Jamie I.D. Campbell (Ed.)

Handbook of Mathematical Cognition

508 pages. ISBN 1-84169-411-8. Psychology Press, An Imprint of Taylor and Francis, 2005.

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The Handbook of Mathematical Cognition (henceforth The Handbook) is an interesting collection of essays on the cognitive and neurological processes that form the core of mathematical ability, particularly numerical ability. The 27 chapters in this book are primarily concerned with the notion of number and address the developmental and biological bases of number concepts and processes, including eclectic studies which investigate causes for mathematical disabilities (developmental dyscalculia) and whether there are similarities between humans and animals in numerical processes.

The reviewer became interested in the contents of this book when researching recent literature on gender stereotypes in mathematics, which resulted in a reference to chapter 14 in this book: Stereotypes and Math Performance (Talia Ben-Zeev et al.). This particular chapter offers a survey of the major work in the area of gender stereotyping and dispels biological explanations for gender differences such as those drawn from Benbow and Stanley's studies with mathematically gifted adolescents. Ben-Zeev et al write:

> *This reasoning is based on the following logic: because this sample of gifted males and females had taken the same math classes and thus shared the same environmental influences, the differences must be biological. This argument is severely flawed. Research has shown that not all individuals experience the same environment in identical ways, especially when individuals are stigmatized in a given domain (pp.235-236)*

This chapter identifies the construct of "stereotype threat effect"- the phenomenon of high achieving individuals becoming targets of stereotypes which allege inferiority and reminders of the possibility of stereotype confirmation. The authors point out to this phenomenon in other minority and historically stigmatized groups and other subtle ways in which stereotype threat triggers. The chapter concludes with recommendations for combating this effect based on the empirical findings of research on performance expectancy, evaluation apprehension, self-handicapping, stereotype suppression, anxiety and physiological arousal.

> *....to turn situations in which students are threatened into situations in which they are challenged to perform to their full potential. (p.246).*

After reading this excellent chapter by Ben-Zeev et al., the reviewer became more curious of the contents of the book and started to pick chapters which looked interesting, particularly those which aligned with the reviewer's current research interests. Eventually having read the whole book over the course of seven months, my conclusion is the book is certainly relevant for mathematics education researchers interested in what cognitive scientists have to say about mathematical thinking and learning. Readers familiar with the book entitled *Cognitive Science and Mathematics Education* (edited by Schoenfeld, 1987), published nearly 20 years ago by LEA, will find that this Handbook has new research based perspectives from cognitive science for the learning of arithmetic, magnitude, ratio and proportion, estimation skills and problem solving, all important areas of concern in mathematics education.

The Handbook is thematically divided into 5 parts. Part 1 consisting of seven chapters deals with cognitive representations for numbers and mathematics. Chapter 1 entitled About Numerical Representations: Insights from neuropsychological, experimental and developmental studies (Fayol and Seron) examines among other things, pre-verbal representations of number concepts in animals and infants. The authors write:

> *…Both newborns and animals seem to be able to mobilize two different systems for the processing of quantities. One of these is precise and is limited by its absolute set size…the other is extensible to very large quantities, operates on continuous dimensions, and yields an approximate evaluation in accordance to Weber's law.(p.5)*

The chapter then addresses non-linguistic processing of "numerosity" in adults, relations between the Indo-Arabic code, verbal codes and semantic representations. The authors point to the fact that many open questions remain on the nature of representations underlying arithmetic and number cognition. The interested reader can read what these particular open questions are. Chapters 2-5 cover number recognition in different formats (verbal, Indo-Arabic and analog); spatial representations of numbers; automaticity in processing ordinal information and computational models of numerical cognition. These chapters are not light reading and assume the reader is familiar with theoretical and methodological foundations of cognitive science. In Chapter 2, Brysbaert reviews and discusses the findings of major studies on number recognition. In Chapter 3, the subsection on developmental and cultural determinant in spatial representations is particularly interesting. Fias and Fischer write:

> *What determines the left-right orientation of the mental number line?...[W]estern participants in number studies typically read from left to right, and this cognitive strategy may transfer from the domain of letter, word, and sentence processing to the processing of digits, numbers and equations. (p.51)*

Chapter 4 by Tzelgov and Ganor-Stern exposes the reader to notions of “distance effect” and “SNARC effect”. In Chapter 5, Zorzi, Stoinav and Umiltà describe computational modeling as an interesting modeling tool from cognitive science “to evaluate or compare existing verbal theories and to make novel experimental predictions” (p.67). These authors give illustrative examples/simulations of number comparison and simple arithmetic using a connectionist theoretical framework. Chapter 6 by Brannon is one of the more esoteric works in this Handbook, interesting in its own right, it summarizes a 100 years of research studies on animal numerical competencies and analyses similarities and differences between humans and species such as pigeons, rats and monkeys in numerical representations and the operations we carry out. Chapter 7 by Nunez and Lakoff on the cognitive foundations of mathematics is familiar territory to mathematics education researchers. In this chapter the authors present the main theses from their book *Where Mathematics Comes From*, namely the role of conceptual metaphors in the structuring and organizing of mathematical ideas.

Part 2 of the Handbook, consisting of seven chapters focuses on Learning and Development of Numerical Skills. These chapters are very relevant to the mathematics education community. In Chapter 8, The Young Numerical Mind: When Does It Count?, Cordes and Gelman offer a provocative analysis of differing theoretical perspectives on the development of counting skills in relationship to the structure of arithmetic. These authors highlight the role of non-verbal mechanisms in young children and how it leads to the counting principle (based on cardinality).

> *Paradigms in which assessments of counting are combined with its role in arithmetic are much less prevalent…[G]iven the assumption that young children do not understand their own counting, it hardly makes sense to ask them to relate counting to mathematical operations. For example Piaget’s theory is a set-theoretic one that grounds the understanding of cardinality in the operations of one-one correspondence and logical classification. For him counting in pre-operational children is done by rote and without understanding…[t]his class of accounts rejects the view that pre-schoolers have any numerical abilities. (p.128)*

The reader is presented a very interesting accounting of the phenomenon of early counting which assumes phylogenetic and ontogenetic continuities. The authors ask the community to critically examine the nature of tasks/assessments for the study of young children’s (< 3.5 years) numerical abilities to determine whether or not young children understand the notion of cardinality. There are some open questions posed by these authors which are worthwhile for researchers interested in this domain of research.

In Chapter 9, Bisanz et al., survey the development of arithmetic skills and knowledge in pre-school children. In chapter 10, Miller et al., present cross-cultural insights on the nature and course of pre-school mathematical development in China and the United States. Given the recent debate over Liping Ma’s (1999) book *Knowing and Teaching Elementary Mathematics*, this chapter presents a complementary accounting of how early learning experiences in arithmetic in China and the United States, differences in parental beliefs and practices before formal schooling, results in significant differences on the ensuing mathematical development of children in these two countries. Chapter 11 by Noël et al examines the development and dysfunction of magnitude representation in children. Chapter 12 by Siegler and Booth reviews the state of affairs in the development of numerical estimation skills. This chapter has implications for teachers wishing to initiate problem solving involving the generation of numerical estimates at the elementary school level. Chapter 13 by Fuson and Abrahamson analyzes understanding ratio and proportion as an example of the apprehending zone and conceptual-phase problem-solving models. This chapter provides some new fodder for those already familiar with the findings of the Rational Number Project. Finally chapter 14 by Ben-Zeev et al., looks at stereotypes and math performance, which was described at the beginning of this book review.

Part 3 of the Handbook examines learning and performing disabilities in mathematical and number processing with chapters on: Learning Disabilities in Arithmetic and Mathematics: Theoretical and Empirical Perspectives (Geary and Hoard); Math Performance in Girls with Turner or Fragile X Syndrome (Mazzocco & McCloskey); Number Processing in Neurodevelopmental Disorders: Spina Bifida Myelomeningocele (Barnes et al.); Math Anxiety and its Cognitive Consequences: A Tutorial Review (Ashcraft & Ridley). This section is relevant for those offering graduate courses for teachers of students with special needs and provides the state of the art of what the research says on learning and performing disabilities.

Part 4 of the Handbook provides reviews of issues in calculation and mathematical problem solving. The chapters in this section are:

- (Zbrodoff and Logan): What Everyone Finds: The Problem Size Effect
- (Campbell and Epp): Architectures for Arithmetic
- (LeFevre et al.,): Mathematical Cognition and Working Memory
- (Dixon): Mathematical Problem Solving: The Roles of Exemplar, Schema, and Relational Representations
- (Duverne and Lemaire): Aging and Mental Arithmetic
- (Pesenti): Calculation Abilities in Expert Calculators.

Again, there are numerous topics of interest in Part 4 for the mathematics education community. Researchers familiar with findings in analogical problem solving; and the recent work on mathematical and analogical reasoning (see, English, 2004; Sriraman, 2005) will find many of these aforementioned chapters of interest.

Part 5 of the book entitled Neuropsychology of Number Processing and Calculation with chapters by Dehaene et al. on three parietal circuits for number processing; by Butterworth on developmental dyscalculia and Lochy et al., on rehabilitation of acquired calculation and number processing, gives the reader findings from neuropsychological research in mathematical cognition. These three chapters are tersely written and are challenging reading for one unfamiliar with this domain of research.

Overall, a book which surveys the extant work on mathematical cognition from such a wide variety of theoretical and research perspectives (cognitive science, computational science and neuroscience) with emphasis on the developmental phases of numerical and mathematical ability is a major contribution to the field of research and certainly of potential interest to the community of mathematics education researchers. The Handbook presents opportunities to tailor graduate seminars in the area of mathematical cognition focusing on particular areas of ongoing research inquiry for doctoral students. The Handbook also sets a good example of the time, effort and teamwork needed for coherent research consolidation in particular areas of mathematical cognition, which sets an interesting precedent for consolidating the state of affairs in mathematics education research.

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