Materialization and Organization. Towards a Cultural Anthropology of Mathematics [*)]

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Abstract: *This summary of six articles which have been written in the past fifteen years focus on the question of the social relevance of mathematics on a principal level. The main theses are: Mathematics provides materializations of abstract issues, thereby it supports mass communication. The principles of mathematics are basic for our social organization. The limits of mathematics are limits of organization. But they can be overcome by emphasizing the reflexive potential of mathematics.*

ZDM-Classification: A40, D30, E10

During the past fifteen years I wrote some papers about the social relevance of mathematics. They have been collected in the book "Materialisierung und Organisation. Zur kulturellen Bedeutung der Mathematik". (Fischer 2006) The present article provides a survey on these papers.

Introduction

There is a difference between the meaning of mathematics for somebody who does mathematics actively (may he be a calculating pupil in elementary school, a secondary student solving a problem in algebra or a mathematical researcher) and the meaning for somebody who is affected passively by mathematics, its applications, its power of regulation and order. I will deal with the second kind of meaning.

Another important difference concerns the contribution of mathematics to our culture. I make a difference between the contribution to the implicit culture, that means to our norms, value systems, patterns of organization, especially to civilization and regulation of modern societies, and the contribution to our explicit culture, which is processed in the consciousness of people, about which we speak, negotiate and make decisions according to important / unimportant.

Firstly I will speak about the implicit culture, which need not be unconscious, but which is in a certain sense a prerequisite for our daily life (also professional and political lives). Later on I will come to the role of mathematics in our explicit culture.

Materialization of the Abstract

The first thesis is, that mathematics is important, because it is *materialization of abstract issues,* namely those not directly perceivable by the senses. In addition to pure thinking, mathematics provides systems of signs, which eventually are materially fixed and by which abstract issues can be represented and manipulated. These sign-systems begin with calculation stones, with marks for numerating, digits, algebraic notation, graphs of functions, graphs with vertices and edges, flow charts and end with the fixing of abstracts in electronic computers. The laws of physics, starting with the fact that stones do not increase by themselves and ending with the laws of quantum physics, are used for an "outhousing" of thinking – *mathematics as applied non-living nature,* in a certain sense. The represented abstract issues are numbers, relations, structures, probabilities etc. Behind all these concepts there stands a view of the world, according to which abstract issues are important, especially relations and processes, perhaps more important than those "substances", to which the abstracts are related. This world view is expressed in the Pythagorean dictum *"All is Number"* and its modern pendant *"All is Structure"*.

The claim to the importance of materializing for mathematics itself is not new, recently it is considered in an intensive discussion about the *semiotic character of mathematics*, partially following the philosophy of Charles S. Peirce (see Otte 1994, pp. 382, 383). For this discussion the doing of mathematicians is the starting point. Therefore one has to add that materializations are especially mathematical (in difference to pictures or schematic representations) if they are accompanied by a system of rules for ma-

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nipulating the materializations. The actions of an active mathematician – again from a calculating elementary student up to a mathematician doing research – is an interplay of representing and operating (with the representations), with more or less creative steps. The point is that it is not necessary to be creative all the time, not even thinking, but it is possible to rely on the rule system. It can also be shown by examples that the prevailing systems of notation – i.e. forms of materializing – have an influence on the kind of theories within even pure mathematics.

The Benefit of Materializing

Materializing makes the abstract concrete and thereby perceivable by senses. Thereby it facilitates the process of abstraction and *gives reality to the abstracts*. In practical life this fact is important, if decisions are to be made. Decisions are, if they are not clear a priori but require deliberation of various arguments, facilitated by abstraction. One formulates abstract principles and criteria, one classifies the concrete issue with respect to an abstract pattern, one possibly evaluates single factors by assigning points and then makes thereby the decision. In this process it is important to forget. One cannot bear in mind all the pros and cons, one has to put aside some aspects of the issue, one has to concentrate on those relevant from the abstract point of view, in order to come to a decision. Material representations facilitate this process. If, in addition, the material representations allow for manipulations and transformations, which generate condensation (if, for example, a mean value is calculated) or directly show the best alternative (in, for example, an optimization problem), the decision process is additionally supported. Of course there exists a back side of the coin: Important aspects can be forgotten, a questionable force of circumstances can be constructed.

What has been said about the importance of materializing so far can be understood as follows: the *single human*, the individual, is supported in his/her thinking, abstracting and decision making. But materializing the abstract and mathematics become still more important, if *social systems or whole societies* are under consideration. Firstly abstract issues are of eminent importance for social systems, in which face-to-face-communication is not possible because of their size. Self-description and self-perception of such systems and their ability to act require the development of appropriate abstracts such as the number of people in various subsystems, environmental conditions, consumption of energy, welfare, etc. Gross domestic product, tax- and pension fund systems are materialized abstracts, which cause discussions, decisions and payments. The problems of these reductions are well known, nevertheless modern societies cannot live without them. But even before this the processing of such systems, that means the performing of the interactions between the members of the system, makes certain abstracts necessary, for example money as the materialized potential to get goods or services.

The materialization of these abstracts is important, because thereby they gain that status of existence, which makes them able to be content of communication among many people. Materializing has the function to give existence to the abstracts by borrowing existence from matter – that entity about we have the highest common security of existence. It helps to establish *communicative stability*, materializing thereby contributes that people have the impression to know what they are talking about. In this specific sense mathematics is an important medium of *mass communication*.

Systemicity

Besides materialization there is another aspect of mathematics which is essential for its societal importance. For mathematics is not only an offer of material “tools” and certain transformation rules, it moreover offers a *system of concepts, theories, theorems, proofs etc.* – partially in form of the transformation rules, but going widely beyond. Even if one, with respect to social relevance, emphasizes the material forms of representation the fact that there exists a system of connections with the claim to totality is significant. Certainly this “total connections” have not been worked out explicitly in all details – mathematical research is working and there is no end in sight – but there are intensive efforts of mathematicians to establish these total connections. Compared with other disciplines in mathematics the striving for connections is rather strong, though there are also strong tendencies to specialization. Additionally there occur permanently enlargements of the system by inventing new subjects, concepts and forms of representation.

One principle to be regarded in this system of total connection is the *avoidance of contradictions*. Certainly it cannot be guaranteed but it has absolute priority in case of a concrete contradiction. The elimination of contradictions is always possible in mathematics, since mathematics is not obliged to a fixed range of objects outside of it – e. g. nature – that means that one can avoid the contradiction by cancelling parts of the content.

What is the relevance of the “systemicity” of mathematics for the society? Besides materialization, the *contradiction-free systemic network is a second factor of security.* As far as the society tends to consensual synthesis – what of course does not include all aspects of society – mathematics as an intentionally contradiction-free system offers a basis for a minimal consensus.

A further property of mathematics as a system is its *beauty*, expressed for instance by minimality of descriptions. Maybe the beauty is at the present only accessible by those who do mathematics on a higher level. But, as I think, there is the potential to gain societal relevance also from this aspect of mathematics. Aesthetic appeal is a necessary prerequisite for perceivability. Abstract issues, especially those which arise from complex phenomena, need form in order to be perceived, especially if they shall be perceived not only by individuals but by collectives. This is a category which we seldom take into consideration when social systems are designed; today democratic negotiations of interests dominate. The *law system*, for example, could be improved if aesthetic categories would be applied, with the benefit of more collective perceivability and thereby of more legal security. One should try out whether mathematics or mathematicians, for whom beauty is important, can contribute.

Means and System

At the end of my deliberations about the contribution of mathematics to our implicit culture I point to a *duality*, which describes the impact of mathematics onto our society: Mathematics is a *means* which we can use, and simultaneously it is a *system*, to which we are subject. I now use the term “system” in a slightly different sense than before, when I wrote about the “systemicity”, meaning the fact that mathematics is a coherent building of thoughts. Now I mean by “system” the socially implemented system of norms, principles, conventions of representations, rules of decision and forms of organizing, which have to do with mathematics and have become indispensable for modern life. This system comes to expression by the fact, that numbers play an important role, that measuring and calculating occur in many fields, but also in the fact that logical reasoning, formalizing, the generating of rule-oriented and/or hierarchical structures are of high importance in our life. By this importance also those people are affected, who are not dealing with mathematics, they are subject to the system.

In modern societies *people are socialized* in a way such they submit themselves voluntarily: They pay the bills, they fill out questionnaires, they accept decisions based on statistic data, they trust in the computer. Especially the rise of the computer would not have been possible if mathematics would not on the one hand have provided means for development of technologies and on the other hand have had an impact towards a disciplined society with competence in formal thinking.

What is means and what is system in ambiguous in special cases: The index of prices is a means of description of parts of the economy, simultaneously it is part of the system, if it is used as a parameter within contracts, for example. More precisely: There is a *circularity.* Permanently new means are developed, especially to handle the complex system, and exactly these means can become parts of the system, thereby increasing its complexity. The interplay of developing instruments of description for economical processes and the implementation of these instruments in legal regulations is an example. In the field of finance markets it is not even necessary to implement into a rule system, it suffices that all, or at least the most important, participants suppose that the others use the mathematical instruments of analysis.

Value of Meaning and Value of Using

Now I come to the contribution of mathematics to our explicit culture, for the first with a negative diagnosis: The complaint of friends of mathematics about the minor role of mathematical (and in general scientific) contents in our understanding of good education, compared with contents of literature and history, which often can be heard, is legitimate. One has to know the story of Hamlet, the main theorem of calculus can be forgotten after school, if one has it perceived as such actually. But, and now it comes still worse: the complaint is not legitimate in that sense, as it complaints about a fact, behind which these stands a certain reasonableness. What do I mean? *Functioning renders discussion superfluous.* Mathematical procedures are functioning without to be understood. One can use them – from a simple division-algorithm up to a sophisticated mathematical software – if one correctly handles the "user interface". The reliability of mathematics, based on objective validity, combined with the outhousing by more or less comprehensive materializations, makes possible a division of labor between the creators and the users. This kind of division of labor is not possible in other fields of cultural creation, especially in the humanities. Because of this division of labor it is not necessary to become acquainted with the contents of mathematics as it is with contents of literature, history etc., and can nevertheless get benefits from them.

Further because of the fact that mathematics avoids contradictions and controversies it offers only a small potential for conflicts as motivation for discussion. A cultural asset, which stands as an objective, unassailable block of knowledge, is not interesting. It is clear that for the active mathematician the case is otherwise. It is difficult to get both: acknowledgement for reliable results and discussion about them.

These deliberations are supported by more general considerations in the sociology of knowledge. The German sociologist F. H. Tenbruck (1975) makes a difference between the *value of meaning* and the *value of using* of a discipline, where by “meaning” he means something independent of instrumental using, namely the power to give sense and orientation to humans and the society. He states a “law of trivialization” according to which in the progression of any discipline its value of meaning decreases, though the value of using may increase. Tenbruck gives evidence to his thesis by considering the development of modern natural science, which, in his presentation, had a high value of meaning in the 18th and 19th century, which step by step decreased. For mathematics the situation is a bit more complicated, but in principle the smallness of the contribution of mathematics to our explicit culture can be understood in these terms.

Meaning of Using

What I have said so far gives no pleasure to all who would like to see a larger contribution of mathematics to our explicit culture. But one can develop a more optimistic perspective and this is the focus of the rest of the paper. The basic idea is the following: For any discipline with a high value of using a new meaning in the sense of Tenbruck can be made accessible, if *humans refer self-reflexively to the using.* To say it otherwise: If mathematics is relevant for our lives, as a system of representations and procedures, as patterns of thinking, as scheme of organization, then we can learn about ourselves by studying mathematics, especially about our social lives. *Mathematics as a mirror of mankind* in a certain sense.

The central questions of reflection are of the following kind: *What does the using of mathematics tell us about us, our intentions, our predecisions?* What is our will, maybe unconscious, when we use mathematics in a certain way? The aim is self-recognition with the opportunity of new options of action.

Some examples how these very principal questions can be put into more concrete terms:

- What means *measuring* – what do we gain, what do we loose by it?
- What are the opportunities, what the limits of *(algebraic) formalisms?*
- What are benefits and the costs of *reifying visualizations?*
- What means *linearization?*
- What is the benefit *of statistics,* what are the limits?

Partially these questions are dealt with in special disciplines (of mathematics or philosophy), but usually not with that principal openness, which would be necessary in order that the discussion could become part of our explicit culture. For this purpose the questions must be transformed into thesis and anti-thesis which can be understood by educated laymen. I think that this should be possible; it should be a task of schools to introduce into this kind of discussion – its most important task at the upper level.

Rule-oriented Social Systems

More fundamentally than in the above mentioned examples of reflective questions one can learn from *analogies connecting the mathematical way of thinking and social organization.* For me papers and lectures of the Viennese philosopher G. Schwarz (1985) were and are an interesting source of ideas. He established an analogy between Aristotelian logic as a system of thinking and hierarchy as a system of social organization. I extended this analogy by letting mathematics correspond to a type of organization which I would call rule-oriented. The paradigmatic example for this type is bureaucracy; but also the market (in economical sense), large parts of the organization of a modern state and increasingly international networks are *rule-oriented* systems. All these systems have in common that they are largely governed by rules, maybe even kept together by rules. This is accompanied by the fact that frequently well-defined procedures play an important role and that – partially in order to avoid arbitrariness – the issues are de-personalized and objectivized. Thereby a *separation of rule system and motives* is established in a way that the rule system is the invariant, which represents the structural framework for various motives, which are brought in by people. The two characteristics of modern societies, namely to allow for individuality and variety at the one hand and to establish commonality on the other hand are realized in this way.

One specialty of this type of systems is that *nobody has to care for the "whole"*, it suffices that the individuals care for their issues – for instance by maximizing their profit as "homines oeconomici" in a market – and observe the rules. The rest is done by an "invisible hand", of the market, but also of a bureaucracy or of political negotiating. That by organizing in this manner not always best results arise, can be seen by considering thought experiments like prisoner's dilemma (see A. Sen 1982, p. 62). I call systems, in which their wholeness is not collectively reflected, *"systems without consciousness"*.

The "Logic of Functions"

In the following the analogy between mathematics and rule-oriented social systems shall be illustrated by an example. The aim will be, that the limits of both parts – mathematics on the one and rule-oriented social systems on the other hand – should become more obvious.

The example starts with the mathematical concept of function: This concept requires two separate entities: a rule for assigning and the area of objects to which this rule is applied, usually called the domain of the function. These entities have to be separated, especially *it is not allowed that the elements of the domain define the rule*. Certainly by introducing additional parameters and thereby enlarging the domain, it can seem as if this were possible, for instance:

$$f\ (x) \quad = \quad x^2$$

is enlarged to

$$f\ (x, n) \quad = \quad x^n$$

But such an enlargement will never be exhaustive, always an additional (meta-)rule will be required, which is not defined by the elements of the domain. These are in a certain sense "subject" to the rule.

By this phrasing I have already done one step towards social organization. Rule-oriented social systems are designed in a way, that there is a rule system which is put before the members of the system (corresponding to the elements of the domain). This rule system regulates collaboration, for instance with respect to the production of goods and services, and may not be changed, at least not in the last resort, by the members of the system. At least there must exist an invariant kernel of the rule system which can only be changed by an authority standing outside (or above?) the system.

The such described idea of organization is determining for large parts of thinking and acting in business administration. Especially the question, *how to govern organizations,* usually is answered on the basis of this idea, even if one thinks to have leaved rigid bureaucratic concepts of organization. The “logic of function”, as I call it, seems to be compelling: How else should one be able to govern a system, if not some components of the system are fixed? How else could the identity of a social system be constituted, if not by abstraction from the elements towards an invariant mechanism of processing the system? And should not mathematics earn the merit, that by its way of thinking this abstract invariant can be named and perhaps even represented by appropriate concepts?

Irreflexivity of mathematics

Such we have arrived at a fundamental limit. In the field of designing social organization (the disciplines are called "theory of management" or "organization development") since some decades there exist efforts to invent and try out new, alternative models of governing, which are not based on the "logic of function", on the separation of rule system and motives. One speaks about the "eigen-logic" of a system, about "learning systems", about the competence of self regulation, about "coupling up" and "irritating" instead of governing, about "evolutionary management" etc. The aim is to give "more rights" to the systems, to view it not only as subject to the will of a "governor". Most of these concepts integrate the motives in the rule system and simply cancel the elements as not belonging to the system. My approach is different and ends, as a theoretical problem, up to the question, whether *another relation between elements and the whole* can be conceived, respectively put into practice, than that which is suggested by mathematics respectively brought to the point by mathematics: The elements are less than the whole, the whole arises only when some structure is implemented, at least the comprehension of some elements. Behind this there is a principle of set theory, namely: elements become a set only by being integrated by somebody else, they cannot do it by themselves. I call this the *irreflexivity of mathematics.*

Is it really possible to think otherwise? One hint stems to Thomas Kuczynski (1987) who in a lecture in Klagenfurt 1987 pointed to Karl Marx's concept of the individual as the "ensemble of social relations" and to Werner Heisenberg's idea of elementary particles consisting of the relations with all the other particles in the universe. In both cases the point is a *dialectical relationship between elements and the whole,* a relationship according to with the element is not subject to the whole but, metaphorically spoken, stands face to face with the whole with equal rights. In a certain sense then, not only the element is contained in the whole, but the whole is also contained in the element. This is a relationship which is not allowed in mathematics.

Keeping vs. Overcoming

On the base of a concept of humans, according to which individuality and sociality are related dialectically, mathematics is inhuman. But one has to add that almost no discipline makes contributions to dialectical organization. So mathematics is not in bad company. Still more: In its fundamental considerations it brings the basic assumptions of the dominant disciplines to the point. Going beyond this defensive diagnosis, I dare to claim still more: If one takes the character of mathematics as a means of reflection for serious, one could expect from it to *foster dialectical organization.*

I mean this in the following way: One can use descriptions with mathematical means in order to keep the described fix or in order to overcome the described. For instance the *organigram* of a firm is usually used to fix the structure. On the other hand one uses *sociograms,* that are representations of the relations in a small group (of humans), as an instrument, by which the described structure is changed, namely simply by confronting the group with its image in the mirror named sociogram.

Transferred to mathematics as a whole this means: By its property as a powerful means of representation, which brings to light structures very precisely and clearly, in some cases by transforming the representation letting consequences come to light, mathematics can contribute to change these very structures; namely *by provoking decisions which lead to change.* Its "decidedness" and precision serve for sharpening, so the limits of given conditions can become obvious. Just by this potential mathematics can, if it is not used in order to dogmatically legitimate the given, contribute to its overcoming. In my opinion, like no other discipline *mathematics has the potential to overcome itself.* It thereby can foster a process which I would call *consciousness of the society.*

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(for more references see Fischer 2006)

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