

MATHEMATICS AS A CONSTRUCTIVE ACTIVITY: LEARNERS GENERATING EXAMPLES

Anne Watson and John Mason
Mahwah, New Jersey: Lawrence Erlbaum
Associates, Publishers, 2005.
xiv+228 pp. \$27.50 (Paper) ISBN 0-8058-4344-2

In his address *On the Study Methods of Our Time*, Giambattista Vico said “Geometrica demonstramus, quia facimus” (Vico, (1708) 1911), “We demonstrate geometry because we make it,” and, indeed, he believed that what we know is proportional to what we construct. That belief has become emblematic for modern thinking as a whole—so much so that philosopher David Lachterman referred to “construction as the mark of modernity” (Lachterman, 1992). In mathematics education too, we have come to emphasize learners’ active engagement in constructing their own knowledge; we have come to believe that when learners generate their own representations, questions, and problems, they probe and crystallize their mathematical knowledge more deeply than when they are given ready-made facts. Completely in line with this spirit, and in a way a celebration of it, is Watson and Mason’s new book, *Mathematics as a Constructive Activity*.

As the subtitle of the book states, *Mathematics as a Constructive Activity* focuses on learners’ generating their own mathematical examples. It counters the still common classroom practice in which examples, like questions and problems, are presented to learners as given; indeed, learners are not typically led to believe that they *can* create and explore mathematical examples, at least not significant ones. In its inverting the usual order of things, asking learners to produce mathematical objects that are normally provided in advance, one is reminded of other teaching strategies in the constructivist mode, such as ‘problem-posing’ (as described, say, in Brown and Walter, 1990). But the purpose of the present book is not just to display ‘learner-generated examples’ as another instance of a genre of strategies in which knowledge is assumed to derive from learners’ own activity. As one reads *Mathematics as a Constructive Activity*, one soon realizes that there is also bigger game at hand.

A hint of this can be found already in the first chapter where the authors define an ‘example’ as

“...anything from which a learner might generalize” (p.3) and the process of ‘exemplification’ as “...any situation in which something specific is offered to represent a general class with which the learner is to become familiar—a particular case of a generality” (pp.3-4). Thus, although there is mathematical delight in the specific properties of specific mathematical objects—a delight not to be forgotten or belittled—Watson and Mason stress that mathematical examples always point beyond their own particularity; they make it clear from the start that the process of generating of examples is, in fact, the obverse of generalization. The intimacy of these two processes, taken *as* processes, is a constant theme throughout the book, one that is both theoretical and normative. Thus, for example, we find in chapter 5, “As with many other elements of mathematical thinking, it is the search for examples and not the finished product that promotes learning, and it is important for learners to establish a dynamic between exemplification and generalization wherever they start” (p.100). Statements such as this, as well as many wonderful statements by and about mathematicians in the historical appendix (which, in my opinion, really should have been a full-fledged chapter) continually remind the reader, furthermore, that not only are the processes of exemplification and generalization profoundly related but also *together* they are at the heart of what it means to learn mathematics.

Theoretically there are of course great and ancient difficulties in the relationship between the general and the particular, the universal and the individual, generalization and exemplification. In this regard, I was glad to see the name Plato crop up often in the book. Indeed, anyone who has read Plato’s *Republic*—which Rousseau called the “finest treatise on education ever written”—knows that at the center of the dialogue, in a section where the learning of mathematics is prominent, Socrates speaks of the passage up from particular concrete instances to their unchanging principles and back down again. In modern mathematics education research, concern with the general vs. the particular and particular vs. general has many manifestations, ranging from students’ ‘empirical tendencies’ in proof to metonymic aspects of mathematical thinking. As for the latter, I should mention that I was surprised to find no mention of work done on metonymy in mathematics (e.g. Presmeg, 1997) for the very definition of a metonymy is close to authors’ definition of exemplification. That small criticism aside, Watson and Mason do, in my opinion, make a significant addition to this old but hardly exhausted discussion.

Their contribution lies in their main theoretical construct—the notion of an ‘example space’. In the preface, they introduce this notion as follows: “...examples learners produce arise from a small pool of ideas that simply appear in response to particular tasks in particular situations. We call these pools *example spaces*” (p.ix). This rough definition recalls, *prima facie*, the idea of *topoi* from classical rhetoric. For *topoi* were also pools of examples, stock issues, from which arguments could be developed or into which they could be analyzed; they were sources of invention, themselves examples and sources of further examples. But although the comparison with *topoi* is a useful first step towards understanding example spaces, it is not the whole story. Under the heading, “Examples can be perceived or experienced as members of structured spaces,” the authors write:

Examples are usually not isolated; rather, they are perceived as instances of a class of potential examples. As such they constitute what we call an *example space*. In terms of the observations so far, learners experience access to an example space that emerges in response to the situation, prompts, and propensities...Example spaces are not just lists; they have an internal, idiosyncratic structure...and it is through this structure that examples are produced. Their contents and structures are individual and situational; similarly structured spaces can be accessed in different ways. (p.51)

Thus, example spaces, in contradistinction to *topoi*, have the following basic characteristics: 1) They are dynamic, that is, they can develop and change; 2) They have an internal structure; 3) Their structure is personal and situated.

Watson and Mason suggest various metaphors for example space: a landscape, a toolshed, a larder, a directed graph. They prefer the larder metaphor in which searching for an example is compared to searching through a larder for an item needed for some purpose. That metaphor highlights the fact that in searching for an item one simultaneously reorganizes and recategorizes the other items in the larder (p. 131), suggesting that the search for examples brings with it a restructuring of the example space. My own preference is the landscape metaphor. According to that metaphor, searching for an example is exploring and searching through the meadows and pastures of a landscape. The

authors steer away from this metaphor since it suggests that the example space is something existing independently of learner (p.61). But this is only so if one thinks of the landscape as a real rather than imaginary one. An inner landscape of the imagination is at once one’s own creation and also open to exploration; it is, accordingly, a topography that is responsive to what one finds or does not find within it. The inner landscape seems to me to bring together best of all the three characteristics of example space noted above.

But whichever metaphor one is drawn to, all show how consideration of examples in conjunction with example space provides a bridge between particular individual instances and broad mathematical generalities. For each metaphor presents examples as never-isolated, never-detached entities; and each presents the search for examples as a process of reorganizing the greater structure containing them.

Example space, as I have said, is certainly the central theoretical concept in the book. Indeed, the book has a more or less symmetrical structure with chapters three, four, and five—the chapters which treat example space most explicitly—forming the actual center of the work. Nevertheless, one should not get the impression that this is a completely theoretical book. On the contrary, one of its great merits, in my view, is the degree to which its theoretical insights are never far from the concerns of real teachers genuinely interested in teaching mathematics at all levels.

It is here that I should point out one of the most enjoyable and enlightening aspects of the book, namely, the more than sixty mathematical tasks sprinkled between its covers. Readers who skip these tasks do themselves a great disservice: they will neither fully understand the teaching strategy proposed by the authors nor fully grasp the theoretical ideas that justify it. Indeed, the teaching strategy itself is largely embodied in these tasks. A few examples garnered almost at random from the book will suffice:

Task 22a Write Down

Write down a number.

Task 22b Write Down Continued

Write down a number no one else in the class will (or will be likely to) write down.

Write down a number that no one else in the whole world will (or will be likely to) have ever written down. (pp.78-79)

Task 48a: What if Something Else?

In a geometric progression each term is a constant multiple of the preceding term. Construct a progression in which the multiplier changes according to some rule. Can you produce a formula for the n th term of for the sum of n terms? (p.127)

Task 61: Folding Shapes

What shapes can be made by folding a piece of paper once, twice, and three times?

What proportions of a sheet of paper would mean that folding in half produced a sheet with the same proportions (thus scaling down) [?] (p.191)

In sum, I found this book enlightening and delightful, useful for the teacher and thought-provoking for the researcher.

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