Theories of mathematics education: Is plurality a problem?

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Abstract: In this developed contribution to the Research Forum, held at the recent meeting of the International Group for the Psychology of Mathematics Education, the theme being "Theories of Mathematics Education", I focus on the call by the organisers: 'the time seems ripe for our community to take stock of the multiple and widely diverging mathematical theories'. I examine empirically the diversity of theories and I draw on the sociological theories of Basil Bernstein to relate the developments to the nature of intellectual communities and their productions. In particular, I suggest that the multiplicity and divergence are not surprising nor are they necessarily damaging to the field. I end by discussing concerns about accountability in relation to research in education.

ZDM Classification: D20

1. Introduction

Today, in many countries around the world, constraints on the funding of Universities together with demands for accountability are leading to restrictions on educational research. In some countries national policy is placing constraints on the kinds of research that will be funded (e.g. the effects of No Child Left Behind policy in the USA), in the name of accountability. On the other hand we can observe, at the same time, that research in mathematics education is proliferating, not just in quantity but also, as in the concerns of the Research Forum held at the twenty-ninth meeting of the International Group for the Psychology of Mathematics Education (PME), in the range of theories that are drawn upon in our research. In my contribution I want to ask: is this surprising, or unusual, and is it necessarily a hindrance to the effectiveness of educational research in mathematics?

In discussing these questions I would argue that we need a specific language that enables an analysis of intellectual fields and their growth, a language that will not be provided by mathematics or by psychology. I will draw on some of the later work of the sociologist of education, Basil Bernstein, in particular his 1999 paper on research discourses (reprinted also as Chapter 9 in Bernstein, 2000), and the results of a recent research study (e.g. Tsatsaroni, Lerman & Xu, 2003). Following that, I will make some remarks about the use of theory.

2. A Language of Research Fields

Bernstein (2000) draws on two notions: hierarchy and verticality. Discourses are described as hierarchical where knowledge in the domain is a process of gradual distancing, or abstraction, from everyday concepts. Hierarchical discourses require an apprenticeship: they position people as initiated or apprenticed, the relationship between the initiated and the expert is a pedagogic one, and these discourses are rich in language (*highly discursively saturated*, see Dowling, 1998). Horizontal discourses, by contrast, are generally acquired tacitly, the relationship will be an economic one not pedagogic ('master' and 'apprentice') and the discourse is consumed at the place of practice, not generalised beyond. Clearly academic and indeed school mathematics are examples of hierarchical discourses. Pedagogic modes are characterised by whether the rules for acquiring the discourse are made more or less explicit by the teacher. Students of school mathematics need to learn to recognise the school-mathematical in the texts they encounter and they need to learn how to realise, or produce, appropriate texts that demonstrate that acquisition. Bernstein describes the traditional, or performance model, as an explicit, or visible pedagogy, whereas the liberal-progressive or reform models are built around implicit or invisible pedagogies. It is not always clear to all students, for example, where the mathematical authority lies in the latter classrooms. In a traditional mode it clearly lies in the textbook and with the teacher. In the reform classroom students are taught to look to each other for confirmation or correction of their answers. The teacher tries to present her or himself as a learner on a par with the students. Whilst most modern mathematics teachers and educators might aspire to such a classroom there are dangers. Where the pedagogy is invisible those students who have acquired a rich language in their home life, what Bernstein calls an elaborated code, find that the rules and regulations both of scientific or theoretical discourses and of the moral discourse are more familiar than those students whose language is more restricted. Acquisition of linguistic capital is differentiated by social class.

For example, Holland (1981) gave young children a selection of cards on which appeared pictures of food and asked them to put the cards into sets according to whichever criteria they chose. Working class children organised them by criteria such as the food they liked and the food they didn't like. The middle class children classified them by criteria such as proteins, fats, animal products and vegetables. Holland then asked the children to put the cards together and arrange them in a different way. The working class children had no other strategy whereas the middle class children could then draw on everyday categories such as the food they liked and the food they did not like. Not only had the middle class children acquired scientific classifications (elaborated language) in their home lives, they also knew which was

privileged by the school. Readers should be aware that the acquisition of privileged linguistic resources is a matter of opportunity, not of innate aptitude. Delpit (1988) makes many of the same arguments in relation to students from African American and other minority ethnic groups. In our own field Cooper & Dunne, (2000; see also Lerman & Zevenbergen, 2004) show that setting mathematics tasks in everyday contexts can mislead students from low socio-economic background into privileging the everyday context and the meanings carried in them over the abstract or esoteric meanings of the discourse of academic school mathematics.

Bernstein's second notion, verticality, describes the extent to which a discourse grows by the progressive integration of previous theories, what he calls a vertical knowledge structure, or by the insertion of a new discourse alongside existing discourses and, to some extent, incommensurable with them. He calls these horizontal knowledge structures. Bernstein offers science as an example of a vertical knowledge structure and, interestingly, both mathematics and education (and sociology) as examples of horizontal knowledge structures. He uses a further distinction that enables us to separate mathematics from education: the former has a strong grammar, the latter a weak grammar, i.e. with a conceptual syntax not capable of generating unambiguous empirical descriptions. Both are examples of hierarchical discourses in that one needs to learn the language of linear algebra or string theory just as one needs to learn the language of radical constructivism or embodied cognition. It will be obvious that linear algebra and string theory have much tighter and specific concepts and hierarchies of concepts than radical constructivism or embodied cognition. Adler and Davis (forthcoming) point out that a major obstacle in the development of accepted knowledge in mathematics for teaching may well be the strength of the grammar of the former and the weakness of the latter. Whilst we can specify accepted knowledge in mathematics, what constitutes knowledge about teaching is always disputed.

As a horizontal knowledge structure, then, it is typical that mathematics education knowledge, as a sub-field of education, will grow both within discourses and by the insertion of new discourses in parallel with existing ones. Thus we can find many examples in the literature of work that elaborates the functioning of the process of reflective abstraction, as an instance of the development of knowledge within a discourse. But the entry of Vygotsky's work into the field in the mid-1980s (Lerman, 2000) with concepts that differed from Piaget's did not lead to the replacement of Piaget's theory (as the proposal of the existence of oxygen replaced the phlogiston theory). Nor did it lead to the incorporation of Piaget's theory into an expanded theory (as in the case of non-Euclidean geometries). Indeed it seems absurd to think that either of these would occur precisely because we are dealing with a social science, that is, we are in the business of interpretation of human behaviour. Whilst all research, including scientific research, is a process of interpretation, in the social sciences, such as education, there is a double hermeneutic (Giddens, 1976) since the 'objects' whose behaviour we are interpreting are themselves trying to make sense of the world.

Education, then, is a social science, not a science. Sociologists of scientific knowledge (Kuhn, Latour) might well argue that science is more of a social science than most of us imagine, but social sciences certainly grow both by hierarchical development (what might be understood as 'normal' social science (Kuhn, 1978)?) but especially by the insertion of new theoretical discourses alongside existing ones. Constructivism grows, and its adherents continue to produce novel and important work; models and modelling may be new to the field but already there are novel and important findings emerging from that orientation.

I referred above to the incommensurability, in principle, of these parallel discourses. Where a constructivist might interpret a classroom transcript in terms of the possible knowledge construction of the individual participants, viewing the researcher's account as itself a construction (Steffe & Thompson, 2000), someone using socio-cultural theory might draw on notions of a zone of proximal development. Constructivists might find that describing learning as an induction into mathematics, as taking on board concepts that are on the intersubjective plane, incoherent in terms of the theory they are using (and a similar description of the reverse can of course be given). In this sense, these parallel discourses are incommensurable. I conjecture, however, that the weakness of the grammars in mathematics education research is more likely to enable communication and even theory-building across different discourses, although I emphasise the term 'building'. It is no easy matter to join together different theories and it is done unsatisfactorily rather too often, I feel.

There is an apparent contradiction between the final sentences of the last two paragraphs. If I am claiming that there is important work emerging in different discourses of mathematics education research, but I also claim that discourses are largely incommensurable, within which discourse am I positioning myself to write these sentences? Is there a meta-discourse of mathematics education in which we can look across these theories? I will comment on this position in the next section.

3. Theories in Use in Mathematics Education

First I will make some remarks drawn from a recent research project on the use of theories in mathematics education. Briefly we (Tsatsaroni, Lerman & Xu, 2003) examined a systematic sample of the research publications of the mathematics education research community between 1990 and 2001, using a tool that categorised research in many ways. I will refer here to our findings concerning how researchers use theories in their work as published in PME Proceedings. The table below is expanded from that analysis, in preparation for a chapter on the development of socio-cultural theories in PME meetings to appear in a book to mark the thirtieth

meeting of PME in 2006. It covers all the Proceedings from when social theories first appeared, in 1985, to the most recent one in Melbourne in 2005. The expansion of theories in use within the mathematics education research community as a whole is almost entirely due to the social turn (Lerman, 2000). The categories are made up as follows:

1. Cultural psychology, including work based on Vygotsky, activity theory, situated cognition, communities of practice, social interactions, semiotic mediation
2. Ethnomathematics
3. Sociology, sociology of education, poststructuralism, hermeneutics, critical theory
4. Discourse, to include psychoanalytic perspectives, social linguistics, semiotics.

These categories mirror those we presented in Lerman & Tsatsaroni (1998).

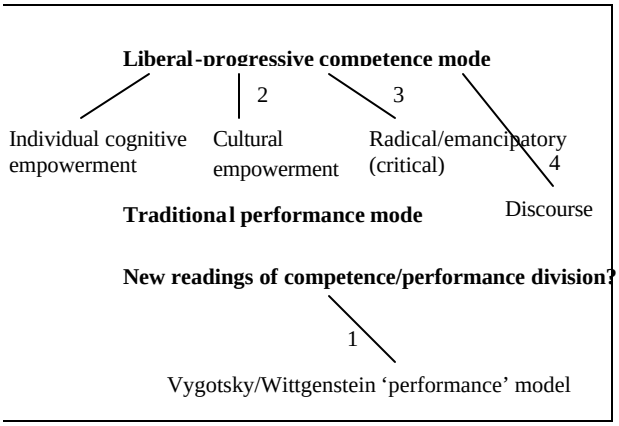

Figure 1: Pedagogic modes

Drawing on Bernstein's description of the turn from traditional performance pedagogy to a liberal-progressive competence pedagogy in the late 1950s, we proposed that this latter could be subdivided into: an individual cognitive focus, that is, Piagetian/reform/constructivism; a social or cultural focus, for example ethnomathematics (as in (2) above); and a critical focus, such as a Freirian approach (as in (3) above). We also suggested that there is evidence of a linguistic turn, to include social linguistics, critical discourse analysis and psychoanalytical approaches (as in (4) above), and, further, an emerging new performance model, quite different from the traditional, based on Vygotskian theories (as in (1) above). If indeed there is a new performance model, we must be conscious of the dangers of the accountability regime in many Western countries. Focusing on performance can be misinterpreted and draw us back into old performance models. This framework formed the basis of our discourse analytic tool (see Tsatsaroni *et al*, 2003), and these latter four constitute the four sub-sections of what I have called socio-cultural theories in the PME analysis. Of course further fragmentation of theories into sub-sections would, in some sense, give us a finer-grained analysis but would also lose both the theoretical rationale provided here and also the possibility of being able to identify trends over time.

I will make some comments drawn from our main study, but extended to take account of the full table.

Our analysis showed that just over 85% of all papers in the proceedings had an orientation towards the empirical, with a further 5% moving from the theoretical to the empirical, and this changed little over the years. A little more than three-quarters were explicit about the theories they were using in the research reported in the Research Report. The theories that were used have changed, however. We noticed an expanding range of theories being used and an increase in the use of social theories, based on the explicit references of authors, in some cases by referring to a named authority.

Table 1: Theoretical fields

PME meeting	Total no of Research Reports	Categories				Total	%
		1	2	3	4		
PME9 1985	76	2				2	3
PME10 1986	82	2				2	2
PME11 1987	153	2				2	1
PME12 1988	73	1				1	1
PME13 1989	102	3				3	3
PME14 1990	111	6	1		2	9	8
PME15 1991	126	7	1	3	2	13	10
PME16 1992	91	10	3	1	2	16	18
PME17 1993	88	9	1	1	2	13	15
PME18 1994	157	15	3	3	2	23	15
PME19 1995	77	12	1	1	2	16	21
PME20 1996	77	9			2	11	14
PME21 1997	122	12		1	7	20	16
PME22 1998	119	8	1	5	1	15	13
PME23 1999	136	7	3		4	14	10
PME24 2000	117	4	1		1	6	5
PME25 2001	171	8		1	4	13	8
PME26 2002	165	7		3	1	11	7
PME27 2003	176	6	3	5	1	15	9
PME28 2004	198	23		2	4	29	15
PME29 2005	130	14	1	5	8	28	22

We might suggest that there is a connection here with researchers creating identities, making a unique space from which to speak in novel ways, but we would need another study to substantiate and instantiate this claim.
We can say that there has been a substantial increase in the number of fields from 1994, although it is too early to say whether this trend will continue, as 1999 to 2003 showed a dropping off, although the percentages from 2004 and 2005 show a return to an increase. What is clear is that the range of intellectual resources, including sociology, philosophy, semiotics, anthropology, etc., is very broad.

In our analysis of how authors used theories we looked at whether, after the research, they revisited the theory and modified it, expressed dissatisfaction with the theory, or expressed support for the theory as it stands. Alternatively, authors may not revisit the theory at all, content to apply it in their study. We found that more than three-quarters fall into this last category, just over 10% revisit and support the theory, whilst four percent propose modifications. Two authors in our sample ended by opposing theory. This pattern has not changed over the years.

Further findings can be found in Tsatsaroni, Lerman and Xu (2003). The development and application of an analytical tool in a systematic way, paying attention to the need to make explicit and open to inspection the ways in which decisions on placing articles in one category or another, enables one to make all sorts of evidence-based claims. In particular, I would argue that one can observe and record development within discourses and the development of new parallel discourses because of the adoption of a sociological discourse as a language for describing the internal structure of our intellectual field, mathematics education research.

4. Conclusion

Finally, I will comment on concerns about the effectiveness of educational research in a time of multiple and sometimes competing paradigms, described here as discourses. ‘Effectiveness’ is a problematic notion, although one that certainly figures highly in current discourses of accountability. It arises because by its nature education is a research field with a face towards theory and a face towards practice. This contrasts with fields such as psychology in which theories and findings can be applied, but practice is not part of the characteristic of research in that field. Research in education, in contrast, draws its problems from practice and expects its outcomes to have applicability or at least significance in practice. Medicine and computing are similar intellectual fields in this respect.

However, what constitutes knowledge is accepted or rejected by the criteria of the social field of mathematics education research. Typically, we might say necessarily,

research has to take a step away from practice to be able to say something about it. Taking the results of research into the classroom calls for a process of recontextualisation, a shift from one practice into another in which a selection must take place, allowing the play of ideology. To look for a simple criterion for acceptable research in terms of 'effectiveness' is to enter into a complex set of issues. Indeed 'effectiveness' itself presupposes aims and goals for, in our case, mathematics education. To ignore the complexity is to lose the possibility of critique and hence I am not surprised by the multiplicity of theories in our field and the debates about their relative merits, nor do I see it as a hindrance. I am more troubled by how those theories are used. Too often theories are taken to be unproblematically applied to a research study. I am particularly troubled by the attacks on educational research as an inadequate shadow of a fetishised image of scientific, psychological or medical research, as we are seeing currently in the USA, increasingly in the UK, imminently in Australia and, I expect in other countries too. Finally, I consider that equity and inclusion are aspects of mathematics education that should be of great concern to all of us, given the role of a success in school mathematics as a gatekeeper to so many fields. I believe that the social turn and the proliferation of social theories have enabled us to examine and research equity issues in ways that our previous theoretical frameworks did not allow.

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