How shall we behave?" Social Reflection can help to point out the consequences of our acts. As a demand on education I want to word the central aim of general education as follows:

Especially the recognition of the strengths and weaknesses of our acts and their consequences on reactions of other decision-makers should be an important aim of education.

Social Reflection – as one form of ethical education – cannot and shall not be seen as rival of religious instruction. However, I plead for a larger perspective and regard of the subject "Cooperation or Denial" also in mathematics classes.

2. The Role of Mathematics

Description of abstract facts enlarges our possibilities of thinking. (Fischer & Malle 1985, p. 227; translation)

This article shall help to show that it is possible to do something with mathematics that not necessarily is associated with it. Mathematics can be a tool to reflect on social circumstances ("*Mathematics as a Means of Reflection*"). This role inheres to mathematics because of the symbolic forms of description that it offers ("*Mathematics as a Means of Representation*").

2.1. Mathematics as a Means of Representation

Mathematics offers us the possibility to make abstract things such as "cooperation" and "denial" more concrete and therefore more manageable. In this context, Roland Fischer talks of the materialisation of the abstract:

Mathematics offers for determined, frequently occurring abstract ideas symbolic material forms of description, that make the abstract visible, manageable and concrete. It establishes a relation between the (more) abstract and the (more) concrete.

(Fischer 2006, p. 33; translation)

In the case of Social Reflection, the materialisation of the abstract with the aid of mathematics consists in the schematic description of the dilemmas as e.g. “The Tragedy of the Commons”. To use mathematical means of representation and to apply mathematical procedures to resolve problems it is necessary to translate the problem examined into the language of mathematics. Before starting the examination, numbers are assigned to the options of acting. Reducing the concrete situation to numbers allows in the following to use other mathematical forms of representation. Numbers e.g. allow the creation of preference-lists of options. *Especially the form of “Matrix” structures the situation “Cooperation or Denial” and makes it therefore more accessible to reflection.* The use of the matrix for visualization serves in the first instance to describe a question and does not primarily aim at a solution. Therein lays an often underestimated value of schematic descriptions:

Some things can only be articulated as question, problem or option if described in a mathematical form. (...)

Laying the emphasis in mathematical education on the aspect of description asks for a change of attitude. It often does not deal with the solution of a problem but with its description. The “result” of an exercise can be a formula, a chart or another form of description and does not necessarily have to be a number. Or even more radical, it can be several forms of description that are presented for interpretation.

(Fischer 1993, p. 43; translation)

2.2. Mathematics as a Means of Reflection

Only materialisation makes the use of mathematics as a means of (social) reflection possible. By materialisation, mathematics facilitates the “Making an object of” – also related to reflection. *Mathematics as a means of representation can depicture structures of social conflicts of interests, can make them more transparent and therefore more accessible for reflection.* Mathematics makes abstraction easier and enables a more comprehensive view that is helpful for reflection². To reach this aim it is possi-

² Ludwig Bauer (Bauer 1990, p. 8) determines different forms of mathematics-related reflection in mathematics education:

ble to use already existing mathematical means of representation. But mathematics can also be a means of representation explicitly created for reflection; if mathematical models that are the basis of reflection have been created with regard of the latter.

The term “Social Reflection” has been attributed a double meaning. *Mathematics itself – as contribution to Social Reflection – can also be seen as a means of Social Reflection in two ways.* On the one hand, it helps us to reflect on social circumstances by being a means of representation and giving us the possibility to look at situations via schematic illustrations; on the other hand, it supports our efforts to do so together with others. Materialisation of the abstract is one of the basic conditions especially for collective treatment of a question:

To be able to handle abstract things in common, we have to replace them by something visible, concrete for all (materialised numbers, pictures). (Fischer 1993, p. 43, translation)

2.3. Demands on Education

The role of mathematics for Social Reflection as described above leads us to the following demands on reflection-orientated³ mathematics education:

Pupils shall

1. be familiar with mathematics as a means of representation (They shall be able to compare mathematical forms of description and to choose the most adequate form(s) of description from different forms presented.

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- A. Reflection during classes
 - B. Reflection on mathematics
 - C. Reflection on the importance of mathematics for oneself

Social Reflection cannot be integrated into this scheme. I would like to delimit Social Reflection from mathematics-related reflection and to emphasize that in the case of Social Reflection mathematics is “only” a means of reflection used to deliberate on extra-mathematical situations.

³ According to Arnold and Schüßler pure data-knowledge is losing importance in education (cp. Arnold & Schüßler 1998, p. 61). Because of computers and calculators pure arithmetical abilities also lose importance. Both facts underline the importance of reflection-orientated education for learning cultures at schools.

They shall also discern/know advantages of mathematical forms of description in comparison to linguistic ones.),

2. know the value of mathematics in situations of decision-making,
3. be able to practise (social) reflection.

Point 1 and 2 are conditions for the third demand. For my classes, knowledge of the importance of mathematical means of representation is taken for granted. The second point is subject of a pilot study done prior to the educational project itself. Social Reflection is in the centre of the education examined.

3. Social Reflection in the Classroom

3.1. The Prisoners' Dilemma

The content of the educational project are situations in which cooperation of all parties involved would show an optimal result but which also have a great appeal to denial for each party involved. The best-known situation is the so called "Prisoners' Dilemma". In a first step, this situation was put in a story of "barter trade" and run through with the pupils:

Let's assume that a tradesman holds a great amount of a good (e.g. money) and wants to acquire diamonds. Therefore he arranges with the only partner disposing of this good a barter trade satisfactory for both sides. For any reason, the barter trade has to be kept secret. Both agree to deposit a bag at an agreed hiding place and take the other's bag from the other's hiding place. Both partners are aware of the fact that they will never meet and will not do any other business together.

(Based on Hofstadter 1983, p. 715)

So now let's have a look at the deliberations that both tradesmen will carry out before doing the deal: Both of course have to fear that the other one leaves an empty bag. Both would be satisfied, if full bags would be deposited; but of course that one that gets the good for free would be even more satisfied. Therefore, the temptation would be great to deposit nothing. The situation from one partner's point of view is the following: "If the other one deposits a full bag and I do not, I will take the biggest advantage possible. If I give money for this, I will not be discontent, but I will have less profit. Assuming

the trader deposits an empty bag, I would be badly off to give money for this. Therefore, I also have to deposit an empty bag, to get the best result possible (which in fact in this case is not barter trade at all)." For the tradesman who also wants to act rationally, the situation is exactly the same, therefore he will also deposit an empty bag. Thus, no exchange of goods takes place, and both partners will be worse off than necessary. Naturally, they would have been more satisfied with a successful exchange.

Both' problem can clearly be described using a "utility-matrix" (fig. 1) where the payments according to the behaviour during the play are enlisted. The structuring of the situation – that firstly is not taken into account – is what mathematics offers. The matrix offers an aid to explain the acts that we observe. The partners are referred to as players. Cooperation means to leave a full bag; denial means to leave nothing. The first number of the pairs stands for the profit of Player A:

		Player B	
		cooperates	denies
Player A	cooperates	(<u>3</u> , 3)	(0, 5)
	denies	(5, 0)	(<u>1</u> , 1)

Figure 1: Matrix of the Prisoners' Dilemma

3.2. The Educational Course

The "Prisoners' Dilemma"-play (barter trade) served as an introduction to an educational project that I made in the 6th grade of a Higher School for General Education (16 year old pupils). The project lasted for 8 educational units:

8 Units

1. Iterated Prisoners' Dilemma – game
2. Discussion of the game/Prisoners' Dilemma
3. Prisoners' Dilemma – Situations
4. Solutions for the Prisoners' Dilemma
5. Tragedy of the Commons – game
6. Tragedy of the Commons – discussion
7. Iterated Prisoners' Dilemma – Tit For Tat
8. The role of mathematics

At the beginning of the project, the teacher divided the class (28 pupils) into groups of four. This division was kept during the whole project. In the following I will outline the contents of the first 4 units.

At the beginning of the first unit, the pupils were presented the matrix of the Prisoners' Dilemma (fig. 1) in a morally "neutral" form. That means, they were presented the matrix shown above without being told what it could stand for. For that purpose, the two possible strategies were not named "cooperate" and "deny" but "X" and "Y" to avoid emotional charges. After that the rules of the game were explained: I assigned for each group (player A) another group (player B) but the assignments were kept secret. The game lasted for several rounds. The number of rounds was not announced beforehand. In each round, the sole task of each group was to choose one strategy ("X" or "Y") and to write it down on a piece of paper. The members of the groups had to agree on one strategy, internal consultations were allowed. After each choice, the group was told the choice of its assigned group and the consequent payment (0, 1, 3 or 5 points) by the teacher. Then the next round started under the same rules. The target was to "collect" as many points as possible. (For further details please consult Orrison 1997.) Only after this first phase of the game, the pupils were presented the story "barter trade" which was followed by a second phase of the game. *The objective of this unit was to consciously live the situation of deciding between "cooperation" and "denial".* This objective was even outreached: The pupils did not only think about their own actions but also about those of the others.

The second unit dealt with the presentation of the results of both game phases by the teacher as well as of a discussion of the result. (During the first phase all groups showed consequent denial. During the second phase sporadic cooperation happened but could not prevail). Homework consisted in reading two texts about the Prisoners' Dilemma that illustrated the origin of the term. Furthermore the pupils were asked to find other examples ("stories") where the persons involved find themselves caught between the devil and the deep blue sea as described in the story "barter trade" and in the two texts.

The content of the third unit was the presentation of the texts read as well as of the exam-

ples found. In addition, each group got another text about the Prisoners' Dilemma that had to be presented to the other groups. At the end of the unit, all pupils got all texts and were asked to find solutions for the dilemmas. By discussing a great number of examples of one and the same situation, the pupils should recognise that many different situations in everyday life can be reduced to the Prisoners' Dilemma. They should see "why things are as they are!" Why does (the desired) cooperation often not take place? The search for a solution of this dilemma led to a more intensive examination of the situation "Cooperation or Denial".

During the fourth unit, common points of the examples discussed were pointed out and proposed solutions discussed. Firstly, the discussions were held within the groups, then within the whole class. The teacher mostly acted as a moderator, but also provided additional information. To conclude the first four units the pupils had to write an essay about the "Prisoners' Dilemma".

The second big subject of the educational project was the "Tragedy of the Commons" already presented above, a generalisation of the Prisoners' Dilemma to several persons. This time again, texts were read and examples from the pupils were presented and discussed. The conclusion consisted in presenting a successful strategy for the iterated Prisoners' Dilemma (Tit For Tat) and a discussion about the role of mathematics.

4. Evaluation

For the evaluation of the project, written work of the pupils (homework, essays, class tests) and sound-recordings (units, interviews) were used. These materials led me to the assumption, that an important aim of the project had been reached:

The project caused a deeper understanding of human decisions and action in the pupils. The classes offered the possibility to reflect. The project was accepted as subject of the mathematics course. *However, a splitting into two groups could be stated regarding the interest in the project and the ability to reflect.* One group was mainly motivated by the games. These pupils liked competition and liked to win. But they had difficulties to discern a universally valid

situation behind the (many) examples given. Therefore, they soon lost interest in analysing the texts. The following statements of pupils that belong to that group are taken from the discussion of the “Prisoners’ Dilemma”-play during the second educational unit and from the homework after the third unit:

- “Sometimes you should play a bit risky, otherwise, the game quickly gets boring.”
- “As we did not know the opponents, no cooperation was possible.”
- “But when we had to read the story, many of us got bored and the group got agitated and loud.”

In contrast, the interest of the second group grew in the course of the project. Those pupils were motivated primarily by the texts read; they saw contexts, they invented stories and most of the time wanted to know more. The comments below were stated by this second group:

- “It is decisive for the strategy, whether you want to win or to get a good result as a group. If you want to win, it is enough to win in the first round, in the other rounds you just chose ‘Y’.”
- Avidity and mutual distrust lead to such problems. (...) The most important thing about negotiations is mutual trust. (...) By learning different examples I could imagine how such Prisoners’ Dilemmas could arise.

It became clear, that only few pupils could be assigned to the second group. In my opinion, an explanation for this fact can be found by building a reflection pyramid out of the levels of reflection.

4.1. One Result: Reflection Levels

The learning objectives of the project have been formulated consciously open. I restricted myself to fix levels of reflection that are meant to be the attempt to classify the different abilities of reflection of the pupils. This seemed obvious as surely not all would reach the same reflection level – I postulated “levels of getting smarter”.

The minimal target was to understand evaluations respectively reflections of others and to have learned something with regard to social circumstances. At the next level, the reflections of others can be questioned and assessed. This

assessment leads to the highest reflection level, creative reflection and assessment:

- *Level 1:* The essential contents of the project have been understood. The comprehension of social circumstances (social behaviour) has improved. Reflections of others are taken up, examples are repeated and correctly classified according to subjects.
- *Level 2:* Reflections seized from others are questioned and assessed independently. First signs of own ideas, own examples are found.
- *Level 3:* Creative reflection takes place leading to an assessment (good/bad, important/unimportant). Own ideas and examples are in the foreground of the considerations. The importance of the learned content and a “bringing into relation” play a central role. The subject is dealt with intense and deepening interest in a measure that out-reaches the quantity demanded (by the teacher).

It seemed obvious to presume that only few pupils would reach the highest level of reflection (Level 3). This led to a layout of the reflection levels in the form of a *reflection pyramid* (fig. 2).

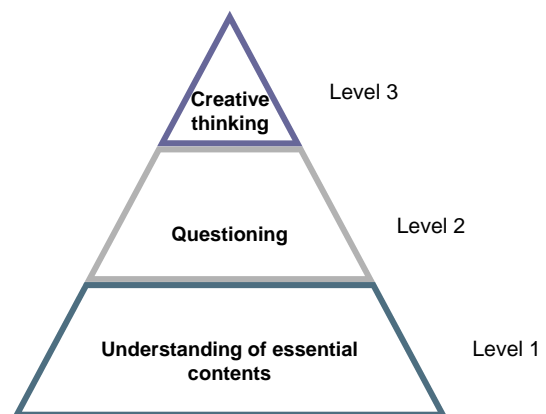


Figure 2: Reflection Pyramid

The form of a pyramid shall underline that the number of the pupils that reach a level diminishes the higher the level is. The lowest level – that is at the same time that basic objective of the educational project – shall be reached by all pupils. By the means of the project, all pupils shall get a more reflected idea of situations in which cooperation or denial is possible. Questioning the learned content and finding own examples seems to be already more difficult and

will probably be reached only by a part of the pupils. Finally education can only offer possibilities to creative reflection; only very few pupils will be able to seize these possibilities.

The presumed form of a pyramid as described was confirmed with the project: The assignment of the essays of the pupils to the reflection levels may serve as an illustration:

- *Level 1*: 11 essays
- *Level 2*: 10 essays
- *Level 3*: 6 essays

The question of different reflection levels plays a central role for the analysis of the project. (It would be interesting to examine the general applicability of the reflection pyramid.) Naturally, the assignment of statements to the reflection levels can only be a rather subjective assessment. The examples below point out the different abilities of reflection of the pupils:

- *Level 1*: “In real life, we act as shown in the Prisoners’ Dilemma. Nobody wants to back down. Everybody tries to cheat. As everybody thinks the same way, they just lose and never win.”
- *Level 2*: “Even though we have dealt so long with the Prisoners’ Dilemma, I do not think that, finding ourselves in such a situation, we would act differently than we did up to now.”
- *Level 3*: “For me, the important of the lessons is, that not every solution, that seems to make the most sense, has to be the correct one.”

It seems to be important, that those who do not reach higher reflection levels do at least know that these exist. That means they know that many thoughts are left to be considered, but that they do not actively take part in those considerations – either because of lack interest or because of other reasons. Pupils who reach the highest reflection level can be called “active reflectors”. One of the targets that these active reflectors can undertake within a society is to set up rules. Rules are an instrument to stipulate cooperation in societies. The results of their reflections, namely the rules, are made available to the general public. On the basis of these suggestions, the society as a whole takes decisions. Individuals that are willing to deliberate more than the average are therefore necessary to

get possible solutions as a basis for decisions to be taken.⁴ Not all individuals have to belong to this group – not everybody has to do reflection to the same extent – since specialization is one of the cornerstones of our social structures. Considering that all citizens have to make decisions that are important for all (as e.g. elections, that in principle are approval or rejection of proposed rules), there should be more Social Reflection.

4.2. Statements of Pupils

In my opinion, the following statements of pupils are reasonable reflections and can illustrate what was achieved with the project. (The statements are taken from the essays the pupils had to write after the project and the answers to a class test question on the subject of the project.)

- “If they had cooperated the dilemma never would have happened. And perhaps this would have been done if on the basis of the Prisoners’ Dilemma possible consequences of the behaviour had been predicted.”
- “But this logic only applies to each player separately. Seen as a whole, the decisions look irrational, because both sides would have obtained a better result by cooperating.”
- “I would tell the people, that the Prisoners’ Dilemma shows, that at the first try we do not trust anybody. (...) We do not want to take any risk, because we are afraid of losing. But on the other hand, we hope that our partner, opponent does exactly take that risk.”
- “If we meet the same opponent several times in the same situation, it is, from the long term point of view, better to build up confidence. At the beginning, you should in consequence behave cooperatively, because if you attack your opponent from behind, he or she will no longer even think of cooperation.”

⁴ During the units, offers for those at the top of the pyramid should be given again and again, to encourage active reflectors. In the presented project, phases of low reflection levels (game phases) alternated with intensive reflection phases (own examples, finding solutions, the role of mathematics).

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(Translations of the titles of the German references are stated in brackets.)

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