Reflections as a challenge

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Abstract: *Reflections on mathematics-based actions and practices bring an ethical dimension to the notion of reflection, and this is the aspect I consider and develop in this essay.*
I elaborate on the notion of reflection by addressing eight different issues. (1) The necessity of reflection *emerges from the observation that mathematics-based actions do not have any intrinsic link to progress by virtue of being mathematics-based. Such actions can be as complex and as questionable as any other actions. (2) Although reflections, from this perspective, are believed to be necessary, one could cite a* functionality of non-reflection*. For example, non-reflection enables the school mathematics tradition to continue to ensure that the future labour force has particular competencies in the right measures to match the social order for which they are destined. (3) Reflections often presuppose* specificity*, as they include general as well as specific reconsiderations with respect to some knowledge, actions and practices. (4) I use* collectivity *of reflections to refer to the observation that ethical considerations can be facilitated through interaction and communication. Often this presupposes that challenging questions be formulated in order to open up the ethical dimension with respect to mathematics in action. (5) Reflections presuppose directedness and involvement, and this brings me to analyse the* intentionality of reflections*. (6) Reflections can address very many different issues, which leads me to recognise the* diversity of reflections*. (7) It is easy to ignore or to obstruct reflections, and when reflections emerge, they can easily be eliminated from an educational context. We should never ignore the* fragility of reflections*. (8) This brings me to recognise the* uncertainty of reflection*. Reflections cannot rely on any solid foundation. Still, I find that reflections are necessary.*

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The notion of reflection is certainly out of control. Its meanings, and here plural is necessary, can be elaborated in very many different ways; it can be used in a variety of situations. I shall try to elaborate on the notion with respect to learning and mathematics only.

Instead of addressing directly what reflection could mean, we could consider what can be addressed by reflections, namely actions. Actions can be selfish, surprising, adequate, impolite, problematic, risky, etc. There is a great need to reflect on actions. What, then, can we think of as being actions? Actions take very many different forms, and we can consider different practices as embodying clusters of actions. By ‘mathematics in action’, I refer to all those practices that include mathematics as a constituting part. It could be: technological innovation; forms of production; automation; management and decision making; financial transactions; risk estimation; cost-benefit analysis, etc.[1] Such practices include mathematics-based actions which can be addressed through reflection.

Leibniz formulated the possibility of establishing logic and deduction in a mechanical form. If an adequate symbolism were identified, one could calculate the conclusion from relevant premises. Logical deduction could then be substituted by mechanical procedures. In this context, ‘mechanical’ refers to the possibility of eliminating the human factor from procedures of reasoning and operating. Instead of having to rely on human intuition or other human limitations, one could simply calculate; one could design mechanisms which are apparently more reliable than those offered by human capabilities. These were possibilities imagined by Leibniz, which in the computer age, have turned out to be more real than he himself could have dared to imagine.[2]

Mechanical procedures can be seen as actions, but by being *mechanical* they appear non-human, and consequently they might tend to escape being addressed by reflection. Mathematics in action can operate in mechanical procedures, but I find that they, too, as actions, merit reflection.

Mathematics is rich with internal reflections: Could this quantity be calculated in a different way? Do we in fact need this concept? How to

1 For further elaboration of mathematics in action, see Skovsmose (2005b); and Skovsmose and Yasukawa (2004).

2 This use of mechanism is also drawn from the mechanical world view as, for instance, formulated by Descartes, and exercised by many others. Mathematics, logic and computation have been vehicles of this mechanical thinking.

prove this theorem? Such reflections seem essential for mathematical creativity, and they are important. Internal and procedural reflections, however, do not address the actions in the world that lie beyond the conceptual boundaries of mathematics, but which are nevertheless established through mathematical procedures. It is one thing to consider how to develop reliable algorithms to be applied in a pattern-recognition application; it is quite a different thing to reflect on practices, such as facial recognition for state surveillance (or security, depending on one's perspective) in which such algorithms play a part. Thus, reflections on mathematics-based actions and practices bring an ethical dimension to the notion of reflection, and this is the aspect I want to consider and to develop.

We could think of reflections as preventing what could be called the *banality of expertise*. This expression alludes to Hannah Arendt's expression, 'the banality of evil', which she coined when she was covering the trial of Adolf Eichmann.[3] She found, initially to her own surprise, that the 'evil' Eichmann did not appear to express any 'evilness'. Instead, as a careful bureaucrat, he had apparently done his best, although in this situation, 'his best' had taken gruesome proportions. Still, from a certain perspective, 'his best' reflected obligations situated somewhere within a bureaucracy. When one, in this way, loses track of the impact of what one is doing, and situates one's actions with reference to particular bureaucratic priorities, then one's expertise becomes tied to a tunnel view of one's obligations. The expertise underlying the actions escapes any self-reflection, submitting itself to 'grander' projects, the nature of which is assumed a priori. In this way, expertise might ignore its own humanity and become mechanised.

1. The necessity of reflection

Are reflections with reference to mathematics necessary? Although I am far from sure that this question is at all well-defined, I will try to present arguments indicating why such reflections are necessary. I want to prevent any banality of expertise within the domain of mathematics.

The world became 'modern' in accordance with the idea that science operated as the motor of social progress. A prosperous future awaited everyone, and the force incorporated in scientific development was seen as taking people safely in the direction of 'progress'. Thus, throughout the first part of the 20th century, technological progress was celebrated. Accompanying this scientific optimism, an epistemic position pointed out that reflections and critical considerations were needed to address possible misconceptions that could obstruct the further development of science.[4] This conception of critical reflections is exemplified by the elimination of metaphysics, which was meticulously carried out by proponents of logical positivism.

When we consider the insights established through the discussion of risk society (as coined by Beck), the interaction of power and knowledge (as explored by Foucault), and the ways in which scientific knowledge has come to operate in the market (as discussed by Gibbons and many others), the relationship between scientific progress and socio-political and economic progress in general appears much more problematic.[5] The science-based route towards a better future appears troubled and uncertain, far from a one-way Fifth Avenue. This means that we ought to think of the importance of critical reflections in a different way; it might be that scientific development is problematic in itself, and that it becomes necessary for the substance of this development to be addressed though critical reflection. To delineate reflection simply in terms of a tool for preparing mechanical procedures would be a (positivist) illusion of how science is operating.

Through my analysis of mathematics in action, I have tried to illustrate that mathematics can be a powerful ingredient in a variety of practices. It can be explicitly applied, but it can also operate behind the scene, for instance in computer systems which facilitate particular ways of addressing a problem, such as the security of information and communication. Mathematics-imbued practices are not inherently 'good' or 'progressive' due to some intrinsic qualities such as technical reliability of the mathematical

[3] See Arendt (1977).

[4] In this context I do not try to distinguish properly between 'reflection' and 'critical reflection'. Sometimes I use the notion 'critical' in order to emphasise that I have some more profound reflections concerning the conditions for knowledge development in mind. Sometimes I use 'critique' to highlight socio-political aspects of reflection.

[5] See, for instance, Beck (1992, 1999); Foucault (1989, 1994); and Gibbons et al. (1994).

algorithm, or the level of precision that can be achieved by some method. Like any social practice, mathematically-based practices require critical reflection. Such reflection must also address the expertise embedded in the practices. I find it to be an illusion that mechanical procedures can be lifted out of the practice in which they are imbedded and dispensed from critical reflection. A mechanical procedure is a human creation, and as such, it is also a questionable creation.

2. The functionality of non-reflections

One could see mathematics education as an important preparation for establishing and maintaining social 'functionality'. We can no longer think of a well-functioning society without assuming a differentiated work force, that is, a society in which a variety of competencies are distributed in proper measures to different groups of people: some must be dentists; others lawyers, teachers, doctors, bus drivers, butchers, etc.; some must serve society's need for unskilled workers, ready to accept any job that might emerge; while still others become labelled as disposable.

Education, as a socio-economic process of structuring, can be a way of 'streaming' students according to a matrix of demands. They can be *disciplined* in at least two ways. First, through an educational process (in this context, the notion 'instructional process' may be more appropriate), students may master an academic discipline, separated from other academic disciplines, as well as from the context in which the knowledge from the discipline operates. So in one sense, disciplining might refer to the enculturation of students into a particular field of knowledge which they master and separate from other areas of knowledge. Disciplining may include a particularisation or fragmentation of knowledge. Secondly, disciplining means forcing submission to some authority expressed through some disciplinary procedure. As part of an instructional process, students submit themselves to criteria of relevance and rigor that rule the academic discipline they are studying. In this sense, both the processes of mastery and submission become elements of disciplining that lead to the constitution and production of expertise.

Mathematics is present in a variety of disciplines, such as the physical sciences, engineering and economics, by operating implicitly as part of those mechanisms which represent the rationality of the discipline, but also by forming the discourse through which expertise in those disciplines is expressed. However, mathematical mechanisms do not operate only within the domain of academic or professional expertise. A variety of everyday practices are imbued with mathematical mechanisms. Although the mechanical rationality that emerges from these mathematical mechanisms is not often visible, many of these everyday practices depend on the fact that people accept the basic logic of the situation. One example is submitting to entering a pin number to draw money out of an automated teller machine because the mathematical algorithm requires this, but also because it apparently protects the money from being accessed by unauthorised persons. This draws us towards the insight that one function of mathematics education, albeit not explicit, might be to prepare people to accept the operation of 'mechanical rationality' in their lives.

The education systems must ensure a supply of people with competencies according to a matrix that represents society's demand for competencies. Some groups must be well-educated in mathematics; some must be able to operate with certain mathematical techniques; some must be able to read diagrams; some must know the mathematics included in instructions; a great many must know the mathematics necessary for shopping and dealing with payment and bank transactions. Like literacy, mathematical literacy can also be seen as a 'basic functional skill' of a well-functioning society. In particular, we could think of mathematics education as disciplining students to follow rules that apparently maintain order in society. This could be the real, although not explicitly intended, meaning of submitting students to the long sequences of exercises and tests so characteristic of the school mathematics tradition. Mathematics education might serve as one of society's disciplining regimes, which ensures that powerful mathematics-based actions are kept within strict control.

These observations bring me to consider that neither mathematics nor mathematics education contain an essence that ensures that they, by their very nature, are 'good' in and of themselves. Both mathematics and mathematics education can operate in many different ways, depending on the context, and as a consequence, they must be addressed through critical reflection. The question then becomes: What is meant by reflection?

3. Specificity of reflections

Reflections can address very specific issues. I emphasise this in order to refute the idea that reflections, in general, do not presuppose a specific insight. Reflections include general as well as *specific* reconsiderations with respect to knowledge, actions and practices.

Let me illustrate how reflections could be encouraged and facilitated in secondary mathematics education. I have referred to the following example of project work in several other situations.[6] It illustrates nicely what I have in mind when I talk about the specificity of reflections.

One aim of the project was for students to experience issues related to statistics and probability, and in particular, the reliability of samples: Can we trust information obtained from samples to draw conclusions regarding a population? The activities were contextualised with reference to salmonella-infected eggs. The whole population of eggs was brought into the classroom on a trolley. Film cases simulated the eggs, and could easily be opened for inspection. Some "eggs" contained a healthy yolk, in the form of a yellow piece of plastic, while others contained a blue piece, indicating salmonella infection. The first task given to the students was to select a sample of 10 eggs each from the trolley. It was known in advance that 10% of the eggs in the trolley were infected with salmonella. Each student sampled 10 eggs and counted how many eggs in the sample were infected by salmonella. They might have expected one egg out of the sample of 10 to be infected by salmonella. The question was to what extent the selected samples did in fact reflect the 'real' degree of salmonella infection. The classroom exercise showed that the story emerging from the samples was far from the case for the population. Less than half of the samples contained one and only one salmonella-infected egg. How could that be? Were the eggs on the trolley not mixed properly? Does this mean that samples are rather unreliable messengers regarding properties of the whole population? Would that always be so? What would that mean for all those situations in which our knowledge regarding the whole population is based only on samples? Such cases are most common; so the students experienced an authentic dilemma which emerges in any form of quality control. In this way, the project led to a broader discussion of the reliability of information provided by numbers. The discussion of reliability not only concerns samples, but any situation where mathematics is brought into action.

During a later phase of the project, the students were put in a situation where they would make decisions based on numbers, and in this way, experience a mathematics-based action 'from the inside'. Two trolleys were brought into the classroom. On one, they found what was referred to as eggs from Greece; on the other, they found eggs from Spain. Each group of students was asked to think of themselves as representing an egg-importing company. The overall decision to be made was: From which country should they import eggs? Both eggs from Greece and Spain were infected by salmonella, but to different degrees not known by the students – and in fact not known to anyone, as the teacher had just randomly added some infected eggs to each trolley.

The students received information about prices, and it just so happened that Greek and Spanish eggs could be imported for the same price, the only possible difference being their quality. The students also got information about the prices of quality control (not very inexpensive), and they were informed that eggs opened in the process of quality control were destroyed. As a consequence, a painstaking examination of each and every imported egg would leave no eggs to bring to the market. It was not difficult to see that they had to investigate the quality of Greek and Spanish eggs through samples. But how many samples would be needed in order to make a well-justified decision? The students had to make a budget that clarified the economic aspects of their business strategy. The students faced the dilemma that the more reliable the quality control, the more questionable the whole economic business would become. The point of this part of the project was that the students would experience a common situation in business: if one wants to ensure the best quality of the product to be put on the market, one has to invest in quality control. So what does it mean to act responsibly in such a situation?

[6] The example was developed in collaboration with Helle Alrø, Morten Blomhøj, Henning Bødtkjer and Mikael Skånstrøm and was described in Alrø and Skovsmose (2002).

The issues of *reliability* and *responsibility* are of general significance for addressing mathematics in action, and I find it important that mathematics education attempt to explicitly address such issues. Thus, the example about salmonella infection may illustrate the specificity of reflections needed for addressing mathematics in action.

4. Collectivity of reflections

What could it mean to facilitate reflections? In an educational context, I believe it to be a permanent obligation to try to prepare for reflections on any mechanism in which mathematics may take part. But it is a difficult challenge.

In the project 'Caramel Boxes', the students were engaged with the question of the adequacy as well as the limitations of proportional reasoning.[7] The task was to construct caramel boxes; in this process, the relationship between the length factor, *l*, the area factor, *a*, and the volume factor, *v*, for different boxes of the same proportion could be addressed. However, the possibility of exploring these relationships was not stated explicitly as a task for the students. They were invited to design boxes for caramels. What should a bigger box look like if it contains double the number of caramels contained in a smaller box? What would happen if one uses double-sized paper to construct a box of the double size? What does 'double size' in fact mean? What would happen to the volume if the length of the sides of the box were doubled? During the project, these questions were not posed directly, but students were able to arrive at the insight that if the length factor for two proportional boxes were l, then the area factor a would be l^2, and the volume factor v would be l^3.

The students, however, were not challenged more directly, in a way that could draw their attention to such observations. Not pointing out any such mathematical insight was part of the planned strategy, as we were interested in clarifying to what extent reflections of such type might develop spontaneously from the students' activities, and to what extent it was necessary for reflections to presuppose a challenging input.

Based on our observations in the 'Caramel Boxes' activity, we found that communication and challenging questions are important to facilitate and to provoke reflections, both mathematics-related reflections and reflections addressing, say, reliability and responsibility. Active communication between teacher and students and among students is important for establishing reflections. Reflection might be an expression of an interaction, and not of an individual process. Still, we could not claim that personal reflections do not exist; but in order to address profound questions concerning a mathematical insight and mathematics in action, communication and interaction appear relevant.

This brings me to the conclusion that if we are interested in developing reflections that address the mechanism operating in many forms of expertise, and which may be established through mathematical calculations, then one must be careful to establish communicative elements as part of the educational processes. I find that reflections presuppose a dialogical basis. If we want mathematics education to facilitate reflections on mathematics in action, then we must work towards establishing learning environments in which reflections can be stimulated through dialogues. Such stimulation is influenced by the way the teaching-learning is organised and contextualised: the 'Caramel Boxes' example illustrates one possibility, and 'salmonella infection' another. Through the discussion of such 'landscapes of investigation', I have tried to be specific about what it could mean to invite students to explore situations and make it possible to establish a collective basis for reflections.[8]

5. Intentionality of reflections

Intentionality refers to directedness, and I see intentions-in-learning as a characteristic of learning that involves and engages students.[9] Rather than being involved, we often observe students withdrawing from the learning process, doing what is asked of them as a 'forced' activity, submitting themselves to the school-logic as administrated by the teacher and the textbook.

Students can learn many things as a forced activity; they can learn to solve equations following the proper procedures. However, if we

[7] The project is described in Alrø and Skovsmose (2002). It was carried out by Henning Bødtkjer.

[8] See Skovsmose (2001).

[9] The relevance of intentions-in-learning has been discussed in Skovsmose (1994) and in Alrø and Skovsmose (2002).

think of students as being involved in addressing mathematical procedures and results through reflection, it is more difficult to view this as a forced activity. Reflections presuppose the involvement of the students. In this sense, intentions must be an integrated part of reflections.

These considerations have led me to consider the importance of 'landscapes of investigation', as I referred to previously. I find it important to challenge the school mathematics tradition. One way of doing so is to reconsider the extensive use of exercises. This is a defining aspect of this tradition, and for many, mathematics exercises represent the essence of mathematics. The exercises serve as an ongoing test of a student's mathematical capabilities, and the disciplining aspect of mathematics education seems to be linked to these exercises.

The paradigm of exercises can be substituted in several ways, one of which is doing project work. The discussion of project work has been elaborated, incorporating more specific terminology, but in order to formulate the idea of students' participation in a more general way, I have chosen to talk about 'landscapes of investigation'. Such landscapes could be explored through mathematics. The landscape can be defined in mathematical terms (having to do with samples and the hyper-geometric distribution); it can be formulated in terms of some more-or-less realistic references (having to do with salmonella-infected plastic eggs); it can be described as real life situations (having to do with a real quality control process). The important point is that landscapes of investigation are *not* explored on the basis of a prefabricated list of exercises. Instead, the explorations take place in terms of a 'learning journey' where the students have the opportunity to point out directions, formulate questions, ask for help, make decisions, etc. However, it is the students who travel through the learning landscape, rather than the teacher or the textbook writers who design a pre-packaged tour in the form of exercises that leave no time or option for alternate routes.

Establishing learning landscapes is a way of inviting reflection. The students have the opportunity to reflect on mathematical procedures in a different way than when they are solving exercises. A mathematics exercise can be answered correctly or incorrectly, and within the school mathematics tradition, much reflection by the students concentrates on the true-false issue: Have we calculated correctly? How to present the solution? How to double-check the solution? However, when students are operating in a landscape of investigation, they need not accept a particular mathematical content as a given; rather, they can consider its relevance for addressing particular issues. Their reflections can be 'similar' in some ways to reflections crucial for mathematics research. However, it is also possible to open up landscapes of investigation for reflection concerning ways in which mathematic is applied, and how it may operate in a context of application. Such reflections can address questions of reliability and responsibility, as mentioned previously.

A learning landscape might help to establish students as 'owners' of their learning. This is important for reflection. I find that reflection should, in a profound way, address the content of learning and its possible relevant application, not to forget its usefulness for the future of the students. This cannot be a forced activity. Instead, I see students' intentions as an integral part of reflections. Reflections without intentions are pointless.

To attempt to eliminate some of the disciplining elements of mathematics education, it is important for students to address what they are learning, how they are learning, and the relevance of what they are learning. They must have the opportunity to reflect on the discipline and on their acquired knowledge and insights, maybe turning them into a form of expertise. I see reflections as important for counter-acting the disciplining elements that might be included in the school mathematics tradition. Reflections might counter-act the development of blind expertise.

6. Diversity of reflections

In some theories of learning, reflection is described in more-or-less straightforward terms. I shall not, however, try to make any attempt to point out the essence of reflection. I find that in theorising about learning, it is unnecessary to search for any simplification of reflection, but that it could be helpful instead to try to grasp the *diversity* of reflections. For me, it is important to illustrate that reflections addressing mathematics and mathematics education can take many different forms.

Let us look again at an example of content-oriented reflections. One can imagine young students being engaged in exercises involving division. Based on their previous experiences, they know that division makes a number smaller. This is based not only on extensive experience with solving exercises (which in some grades, are designed only to demonstrate this particular property of division), but also on the fact that the very notion of division signifies that when 'something' is divided, the resulting pieces must be smaller than the original totality. However, students could be engaged in a sequence of exercises: dividing a given number by 5; then by 2; then by 1.22; then by 1.02; then by 1; and why not then by 0.98? Such an activity could result in reflections that challenge already established mathematical insights. Older students could come to consider a population of 12 units of which 8 are one type while 4 are of a second type. Thus, the proportion between the two types is 2:1. Let us take a sample of 3 units: What is the probability that the sample reflects the 2:1 proportionality? Let us imagine that we consider 120 units of which 80 are of one type while 40 are of a second type. What is the probability that a sample of 3 reflects the 2:1 proportionality? What will happen if we start with 1200 units? What is the probability that a sample of 6 will reflect the 2:1 proportionality?[10]

One can see many different examples of reflections with similar mathematical content. This form of reflection addresses the very understanding through which mathematical insight develops. However, reflections concerning reliability and responsibility are opening in a different dimension of reflection. They address the actions and practices that can operate outside the realm of mathematics, but which might be constituted through mathematics-based mechanisms. For me, this kind of reflection is important if we are to prevent an ethics-blind operation of mechanical procedures.

But reflections could still take many and completely different routes.[11] A student could consider what could be the purpose of being engaged in a project about salmonella-infected eggs: Is this project part of the curriculum that is relevant for the exam? How are the groups going to be organised? Do I have the possibility of getting into the same group as Sanne? Or has the teacher formed the groups in advance? What is the point of learning this? What is the point of *me* learning such things? Such reflection may not concern the content for learning directly, but they certainly concern the learning situation. Such reflections have implications for how students may engage (or disengage) themselves in the learning. Students' approaches to learning come to reflect very many different considerations beyond an understanding of mathematics.

Students reflect on their possibilities (or lack of possibilities) in life. The students' foreground is an important framework for reflection.[12] One could see their foreground as partially constructed through reflections. What can one dream of becoming? What can one, realistically speaking, expect to become? Such possibly conflicting considerations can make reflections a way for students to attribute meaning to what they are doing. Students might also, as a result of reflection, come to realise that the activities presented to them in school have no point, at least from their perspective. Reflections and meaning construction are related; reflection has the added potential of provoking meaning destruction and realisation of meaninglessness.

As mentioned earlier, mathematics education might be a functional structuring procedure for classifying and distributing students in a supply matrix, which might fit the demand-matrix of the labour market. This possible functionality of mathematics education can also be addressed by students, in the form of blind compliance in the absence of reflection, or conscious compliance or resistance as a result of reflection. Their foregrounds are heavily structured by the social functioning of mathematical disciplining.

7. The fragility of reflections

Mathematics can serve as a mechanism for reasoning about a set of problems, but I find it problematic when mathematics is taken as a mechanical given for a practice. A mechanism

[10] The last example, however, can also open up for discussion how to ensure the proper proportionality of ingredients in medical doses. How to make sure that all pills produced also contain the different ingredients in the right measures?

[11] I have explored some of this diversity in Skovsmose (1994), where I discuss 'reflection' as an open concept.

[12] The notion of student's foreground has been presented in Skovsmose (2005a).

becomes a procedure which one administrates without having to take responsibility for the impact of this procedure on what it is operating on. The banality of expertise signifies a situation wherein one executes actions for which one is not responsible because one is following a priori procedures. This general observation leads us to recognise the fragility of reflections.

This fragility can be experienced in education at the university level. As an indication of what I mean, let me refer to risk estimation, which is an important topic in engineering education. How is the appropriate structural stability of certain buildings determined, for instance, in the case of constructing earthquake-proof buildings? Technically, it may not be that difficult to construct an earthquake-proof building. However, it would be extremely expensive, so we are immediately faced with the challenge of finding the balance between what can be considered 'reasonably safe' and 'reasonably cost efficient'. But what does it in fact mean to balance such different forms of reasonableness? How can reasonableness about safety be balanced with reasonableness about capital outlay? It is a question of the type of expertise required to make such judgments, but what kind of expertise could be brought into operation in this case?

Put in simplified form, the risk associated with an event, A, can be expressed as $R(A) = P(A)C(A)$, where the risk is calculated as the product of the probability that A will take place and the severity of the consequences of A taking place. From a mathematical point of view, there is nothing surprising in this formulation. However, the probability of A taking place, $P(A)$, can be difficult to estimate. If event A signifies the collapse of a building, then we can imagine many different calculations in order to arrive at this number, such as statistics on previous occurrences of earthquakes, the number of similar buildings being submitted to an earthquake, etc.[13] Then we come to the estimate of $C(A)$, namely of the cost of event A in fact taking place. This estimate could be based on what it would cost to rebuild the construction. However, there are in fact many different factors to consider. People could be killed or injured because of the collapse. How does one estimate the cost of this? What is the price of a person? What could such an estimate possibly signify? One could view the estimate from the perspective of insurance; how much money must be paid if someone gets killed? Or one could think in terms of the average value of the productive output of an average person during their estimated remaining working life.[14]

In order to carry out such cost-benefit calculations, which are applied in all kinds of decision making, one has to resort to mathematics. Mathematics provides a specific way of seeing the world. Some aspects are pointed out as important, others are eliminated. However, it is important to note that an aspect of reality is not represented in mathematics (in the sense of being copied or mirrored), but rather is re-presented, in the sense of being presented in a different format. Mathematics provides a reconfiguration of the world. In philosophy, the rationalist tradition assumes that there is a coherence between rationally and reality. According to Descartes, one could come to grasp the structures of the material world through a rational process. Contrary to this, it could be claimed that we should not expect mathematics to represent any reality. Nevertheless, when mathematics is brought into action, it is easy to act as though what mathematics is re-presenting is in fact a representation. This is an illusion, but a powerful one, which needs to be addressed through reflection. The mathematical *transposition* establishes a new world in terms of the so-called description, and this new world can apparently be addressed adequately though calculations. In particular, a complex decision making situation can be directed into a cost-benefit analysis. Performing such a mathematical transposition presupposes a mastery of mathematical mechanisms, and the education of expertise often tends to concentrate on handling this transposition. As an example, in much engi-

[13] Let me just mention that estimations similar to this are included in considerations of the risk of an accident taking place in an atomic power plant. Applied probability theory also becomes stretched to its limits here.

[14] A variety of issues are approached through similar forms of cost-benefit analyses. For instance, health economics calculates the gain of applying one particular health approach compared to another. Here the effects can be measured in life-years gained. And immediately one can ask if life-years gained is the most appropriate unit for measurement. One could also measure in productivity gained, implying that it is more profitable to invest in particular skilled workers. A similar type of cost-benefit analysis can address any humanitarian aide programme.

neering education, the mastery of cost-benefit analyses and of risk estimation are exercised as a calculatory efficiency, while the ethical dimension of mathematics-based actions may not be addressed as part of their education to become experts. In this sense, the disciplining demands included in the education of experts reveal the fragility of reflections.

I find it important to consider the development of expertise not only in terms of developing the capacity to handle and operate certain complex mechanisms, but also the capacity to reflect on what such an operation could mean. It is my hope that any expertise comes to include self-reflection as a defining element.

The type of reflection that took place in the project about salmonella infection can be practiced in the process of developing any expertise in which mathematic plays an important role. Reflection practiced in secondary education has the potential to address fundamental elements of mathematics in action. In this way, expertise-relevant reflections can be prepared for at more elementary levels, although I do not see this being done in any profound way in mathematics education. In fact, one could find serious obstacles to such reflections created by the school mathematics tradition. Those students who come to acquire mathematical proficiency might be well prepared for seeing mathematics in a de-contextualised way, and to perform context-blind applications of mathematics. They could turn into 'functional' carriers of expertise. Those many students who do not come to master mathematics, and who the school mathematics tradition labels as having difficulties with mathematics, may also acquire a 'functional' competency suitable for their designated positions within the social order. For if 'blind' expertise should function, it is important that a broad majority accept the effects of this expertise, and who could do that better than an audience that has been disciplined to see and to accept that mathematics is not for them?

8. Uncertainty of reflection

Reflections are uncertain. This has at least two meanings. Epistemic foundationalism assumes that it is possible to identify some kind of foundation on which knowledge can be properly erected. This foundationalism was presented by Descartes, and many other have followed this approach by making suggestions about the basis upon which to establish knowledge. A different variation of foundationalism is established through the attempt to produce a transcendental philosophy as exemplified by Kant's *Critique of Pure Reason*. He tries to disentangle some elements of philosophical analyses from the stream of everyday life, and from a particularly detached point of view, he specifies a priori conditions for establishing knowledge.

All attempts to formulate something about knowledge before knowledge is established can be seen as an attempt to eliminate uncertainty from the epistemic arena. It would be wonderful if one could formulate an insight into the structure of mathematics and identify the essence of mathematics, and in this way reveal how the power of mathematical rationality ensures an all-embracing dynamic of progress. It would be wonderful to do the same for science. This would ensure that reflection could be restricted to facilitating the operation of rationality. Much uncertainty would be eliminated.

I find that such epistemological aspirations resemble philosophical daydreaming. Instead I find it important to face *uncertainty as a human condition*, also with respect to the socio-political functioning of science and mathematics. This could, however, be interpreted as advocating for absolute relativism, which would render reflection irrelevant. Why reflect if one action could be as good as any other action? However, I try to develop reflection as a resource for facing uncertainties. I do not try to establish any foundations for reflection, but I refute the notion of absolute relativism. In fact, if I had assumed absolute relativism, I could hardly have started this paper by addressing the necessity of reflection. To relate reflections to both uncertainty and necessity might be inconsistent, but so be it.

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