

Towards a didactical theory for mathematical modelling

Gabriele Kaiser (Germany)
Morten Blomhøj (Denmark)
Bharath Sriraman (USA)

Introduction: The issue of theory building

Mathematics education research has been somewhat short of its own paradigmatic theories (see ZDM, 2005, issue 6 and ZDM 2006, issue 1 for an ongoing discussion). Theories are often borrowed from the background sciences and applied to the field of mathematics education e.g. general learning theories from pedagogy, sociology, psychology etc. This has led to our field being a subject to a host of criticisms (e.g., Lerman, 2006; Lester, 2005; Sriraman & English, 2005, 2006; Steen, 1999). Therefore it is relevant to look for areas in mathematics education where theories may emerge from studying the processes of teaching and learning mathematics. In this issue of ZDM we present and discuss what we consider to be an example of a developing theory within the field of mathematics education research, namely a theory for the teaching and learning of mathematical modelling. During the last decades a coherent theoretical understanding of mathematical modelling processes and connected teaching and learning processes has been emerging. This has happened through a close interplay between the development of curricula, teaching practices, experimental teaching, theoretical reflections and research. Together with the use of information technology, the introduction of mathematical modelling and applications is a prominent general feature of the recent developments in the practice of mathematics teaching, especially with regard to secondary level teaching. In this development, didactical research has already played an important role and we find it of general interest for mathematics education research to examine the development of a theory for mathematical modelling as a paradigmatic case for developing theory in close interplay with teaching practice.

Towards a theory of teaching and learning in the domain of mathematical modelling

It is an open discussion to what degree we actually have a theory for teaching and learning mathematical modelling and indeed this discussion depends on which notion of theory we are using.

However, we would like to claim that we do have a global theory for teaching and learning mathematical modelling, in the sense of a system of connected viewpoints covering all didactical levels: learning goals, fundamental reasons for pursuing these goals at different levels of the educational systems, tested ideas about how to support teacher's in implementing learning goals and recognised didactical challenges and dilemmas related to different ways of organising the teaching, theoretical and empirical based analyses of learning difficulties connected to modelling and ideas about different ways to assess students' learning in modelling activities and related pitfalls.

The 14th ICMI-study on applications and modelling in mathematics education, which held its conference in 2004, documented the state of affairs in this field of research and contributed significantly to a picture of a discipline with growing coherence in the overall theoretical framework (Blum et al. 2006).

Obviously the theory of teaching and learning mathematical modelling is far from being complete. Much more research is needed, especially in order to enhance our understanding on micro levels, meaning teaching and learning problems which occur in particular educational settings where students are engaged in modelling activities. This is exactly the focus of many of the papers presented in this issue. As a common feature, many of the papers develop and use a detailed general description of a mathematical modelling process as an analytical tool for identifying the competencies involved in modelling, the blockages that students experience working with modelling or the related challenges for the teaching of mathematical modelling.

Perspectives in this issue

The idea about a general mathematical modelling process is in fact one of the main components of the theory for teaching and learning mathematical modelling. In principal there is a modelling process behind every mathematical model. This means that someone explicitly or implicitly has gone through a process of establishing a relation between a real life situation and mathematics. In other words, in order to create and use a mathematical model it is, in principal, necessary to carry through a modelling process. Analytically it is possible to describe a mathematical modelling process as a cyclic process involving five or six sub-processes as it is done in several of the papers. The different ways of describing the general modelling process give raise to interesting debate. The paper by Borromeo Ferri

addresses this issue and in the paper by Burkhardt (supported by Pollak) we find some reflections on the role of modelling diagrams in the history of mathematical modelling.

However, from an overall perspective on the theory for teaching and learning modelling the important thing is the commonly accepted idea about a general mathematical modelling process. Different models of the general modelling process can be used and are being used for many different research purposes as illustrated in this issue. We are offering the following list with six different purposes for using a general modelling cycle as an analytical tool, which are represented by papers in this issue.

(1) Analysing authentic mathematical modelling processes retrospectively in order to understand and validate the process that might form the basis for important applications and decisions. Such analyses can serve as a basis for selecting and designing modelling problems and situations that are representative in some way for authentic mathematical modelling and authentic applications of models. The paper by Lingefjärd presents and discusses different types of authentic models with relevant historical and contemporary examples of modelling problems which have intrigued mathematicians over the centuries.

(2) Identifying key elements in mathematical modelling competency. This is the focus of the paper by Maaß, which applies both a theoretical and an empirical approach.

(3) Analysing students' modelling work retrospectively in order to determine which part of the cycle they have been working with, which paths they have taken and the types of difficulties they have experienced during their modelling activities. The paper by Galbraith and Stillman analyses the modelling process in detail in order to describe and further analyse the blockages that the students' experience in modelling activities. This paper connects with Borromeo Ferri's analysis.

(4) As a tool for supporting students' modelling work and their related metacognition. In the paper by Kaiser & Schwarz a simplified didactical description of the modelling process it used to support the students' work in a modelling course. Maaß is explicitly discussing this way of using the modelling cycle in her paper.

(5) As a didactical tool for planning modelling courses or project. Which part of the modelling cycle should the students work with? How can they be challenged according to the design of the didactical situation or interaction with the teacher

during the activity? And where in the cycle may they need support? This method of using the modelling cycle is behind the design and analysis of the in-course described and analysed in the paper by Blomhøj & Hoff Kjeldsen.

(6) As a way of defining and analysing a curricular element in mathematics teaching. Many of the papers refer to and discuss the official guidelines and regulations concerning the teaching of mathematical modelling in their particular context and in doing so they use a description for a general modelling cycle as a point of reference.

Dissecting the Contents

As described previously the papers in this issue discuss the teaching and learning of modelling at various levels with particular emphasis on the conceptual difficulties and the cognitive processes which play a role during the modelling process. Borromeo Ferri takes an in-depth look at the different modelling cycles in extant descriptions of the modelling process and re-constructs these cycles from a cognitive viewpoint, i.e., a psychological viewpoint. Based on the results of the COM²-project (Cognitive psychological analysis of modelling processes in mathematics lessons), Borromeo Ferri empirically describes the cognitive phases of the modelling process and the difficulty in accurately differentiating these phases. This paper also serves as an accessible introduction to the lay-reader on different descriptions of the modelling cycle in the literature.

Lingefjärd's paper reflects on the teaching of mathematical modelling at the early university level. By examining historical perspectives on modelling from the domain of applied mathematics, Lingefjärd attempts to show that mathematical modelling is inextricably linked to phenomenon in the natural and physical world. By conveying this to students, the hope is that university students will appreciate the need and purpose of mathematical modelling during the course.

Maaß argues that although modelling and applications are increasingly being viewed as important in mathematics lessons, little is understood about the often-used term 'modelling competencies'. Based on extensive empirical data collected in daily classroom teaching, Maaß adds to former descriptions of modelling competencies and presents her research based on definition of modelling competencies. Other questions explored in this paper are whether mathematics lessons which focus on modelling really enable students to carry

out these processes on their own.

Galbraith & Stillman identify in extensive empirical research student “blockages” that occur during transitions when they are engaged in a modelling task by using a framework developed by Galbraith et al. (2006). The paper provides a useful framework for implementing modelling tasks in daily teaching, because it draws attention to possible blockages and learning barriers, which might occur during modelling tasks. In addition it provides a tool for the assessment as well as the design of appropriate modelling tasks at the early secondary level.

Blomhøj & Hoff Kjeldsen analyse experiences from developing and running an in-service course in project work and mathematical modelling for mathematics teachers at the upper secondary level, grades 10-12 (Danish Gymnasium). These authors report that teachers’ reflections on course projects and related classroom discussions reveal concerns on how one teaches modelling in the secondary classroom. In particular teachers reported difficulties in orchestrating the ‘setting the scene’ stage and initiating dialogues with students to support and challenge them during their work on a modelling project. This paper complements the works of Maaß and Galbraith & Stillman because it conveys the teachers’ perspective and difficulties in implementing modelling activities in the classroom, and helping students’ successfully transition through difficult stages in order to become competent mathematical modellers.

Burkhardt, based on his dialogues with Pollak, describes the development of mathematical modelling as an element in school mathematics curricula and assessments over the past 40 years from the perspectives of two modeller-mathematicians. Burkhardt discusses the place of modelling in the school curriculum with respect to current international goals of creating a citizenry that are mathematically literate. This paper presents an overall perspective on institutional resources and mechanisms necessary to initiate large-scale implementation of innovations, particularly modelling, and develops an impression for the major periods of the history of development and the lessons they have for now.

Kaiser & Schwarz report on university seminars on mathematical modelling which are conducted with co-operation from mathematicians, mathematics educators and schools in Hamburg. This project is unique because it creates a setting where pre-service teachers and upper secondary students are afforded

the opportunity to work on authentic problems which applied mathematicians tackle in industry. This paper gives a glimpse at student solutions on authentic modelling problems and the parameters for those attempting to structure a similar course or seminar for pre-service teachers at the university level.

Looking ahead

Part B (vol.38, no.3) of the special issues on modelling will include other worldwide perspectives on current research in mathematical modelling including the adaptation and effectiveness of model eliciting activities (Lesh & Doerr, 2003; Lesh & Sriraman, 2005) in different geographic locations. Papers in this issue will also examine different theoretical perspectives arising from the modelling literature and address whether compatibility of theoretical perspectives can be achieved. In the spirit of recent ZDM issues, we also will include some commentaries on articles appearing in this issue. One of the commentaries will attempt to address the issue of whether or not modelling activities promote mathematical literacy (e.g., Skovsmose & Valero, 2002). If so, what type of literacy do they promote? Do we want students to only develop functional literacy historically linked to keeping a proportion of the population competent for the workplace, thus enabling the status quo, as opposed to modelling activities designed to promote critical literacy which would be a more democratic and important thing to do.

References

- Blum, W., Galbraith, P., Henn, H-W., & Niss, M. (Eds.). (2006) *Applications and Modelling in Mathematics Education*. New ICMI Studies Series no. 10, New York: Springer. To appear.
- Lerman, S. (2006). Theories of mathematics education: Is plurality a problem? *Zentralblatt für Didaktik der Mathematik*, 38(1), 8-13.
- Lesh, R. & Doerr, H. (Eds.) (2003). *Beyond Constructivism – Models and Modeling Perspectives on Mathematics Problem Solving, Learning and Teaching*. Mahwah: Lawrence Erlbaum.
- Lesh, R. & Sriraman, B. (2005). Mathematics Education as Design Science. *Zentralblatt für Didaktik der Mathematik*, vol. 37 (6), p. 490-505.
- Lester, F. K. (2005). On the theoretical, conceptual, and philosophical foundations for research in mathematics education. *Zentralblatt für Didaktik der Mathematik*, 37(6), 457-467.
- Skovsmose, O., & Valero, P. (2002). Democratic

- access to powerful mathematics ideas. In L. D. English (Ed.), *Handbook of international research in mathematics education: Directions for the 21st century*. Mahwah, NJ: Lawrence Erlbaum.
- Sriraman, B., & English, L. (2005). Theories of mathematics education: a global survey of theoretical frameworks/trends in mathematics education research. *Zentralblatt für Didaktik der Mathematik*, 37(6), 450-456.
- Sriraman, B., & English, L. (2006). Theories of mathematics education: European perspectives, commentaries and viable research directions. *Zentralblatt für Didaktik der Mathematik*, 38 (1), 1-2.
- Steen, L. (1999). Review of Mathematics Education as research domain. *Journal for Research in Mathematics Education*, 30(2) 235-41.
-

Gabriele Kaiser
Chief Editor, *Zentralblatt für Didaktik der Mathematik*
University of Hamburg
Faculty of Education
Von-Melle-Park 8
20146 Hamburg, Germany
E-mail: gabriele.kaiser@uni-hamburg.de

Morten Blomhøj
Editor, *Nordic Studies in Mathematics Education*
IMFUFA, Roskilde University
Postbox 260, Dk-4000 Roskilde
Denmark
E-mail: Blomhoej@ruc.dk

Bharath Sriraman
Editor, *The Montana Mathematics Enthusiast*
Dept. of Mathematics,
The University of Montana,
Missoula, MT 59812, USA
E-mail: sriramanb@mso.umt.edu