Theories of Mathematics Education[1]: A global survey of theoretical frameworks/trends in mathematics education research

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Abstract: In this article we survey the history of research on theories in mathematics education. We also briefly examine the origins of this line of inquiry, the contribution of Hans-Georg Steiner, the activities of various international topics groups and current discussions of theories in mathematics education research. We conclude by outlining current positions and questions addressed by mathematics education researchers in the research forum on theories at the 2005 PME meeting in Melbourne, Australia.

ZDM-Classification: D20, C30

1. Hans-Georg Steiner and Theories of Mathematics Education

In the recent history of the *Zentralblatt für Didaktik der Mathematik*, the year 1992 was different from any other year because it included a special 7th issue which focused on mathematics education in the Federal Republic of Germany (FRG). This issue, divided into three sections, included a comprehensive report and analysis of the state of mathematics education in FRG with articles addressing the curricular place of mathematics, mathematics teacher education and didactics of mathematics. In the third section, Hans-Georg Steiner co-authored a forward looking paper with Heinz Griesel in which the historical development of mathematics didactics in Germany (specifically the western part) was traced in great detail from pre-1900 onto that moment in time. This ambitious article included descriptions of the roles played by mathematicians, philosophers, educators, policy makers, professional organizations as well as practitioner and researcher journals in the development of mathematics education as a field of research. Griesel & Steiner (1992) also outlined the birth of mathematics education as a research discipline in Germany and the collaborations between working groups of researchers within Europe to develop and internationalize this fledgling field of research.

Hans Georg Steiner[2] (1928-2004)

One of Steiner's many contributions was the formation of an international study group called *Theory of Mathematics Education* (TME) which had held five international conferences until 1992, and was a regular topics study group at the quadrennial International Congress of Mathematics Education (ICME) until the turn of the last century at which point activity seemed to cease. Among the numerous creative products resulting from the groups' activities was the book edited by Steiner and Vermandel (1988) on the foundations and methodology of mathematics education as well as the release of the prominent book *Didactics of Mathematics as Scientific Discipline* (edited by Biehler, Scholz, Strässer & Winkelmann, 1994). The latter celebrated the contributions of Hans-Georg Steiner and the 20th anniversary of the Institut für Didaktik der Mathematik (IDM) in Bielefeld. This book offered a comprehensive survey of how mathematics education was viewed around the world until that point in time. Since then, the International Commission of Mathematics Instruction (ICMI) initiated a study of mathematics education as a research domain, resulting in a book (Sierpinska & Kilpatrick, 1998) published in two parts that juxtaposed the numerous international perspectives on mathematics education research. Several International Handbooks on mathematics education were also published in the last few years (Bishop, 1996, 2003; English, 2002). These handbooks display the numerous traditions of research around the world and the complexities of issues being examined in the ever-changing face of technology, curriculum, and policies. A comprehensive handbook on research design (Kelly & Lesh, 2000) is also part of the general canon. In spite of this Renaissance like flowering of mathematics education research, the field has been criticized for its lack of focus, its diverging theoretical perspectives, and a continued identity crisis (Steen, 1999).

2. Defining the problem

In the first decade of this new millennium, the time seems ripe for our community to take stock of the multiple and widely diverging mathematical theories, and chart possible courses for the future. In these

[1] Dedicated to Hans-Georg Steiner

[2] Photo retrieved from idw-online.de/ pages/de/news95049

volumes (vol 37(6) and vol 38(1)) of the *Zentralblatt für Didaktik der Mathematik* (ZDM) the current international discussion on the topic of theories is presented. The scope of the papers is within the context of the Research Forum on Theories of Mathematics Education held at the 29th Annual meeting of the International Group for the Psychology of Mathematics Education (PME) in Melbourne, Australia. The aim of these special issues of ZDM on theories (2005, vol 37(6) and 2006, vol 38(1)) is to re-initiate the discussion on the critical role of theories for the future of our field.

Our conception and preference for a particular mathematics education theory invariably influences our choice of research questions as well as our theoretical framework in mathematics education research. Our theoretical alliances also influence the content of university level mathematics education courses. One question that repeatedly confronts the field is whether or not mathematics education research is a scientific discipline akin to the hard sciences. If so, we need to consider the important role of theory building and theory usage in mathematics education research.

In recent years, Stokes (1997) suggested a new way of thinking about research efforts in science, one that moves away from the linear one-dimensional continuum of "basic, to applied, to applied development, to technology transfer." Although this one-dimensional linear approach has been effective, Stokes argued that it is too narrow and does not effectively describe what happens in scientific research. In, *Pasteur's Quadrant*, Stokes proposed a 2-dimensional model, which he claimed offered a completely different conception of research efforts in science. If one superimposes the Cartesian co-ordinate system on Stokes' model, the Y -axis represents "pure" research (such as the work of theoretical physicists) and the X-axis represents "applied" research" (such as the work of inventors). The area between the two axes is called "Pasteur's Quadrant" because it is a combination (or an amalgam) of the two approaches.

If we apply Stokes' model to mathematics education research, we need to clearly delineate what is on the Y-axis of Pasteur's quadrant, namely our established theories if we are to call our field a science. For instance, in physics, theoretical terms have to be defined operationally, that is they have to describe nature via theories in which terms are accepted only if they can be defined/backed up via experimentation. Having said this, one question for consideration by the community is whether we can adapt this to educational research? That is, how can learning scientists operationally define observational terms (namely perceived regularities that we attempt to condense into theories, or as Piaget attempted – to phylogenetically evolved mental cognitive operators) (Dietrich, 1991, 1994, 2004). Arguably, given the difficulty of abstracting universal invariants about what humans do in different mathematical contexts, which in turn, are embedded within different social and cultural settings suggests that it is futile enterprise to formulate grand theories. As Henry Pollak once said, there are no proofs per se in mathematics education. However the community has engaged in formulating theories and models in smaller scales that attempt to address fundamental questions. In Alan Schoenfeld's (1999) presidential address to the American Educational Research Association in Montreal, three of the key areas that were mentioned as needing development were (1) theoretical developments in specific "arenas" with considerations for informing practice, (2) synergyising theory and applications and (3) practical development with considerations for informing theory. (Schoenfeld, 1999). The systemic work engaged in by Schoenfeld's research group at Berkeley, California has resulted in a teacher's decision-making model of "teaching in context" which provides a fine-grained characterization of the teacher's decision-making, grounded in the analysis of teacher's knowledge, teacher's goals, and teacher's beliefs (see Schoenfeld 1998, 1999a, 1999b, 2002a).

A different systemic approach to mathematics education research has been the work of the Models & Modelling Group. The emerging perspective of researchers in the domain of models and modelling suggests that when researchers are trying to understand how people engage in real world problem solving, they are in a sense looking at complex systems. There are three kinds of *complex systems*: (a) "real life" systems (or simulations of such systems) that occur (or are created) in everyday situations, (b) conceptual systems that humans develop in order to design, model, or make sense of the preceding "real life" systems, and (c) models that researchers develop to describe and explain students' modeling abilities. These three types of systems correspond to numerous reasons why the study of complex systems should be especially productive for researchers who are attempting to advance theory development in the learning sciences (Lesh, personal communication). This suggests that one aim of mathematics education research should be towards generating models. Rather than thinking in terms of a one-to-one match between research studies and solutions to problems, it's more reasonable to expect that results from many research studies should contribute to the development of a theory (or model) that should have significant payoff over a prolonged period of time.

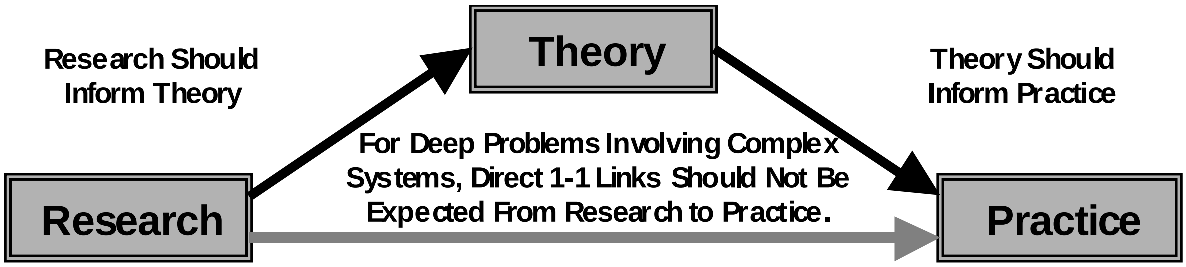

Figure 1. Lesh (in press, a)

In a sense the work and the fundamental questions raised by Steiner and his colleagues were revived in this PME Forum. The primary rationale for such a forum was that in spite of significant advances in mathematics education research, the role of theories and theory development remained in need of attention from mathematics educators. The purpose of the Forum was to stimulate critical debate in the area of theory use and theory development, and to consider future directions for the advancement of our discipline. The issues to be addressed included the following.

- What is the role of theory in mathematics education research?
- How does Stokes (1997) model of research in Science apply to research in mathematics education?
- What are the currently accepted and widely used learning theories in mathematics education research? Why have they gained eminence?
- Is there evidence of marked political structures in the use and development of theories of mathematics education?
- What is happening with constructivist theories of learning?
- Embodied cognition has appeared on the scene in recent years. What are the implications for mathematics education research, teaching, and learning?
- Theories of models and modelling have received considerable attention in the field in recent years. What is the impact of these theories on mathematics research, teaching, and learning?
- How can theories from the general domain of cognition contribute to mathematics education research?
- Is there a relationship between researchers' beliefs about the nature of mathematics and their preference for a particular theory?
- What are the implications of trends in learning theories for the training of future mathematics education researchers?
- To what extent do mathematics theories impact on practice in schools?
- Where are we heading in the development and application of theories in mathematics education research? What is required to propel mathematics education forward?

The Research Forum stimulated critical debate on the above issues Numerous PME members took the opportunity to share and debate their viewpoints, which we hope will sow seeds for significant advancements in our discipline. We also point out that this particular Forum was by no means the first attempt to discuss these fundamental questions as is evidenced in the past initiatives of Steiner and his group.

3. The Past and the Present

The community has tried to address some of these questions and the literature offers multiple, varied viewpoints that constitute the background of the above issues. One plausible explanation for the presence of multiple theories of mathematical learning is the diverging, epistemological perspectives about what constitutes mathematical knowledge. Another possible explanation is that mathematics education, unlike "pure" disciplines in the sciences, is heavily influenced by cultural, social, and political forces (e.g., D'Ambrosio, 1999; Secada, 1995; Skovsmose & Valero, 2002). Mura's (1998) survey of mathematics educators in Canadian universities indicated that Jean Piaget, Zoltan Dienes, and Jerome Bruner were among the most frequently cited theorists. This may indicate a preference for cognitive learning theories over social learning theories, but could also be attributable to geographic location.

Artigue (1999) wrote an interesting article on the state of the research in the *Notices of the American Mathematical Society*, one which articulated a French mathematician/mathematics educator's position on the nature and the state of mathematics education research. In her survey Artigue commented on the influence of Piaget's work on the constructivist phase of mathematics education research from 1980 onwards, one which conveyed a new perspective of learning but also the fact that learning was irreducible to a simple transmission of facts. However the constructivist "model" of learning was criticized for not taking into account the social and cultural dimensions of learning.

Later in this article, using Brousseau (1997)and Chevellard's (1992) frameworks as a starting point, Artigue pointedout the inadequacies of numerous theories at that time to explain the discontinuties in learning that occur for students confronted with advanced mathematical topics in the university setting. According to Lerman (2000), the switch to research on the social dimensions of mathematical learning towards the end of the 1980s resulted in theories that emphasized a view of mathematics as a social product. Social constructivism, which draws on the seminal work of Vygotsky and Wittgenstein (Ernest, 1994) has been a dominant research paradigm ever since. There is also confusion as to whether social constructivism is a cognitive position or a methodological perspective or both.(Noddings, 1990; Ernest 1991).

On the other hand, cognitively oriented theories have emphasized the mental structures that constitute and underlie mathematical learning, how these structures develop, and the extent to which school mathematics curricula should capture the essence of workplace mathematics (e.g., see Stevens, 2000). A recent 'alternative' cognitive perspective on the nature of mathematics is that of "mind-based mathematics" (Lakoff & Nunez, 2000). This view suggests that mathematics is a product shaped by human brains, societies, and culture. Lakoff and Nunez's (2000) concept of "the embodied mind" maintains that the body and brain together with everyday experiences structure our conceptual systems. By adopting the perspective of embodied mathematics, Lakoff and Nunez argued that mathematics can be made more accessible and comprehensible to learners provided they are made aware of how mathematical ideas are shaped (i.e., via "conceptual metaphors"). Social constructivists have been quick to argue that social interaction is a necessary condition in this embodied conception of mathematics.

Underlying these varying perspectives is a particular belief on the nature of mathematics itself, which, one could argue, mathematics education researchers do not openly declare. For example, the social constructivists' preference for Lakatos' (1976) vision of mathematical truth as being subject to continual revision over time suggests a fallible and non-Platonist viewpoint about mathematics; this is in contrast to the Platonist viewpoint, which views mathematics as a unified body of knowledge with an ontological certainty and an infallible underlying structure. von Glasersfeld (1984) interpreted the function of cognition as an adaptive mechanism that helped one organize the experiential world, not the discovery of an objective, ontological "mathematical" reality. Piaget, on the other hand, hypothesized that there was an "ideal" isomorphism between cognitive mental structures and mathematical structures (Beth & Piaget, 1966). Further work is needed to shed light on the relationship between the beliefs of mathematics education researchers about the nature of mathematics and their preference for a particular theory.

In surveying research methods in mathematics education in the last century, Schoenfeld (2002b) wrote:

The flowering of theoretical perspectives and methods continued through the end of the 20th century. Specifically, sociocultural perspectives had long roots. Vygotsky, for example…[h]ad advanced a perspective that, perhaps in too-simple terms, could be seen as complementary to Piaget's. Vygotsky and his theoretical allies argued that learning is a function of social interaction. (p.442)

Schoenfeld's (2002b) conclusions about the state of the field at the end of the 20th century were at best pragmatic:

At the end of the century, one saw a proliferation of perspectives, of theories, and of methods. On the one hand, this was undoubtedly healthy…[O]n the other hand, having let a thousand flowers bloom, it is time for researchers in mathematics education to prune their garden. We must begin to ask and address the difficult questions about theory and methods that will help us move forward. (Schoenfeld, 2002b, p. 443)

Lester (2005) wrote that the previous characterizations of mathematics education as an atheoretical field may have to be revisited.

Mathematics education research is an interesting and important area for such an examination. Although math ed research was aptly characterized less than 15 years ago by Kilpatrick (1992) and others as largely atheoretical, a perusal of recent articles in major MER journals reveals that theory is alive and well: indeed, Silver and Herbst (2004) have noted that expressions such as "theory-based," "theoretical framework," and "theorizing" are common. In fact, they muse, manuscripts are often rejected for being atheoretical. (p.173)

4. Topics addressed in this issue

Numerous extant surveys of the field (Furinghetti, 2003; Lerman, 1998, 2000; Schoenfeld, 2002b) have pointed out that since the 1980's mathematics education research been largely concerned with the cultural and social aspects of learning. While there are geographic variations in the types of sociocultural theories employed by researchers, it is noteworthy that mathematics education researchers in France have by and large utilized Brousseau's theory of didactical situations, which can be viewed as a social theory. In the 1980's researchers in Germany were influenced by the work of Bauersfeld, which also emphasized the social aspects of teaching and learning. The papers in this issue tackle the core questions listed earlier. Frank Lester discusses the reasons why theories are essential to the work of mathematics educators. In doing so, he utilizes Pasteur's quadrant (as described by Stokes, 1997), to help us think about why theory development is so important. Lester also addresses possible reasons for why some researchers either ignore or misunderstand/misuse theory in their work. The three difficult questions addressed by Frank Lester are:

- *What is the role of theory in education research?*
- *How does one's philosophical stance influence the sort of research one does?*
- *What should be the goals of mathematics education research?*

John Pegg and David Tall reflect on neo-Piagetian formulations such as SOLO or Case as a learning hierarchy and assess where they stand in the current climate of research in mathematics education. Different theories are brought together with the aim of delineating similarities and differences. These authors attempt to move beyond these theories and to address some fundamental questions in learning.

Moreno-Armella and Sriraman explore education and mathematics from the viewpoint of an epistemology and theory of cognition that makes sense in the modern world of "natural-born cyborgs." They consider the evolution of cognition and the importance of semiotic systems along this process, including modern technologies. Part of their focus is also on the need to include the role of representations in any theory of mathematical thinking and learning.

The ultimate pay-off of the considerable research in mathematics education is the hope that it will eventually inform practice and shape curricula. Lesh (in press, a) proposes a completely different perspective on the nature and purpose of mathematics education research. Lesh makes a convincing case to abandon making "epsilon changes" to the traditional curriculum, grounded in the 3R's of the 1900's (namely reading, writing and arithmetic) and to re-conceive the nature and purpose of today's mathematics curriculum. He poses several fundamental questions for mathematics education researchers to consider, such as:

- *What is the nature of typical problem-solving situations where elementary-but-powerful mathematical constructs and conceptual systems are needed for success in a technology-based age of information?*
- *What kind of "mathematical thinking" is emphasized in these situation?*
- *What kind of understandings and abilities should be emphasized to decrease mismatches between: (i) the narrow band of mathematical understandings and abilities that are emphasized in mathematics classrooms and tests, and (ii) those that are needed for success beyond school in the 21st century?*

The two inter-related articles on the models and modelling perspective (Lesh & English) and on mathematics education as a design science (Lesh & Sriraman) in this issue convey a different conception on the nature of problem solving, mathematical thinking and learning in the 21st century. These authors argue that in general, when knowledge develops through modeling processes, the knowledge and conceptual tools that develop are instances of situated cognition. Models are always molded and shaped by the situations in which they are created or modified; and, the understandings that evolve are organized around experience as much as around abstractions. Yet, the models and underlying conceptual systems that evolve often represent generalizable ways of thinking. That is, they are not simply situation-specific knowledge which does not transfer. This is because models (and other conceptual tools) are seldom worthwhile to develop unless they are intended to by powerful (for a specific purpose in a specific situation), re-useable (in other situations), and sharable (with other people).

Any journal issue that attempts to adress the issue of theories would be incomplete without an exposition on constructivism and broader issues pertinent to the topic of theories in mathematics education. Numerous journal articles, books and monographs have tackled the origins, use and misuse of the term constructivism in mathematics education research. One of the creative products of working group 4 on theories of learning mathematics at ICME-7 in Quebec City, Canada was the release of an important book for the field called Theories of Learning Mathematics (edited by Steffe, Nesher, Cobb, Goldin, Greer, 1998). This working group was led by Perla Nesher with Leslie Steffe as the consultant and Nicolas Balacheff, Hans-Georg Steiner and Erik de Corte as the co-advisors. Part III of the aforementioned book focusses on constructivism and the learning of mathematics and among others contained contributions from Ernst von Glasersfeld (a proponent of radical constructivism) and Paul Ernest (a proponent of social constructivism).

5. A preview of vol 38(1) on theories of mathematics education

In the next ZDM issue on theories of mathematics education (vol 38(1)), among the various articles we will include a piece by Paul Ernest on the philosophical underpinnings of mathematics education research today and particularly his reflections on the status of constructivism and his assessment of the mutating meaning of this term. Steve Lerman will address theory usage trends in PME reports in the 1990-2001 time period and view mathematics education as a social science. In doing so, he will also address the social turn in our field and sociological perspective on mathematics education, over time. The paper by Törner & Sriraman will be historical in nature and compare trends in mathematics didactics in Germany with trends that occurred in the United States. This paper will illustrate that certain didactical trends seem to occur simultaneously in different region of the world, or they seem to re-occur periodically. Given that the PME Forum only included one European perspective on mathematics education, the next issue also includes a contribution by Sriraman & Kaiser on trends within theory usage in Europe based on a survey and analysis of research papers in seven of the fourteen working groups at the 2005 European Congress on Mathematics Education (CERME4). Proponents of social constructivism ascribe an important role to the teacher as an interface between the content and the student. In other words their role is integral in the classroom milieu. In one of the articles in the next issue, the role of teacher discourse in mathematical thinking and learning will be analyzed.

The next issue will also include critical commentaries on the papers in this issue. Some of these commentaries will introduce the European point of view on the discussion of theories as well as critique the positions of the papers in this issue. Such an enterprise would further Steiner's vision of bridging theoretical traditions which are independently formulated in different regions of the world.

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