The computer as "an exercise and repetition" medium in mathematics lessons: Educational Effectiveness of Tablet PCs

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Abstract: The request of a new educational culture within the classroom goes hand in hand with the introduction of the new Educational Standards. That is they are essentially connected with a paradigm shift. The Project supports this aim via different tasks administered through Tablet PC's within the scope of exercise and repetition phase of learning. The central concern is to find appropriate lesson approaches through computer use in everyday life at school, which are conducive for the math learners and are at the same time easy to effectively implement in other classrooms. In the summer of 2004 the use of Tablet-PCs in school took place in two 9th classes of an Ostalbkreis secondary school in Baden-Württemberg. We report on the effectiveness of this new technology in the classroom.

Zusammenfassung:
Mit dem Einführen von Bildungsstandards und dem damit verbundenen Paradigmenwechsel geht der Wunsch nach einer neuen Unterrichtskultur einher. Das Projekt unterstützt dieses Ziel durch unterschiedliche Aufgabenformen zu verschiedenen Leitideen im Rahmen der Übungs- und Wiederholungsphase. Das zentrale Anliegen ist, durch die Erprobung des Computereinsatzes im realen Schulalltag geeignete Unterrichtsansätze zu finden, die für das Mathematiklernen förderlich und gleichzeitig für alle Beteiligten einfach zu realisieren sind. Durch den Einsatz von Tablet-PCs im Mathematikunterricht wurde eine zukunftsweisende Entwicklung erprobt. Bei dem Projekt handelt es sich um einen Forschungsbeitrag zum Thema Neue Medien im Mathematikunterricht. Der Unterrichtseinsatz fand im Sommer 2004 in zwei 9. Realschulklassen des Ostalbkreises in Baden-Württemberg statt.

ZDM-Classification:

Introduction

Lessons orientated on educational standards are relatively new in Germany since they have only been in existence since 2004 (Bescherer 2005). Essentially they follow the "Principles and Standards for School Mathematics 2000" of the USA. This demands a new lesson structure, which is orientated on competencies. The educational standards formulate common competencies for all students. However as learning is an individualistic process (to an extent), teachers have address the needs of a diverse group of students in their lessons. One of the possible ways is the use of computers.

In the educational-political discussion questions about the use of computers play an important role again and again since many years. A result of those discussions is the binding anchorage of the computer in all school subjects (cp. e.g. curriculum 2004). The rapid increase of the computer as a general medium is still confronted with hesitation in math lessons (Drabe 2002) and has remained a relatively low area of research work for the use of computers in maths lessons. In the federal conferences for the didactics of mathematics, only a relatively few papers addressed the role of new media (Brockmann 2003). Hischer points in this coherence to the discrepancy in the amount of the "catchwords" ("computer-license" etc.) and of the "convincing arguments" for the use in maths lessons (Hischer 2002, p. 8). This discrepancy exists as well between existent technology and the one used in class. C. Laborde describes this as follows:
"Concerning school technology the overall situation can briefly described by saying that there is a gap between, on the one hand the availability of hand held technology for students and the intentions of the Ministry of Education, and on the other hand, the real integration of technology into the practice of teachers." (Laborde 2001)
Actually the expectations in the computer have become more realistic in the last few years: There is no revolution in maths lessons because of the existence of a new medium, but the evolution or a gradual and reasonable integration of new technology can proceed (Weigand, Weth 2002, p. 10). After all a lot of arguments speak for the use of computers in maths lessons as seen in different Proceedings documents (example, Herget, Weigand, Weth. and Bender et al.; Beckmann 2003). That further research and development works are necessary goes without saying. Concretely, the work directly on site of the classes and the work with the consideration of the complicated everyday life at school – in which the use of computers is often absent because of organizational reasons - are necessary. Elschenbroich notes:
It is appropriate to develop stable tasks for a broad and successful implementation in the classroom, which enable the students to develop mathematical activities without too many hurdles (Elschenbroich 2001, p. 172)). Elschenbroich himself developed a number of accumulations of electronic work sheets for working with DGS – predominantly for secondary level – which are self-explanatory and therefore appropriate for independent working. Although there are a wide variety of tasks administered via the use of different software, very few observational studies have been conducted with German students (e.g. Hole 1998, Hoelzl 1999, Kautschtisch 2000, Heinz 2003, cp. as well Straesser 2002 and exercises here for in Giedig 2003). The calculator evaluation study conducted by the Bertelsmann foundation pointed out interesting aspects of computer use through students and teachers. However the study addressed only part of mathematics lessons and is a special case for everyday life at school because of the permanent laptop presence (Schaumburg, H. et al,2002).

The goal of this research effort is to add to the extant literature on the effective use of technology, particularly Tablet PC's in Math lessons. The central concern is to find appropriate lesson approaches through computer use in real everyday life at school, which are conducive for the math learners and are at the same time easy to effectively implement in other classrooms. Although different mathematical task forms for exercises and repetition in direct use by students have been previously

tested; is a forward-looking development achievable by the use of Tablet-PCs in maths lessons? Weigand and Weth (2002) asked: How will the further development of the insertion of new technologies look like? …. The notebooks are always becoming smaller and more manageable. They will be designed so universal that they can be used in all school subjects (p. 11f.).

1. The Tablet-PCs

The basis of this project is 31 Tablet-PCs1, which are available for the PH Schwäbisch Gmünd through a HBFG-offer – total value 90353 €financed one half each by the federal state Baden-Württemberg and the Federal Republic of Germany - since the beginning of 2004.

They are intended and used for the study of lectureship in all subjects and therefore especially for their use in academic practice (See Figures 1a and 1b).

Fig. 1a   Tablet-PC in keyboard function

Fig1b. Tablet-PC in tray form with pencil function

2. The project
Exploratory focus

The project is about individual learning against the background of the educational standards. Exploratory focus is the use of Tablet-PCs as exercise and repetition medium in real everyday life at school.2 The study took place at the end of the school year in July 2004 in two 9th classes of secondary level of the Ostalbkreis in Baden-Württemberg. Therefore we developed tasks appropriate to the educational standards of 9th grade, which students could chose by themselves and solve them independently. For the tasks involving exercise and repetition we consciously choose problems that students were familiar with from class, which were also aligned to various Content Standards.

For the content standard for number powers, students are expected to calculate the negative exponent, for the content standard of measurement they should ascertain the surface of a cylinder and for the content standard of space and form they had to compute the surface area of a centrically stretched figure. The use of a computer to accomplish and integrate these tasks implicitly was new. Connected with it was the integration of automatic checks, support and the possibility to modify output parameters in terms of functional approaches. Moreover the tasks were compiled in such way that the connection between them could be developed in support of deeper understanding. Further tasks were tested, where verbalized formulas had to be evaluated which could be a helpful connector between abstract formula and calculation that are bound to visualization. To us Tablet-PCs seemed to be suitable for these important phases in maths lessons as they probably could contribute to changes and motivation and support important competencies which are demanded by the educational standards. The central research questions were:

1. Are Tablet-PCs suitable for spontaneous exercise and repetition tasks in maths lessons?

2. How should exercise and repetition tasks be structured and prepared so that they are positively received by the students?

3. Are mere computer tasks or tasks connected with work sheets advantageous?

4. Does the computer facilitate independent work? The new German curriculum emphasizes the ability to get information with the aid of the computer independently as an important competence (cp. curriculum 2004, p. 64).

Can the following mathematically relevant activities be activated and assisted by exercise and repetition tasks with the computer? Observing and recognizing mathematical connections, experimental working, an understanding of how formulas work, graphs, terms and notions, depicting and constructing, developing and using new strategies. Which sort of exercise tasks are difficult

and which are easy? Are there particular opportunities that assist the understanding through the use of the computer? Which task aspects are most advantageous?
5. Does the computer motivate students Motivates to check their own results? What are the aspects of (a) independent work? (b) reflection, and (c) self regulation of actions on the Tablet- Pc's?

Conception of the tasks and choice of subject
The tasks that had to be dealt with were offered for free choice at the learning sales counter3. They affect the following subject areas and the connected competences according to the educational standards of 9th grade, namely Plane geometry with Pythagoras and centrically dilation, powers, quadratic equation or function, practical problems with per cent and calculations at the cylinder. The tasks had been provided in different forms, as a file with work sheets that had to be filled out in written form or as a work sheet that had to be dealt with or corrected by means of the computer. The tasks that had to be completely dealt with on the computer differed in two ways. In one task type the results had to be detected and checked independently. In the other one the tasks had to be directly controlled or were supported.

The tasks were oriented towards the concept of functional working of the mathematical contents. The point was not to use exercises to facilitate procedural learning but to enhance conceptual learning, allow insights into successful solution strategies and offer reasons to continue learning. Accordingly, the exercises aimed at a structured consciousness and the ability of a flexible exposure to notions and methods (cp. therefore Hole 1998, S. 60 ff. and Weigand, Weth 2002, S. 32 ff). The opportunity of one's own action and rediscovering will be given.

We now introduce the different task types used in this research. The tasks are textually and methodically based on user software that deals with known problems and features. The preparation and configuration in terms of the research questions of the project are especially new (cp. 2.1). The tasks refer to certain competences, which are formulated in the educational standards and activate appropriate integrations, too. More details of these tasks are found in the Appendix.

Cylinder file
The tasks of the topic cylinder are based on functional connections such as "What if,…" It should be assessed for example by exploring how various parameters such as surface and volume change in a cylinder of constant radius when the height is doubled. Compositional abilities are addressed when the students have to asses if the lateral area of a cylinder is always bigger than its base. The learners have the choice of (a) imagining the

changes spatially, (b) looking at the structure of the formulas, (c) experimenting by putting appropriate values into the table or if they let the appropriate cylinder been drawn. To be able to analyse the work of the students every click on a switch (true / false / support / check; see figure in the appendix of "cylinder file") was registered by a little program4 not visible for the students. The sequence 0h1 recorded in the background means for instance, that the first solution was wrong (0), the support was assessed (h) and then a correct solution was typed in (1) The number of first-attempt correct solutions were: both counted and visible to the students in real time.
Practical problems with per cent file
This file contains 5 tasks that address different uses. Among other things discount, interest, instalment spread and/or added value tax are to consider in dissimilar buying situations. Through the operationalizing of the tasks the knowledge of basic value, part, percentage rate are reviewed. In some cases for example a buying decision has to be made. In addition a catenation with graphical aspects occurs partly, as the students are asked to depict the tasks in diagrams. The file is equipped for the spreadsheet Excel so that the calculations can be carried out by direct formula input. As input assistance a table with the central sizes of the tasks is given and can be used for the input. More support is not intended. The solution has to be handed in on a work sheet. The input of the students is saved in a file for evaluation.

Raise to a power file
The task sequences for raising to a power are aiming at a textual understanding and constructive working. The connection of raising to a power with the operations of multiplying and dividing is made for instance (see figure in Appendix: 2. table for rising to a power). The file contains 4 task sequences in which are tested what effects little changes can cause to a term. The tasks vary only very little In the last task sequence only the logical value of a task as for example $2^{-3}=(-2)^3$ or $2^3 x 4^3=(2x4)^{3x3}$ has to be appointed (see figure in Appendix: 4. Table for raising to a power). The presentation of the tasks is similar as in the file "cylinder", as instant check and support are offered by little background programs.

File group plane geometry
In this complex task four files are located. Three of those have to be solved by variation tasks and one with a construction task. Intention of the variation tasks for centric dilation and theorem of Pythagoras is to develop and deepen the appropriate connections through one's own action, thus by dynamical changing of the basic construction and the observing of invariances. In contrast to paper-pencil geometry the students can make

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3 We choose the slightly laborious way on floppy disk as we were just shortly able to use the PCs and therefore could not utilize all opportunities. The use of the WLAN-function since the ending of the project relieves the work essentially.

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4 The programs in the background are edited in the language Visual Basic for Applications (VBA). VBA is integrated in Excel. The programs can be observed by changing to the **Programmierebene** with the hot key Alt-F11 or by choosing several programs directly using menu Extras → Macro → Macros → **process (Bearbeiten).** There are two examples for the file rising too a power in the appendix.

detections by moving the points (Healy and Hoyles 2001) and use their mathematical knowledge at the same time (Sinclair 2004). In every figure with two similar triangles for instance the stretch factor can be changed with an adequate drawing. At the same time the dependence to the surface area, which is calculated in each case, can be observed. For the task sequence "construction of a Limacon" students were expected to use the theorem of Pythagoras. With the aid of the available dynamic geometry software a construction should be made as well. Therefore the hypotenuse of the triangles have to be calculated and observed first. The solutions have to be recorded and saved in a textbox in each case. Through this they are available for later evaluation.

Quadratic equation/ quadratic function work sheet
The aim of these tasks to establish a connection between equations, features of functions and their graphs and therefore contribute as well to a linked up relation of functions. With it the particular opportunity of the computer, to depict graphical and non-graphical – as well as calculative- aspects of functions at the same time with less extravagance and with great number are used (cp. for that e.g. Heugl 1999, Weth 1998, Beckmann 1996, 1999). Above all the verbal aspect is considered as well by appropriate work orders on the work sheets. With this kind of task type another form of computer work is activated. Starting with a work sheet on which the students have to fill in their solutions the computer is a detection and control medium5. The work sheet is built up like this: At first the knowledge is repeated or acquired. Afterwards the knowledge has to be used in different situations. So the connections between functional equations and graph have to be observed at first and afterwards this knowledge has to be applied to further functional equations or graphs.

2.3 Research methods
In July 2004 the Tablet-PCs were used in each of the aforementioned cases in 4 lessons in two 9th classes of secondary level with 54 students. Usually two students worked together on one computer. Only a few wanted to work alone. For documentation of the activities individual groups were observed closely. Student employees wrote notes of the course, the conversations, particular problems, the noticeable problems etc. Moreover video recordings were made which were transcribed later. Further, we got information directly out of their ongoing work on the tasks on the Tablet PC's (we term this "task machining") as students saved their results after the machining or wrote it down on a work sheet. The results of the plane geometry tasks were recorded in a textbox. The files of the subject areas cylinder and raising to a power received task integrated logging of the student's actions and the student's results. That is, this was the ongoing disguised documentation that was occurring while students were engaged in the tasks.

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[5] The program Matheass was used here because of the easy implementation. Another motive is that the program had already proved it's value in other lesson situations.

Finally conversations with the students about the project took place.

3. Results

3.1. Implementation of the Tablet-PCs
The use of Tablet-PCs in class is easy and uncomplicated. But it has to be mentioned that the background work was quite laborious as each Tablet-PCs had to be packed in bags to be transported to school and be charged there afterwards. Therefore carts with socket strips should be easily and spontaneously be available for everyday use at school. Accumulators last well enough for two lessons.

The use by the students turns out to be positive. Most students negotiated their way immediately without adequate introduction. In doing so they detected independently features like the pencil function. Basically this applies to the user programs, too. With regard to the everyday use in class, "familiar" tasks are best to use, which students can pursue independently without special introduction to the programs. This was or approach in this research. But in a few cases, we had to introduce students to the dynamic geometry software.. The knowledge of the "return"-function turns out to be an important requirement for the machining of the tasks with the instant check. Uncertainties arose when the control button was pressed before the solution input in the table cells was completed with return. The student work at the Tablet-PCs was different. There were some students who alternated. But there were also dominant students who had the mouse the key board always under their control. In exceptional cases the partner was even counted out. But basically the students agreed before they entered each input. Conversations occurred as well. The students who had finished their tasks helped their fellow students in individual cases. Altogether the student liked it that they needn't to write much because of the computer work; they also liked the changing between mere computer work and work sheet supported computer work.

3.2 Task Machining
The students could choose the tasks they had to deal with arbitrary. At first this turned out to be positive as they showed different preferences. Compared with the other files the file "Practical problems with per cent", which has the only application-oriented title, was frequently used as an introduction by the students. On the other hand not all of the students engaged in all of the tasks. Only about one third of the students dealt with the tasks in turn6. The reasons why some of the tasks didn't have solution inputs are probably variable. In accordance with our observation some tasks were just not chosen or omitted. They seemed to be too difficult or they were dealt with but the solutions were probably not set out in writing due to formulation difficulties. This could have been managed and controlled better by a routing slip.

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[6] Another opportunity might be that the several groups that proceeded in turn were not able to finish in time.

Mistakes in saving the solutions are as well possible - a problem which does not occur in traditional lessons. Moreover it has to be borne in mind that expenditure of time was not the same for all tasks. The file Practical problems / per cent in particular and the work sheet on quadratic function as well demanded more time.

	Practical problems/ per cent	plane geometry	quadratic function	cylinder	powers
delivered solutions7	19	22	9	26	14

Table 1. Part of the task sequences

The students coped differently with the machining of the tasks. This could be connected with the different computer knowledge of the students. Some students needed quite a long time for trying to understand the tasks. Other could start working on the tasks straight away. It could be observed that students who read the working and task instructions conscientiously were in the advantage. The students mentioned the following requests in relation to the tasks: not too much text, not too many task sections, a good structuring and numbering of the tasks and buttons for support, retrieving of fundamental mathematical contents such as definitions. Finally the solutions could be directly checked on the computer. This was not considered equally in the conception of the tasks (cp. 2.2). Mathematical background knowledge could only be accessed in part (cp. the files cylinder and rising to a power).

3.3 Task results

Cylinder file
Two-thirds of tasks worked on were solved correctly in the first attempt. Generally there was the tendency of solving those tasks correctly at first go in which a routine calculation was needed. Tasks which were based on quadratic or other non-proportional functions (example task 1f): if one doubles the radius of a cylinder then diameter and surface area of the circle is doubled as well caused difficulties. Table 2 summarized these results:

Task	1a	1d	1c	1g	1b	1e	1h	1i	1j	1f
Task type	C	C	C	N	C	P	N	N	I	N
correct at first go	84 %	84 %	80 %	80 %	76 %	76 %	68 %	68 %	60 %	52 %

C=calculative
N= Non-proportional
P= Proportional
I= independent

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[7] The amount of the handed in solutions is about the solutions per work group. It has to be borne in mind that the half of the students of one class could participate only three of four hours. So, not all 54 students of the classes were present continuously.

Table 2. Proportion of the solutions of the topic cylinder which were correct at fist go subjected to the task type

Practical problems with per cent file
The task file "practical problems with per cent" was chosen quite willingly and was also provided with quite a long activity for the students. One problem was that the students often could not remember the notions and formulas. They also could not develop them independently by means of the tasks. The students managed only in part to become acquainted with the topic by means of the simple issues even though the sequence was structured well. Merely task 1 where the remembering of the basic notions and coherences of calculation of percentage take center stage was - except for one exemption - solved correct. The notional difficulties appear in the form that the table cells below the tasks were often not filled in appropriately according to the headline of the cell. Task 4 for example deals with the buying of a mountain bike. In this task 4 of 7 students quoted 0.7%x6, 0.7%x12 etc. as interest rate for the paying by instalments. This tendency shows that misunderstandings of the notion of the interest as the reason for the incorrect final solutions. Table 3 provides more details of the results.

task/ input	Basic value	Percent age rate	formula discount	Interes t rate	formula interest value
2 (purchase at wholesale)	25 %	92 %	100 %	-	0 %
4 (mountain bike)	100 %	-	-	43 %	17 %

Table 3
Proportion of the correct solutions of the task parts

Mistakes accrued as well as the students did not read and transfer the tasks, notions etc. properly enough. In task 3 the delineation of a diagram takes center stage. This task was solved correctly only on very few occasions. Either the students combined too many jobs or the coherence happened only in art. In task 2 (purchase at wholesale) the bank interest was always calculated for a whole year instead of 20 days.

Raising to a power file
66% (class 1) und 79% (class 2) of the raising to a power tasks were solved correctly at first go by the students who dealt with this task group. In general simple powers with natural exponents 1, 2, 3, … were solved correctly at first go in 90 to 100%. Powers with negative exponent or the exponent 0 caused the most difficulties. Only 25% to 64% were solved correctly at first go.
Table 4 shows the results varied.

The tasks are arranged according to the proportion of the correct solutions.

task	1c	1e	1j	2d	2f	2g	3d	3e	2b	2c
Kind of the expo-nents	>0	>0	>0	>0	>0	>0	>0	>0	>0	>0
Correct at first go	100 %	100 %	100 %	100 %	100 %	100 %	100 %	100 %	100 %	100 %

task	3h	3c	4°	4j	1i	3	3b	4b	1b	1d
Kind of the expo-nents	>0	>0	>0	>0	>0	>0	>0	<0	>0	>0
Correct at first go	100 %	100 %	100 %	100 %	100 %	100 %	100 %	82 %	80 %	80 %

task	4i	4d	2i	1g	4e	4g	2h	3j	4c	3i
Kind of the expo-nents	>0	<0	<0	>0	>0	>0	=0	<0	<0	<0
Correct at first go	77 %	77 %	73 %	73 %	73 %	73 %	71 %	69 %	67 %	64 %

task	3f	3g	1a	1h	4f	4h	2a	2e	1f	2j
Kind of the expo-nents	>0	>0	>0	>0	>0	>0	>0	>0	>0	<0
Correct at first go	77 %	77 %	73 %	73 %	73 %	73 %	71 %	69 %	67 %	64 %

Table 4
Proportion of the solutions of the task sequence raising to a power with were solved correctly at first go.

Plane geometry file
The tasks for plane geometry were based on dynamic geometry software. Only one of the two classes was already introduced to simple construction opportunities. Complicated constructions, macros, term objects and textboxes were not familiar to both classes. As the tasks were conceived in such a way that they could be attempted on the Tablet-PC's without previous knowledge, class two was at no advantage On the contrary, class one solved more tasks correctly. The part of correct answers was only the same in tasks 4 in which construction knowledge were advantageous.

	task 1	task 2	task 3	task 4
crawled tasks	50 %	56 %	67 %	78 %
correct solutions thereof	44,4 %	20 %	25 %	35,7 %
correct, but incomplete	33,3 %	60 %	66,6 %	57,1 % (correct, but not designed)

Table 5
Proportion of the crawled tasks

The conception of appropriate tasks demands that the students acquire certain technical abilities during the machining. It appears that incipient uncertainties in the machining of the tasks are affecting the solution behaviour. Task 1 for example was laborious- as the relation in the solution of task 2 shows - but the solution was not quoted in the textbox. In task 3 (Pythagoras) the most common mistake occurred in the interpretation of the term objects, which calculate the surface area of the squares. 42% of the students assumed that it is about the longitudinal calculation or the surface area of a triangle.

Quadratic function/ equation work sheet
The work sheet quadratic function was mainly chosen by class 2. In this class this topic was currently being covered. The fact that only 9 groups chose the sheet (cp. table 1) can be due to the topic. In summary it can be said, that the tasks were basically dealt with correctly. This occured even if the verbal description of the features of the graph for the construction direction was carried out correctly (task 2 and 3). For instance: all of the 8 students described the graph of the function $y = (x-3)^2-2$ as correct; as upwards opened and the vertex (3/-2) (here it is a exception). Nevertheless two of them chose (3/0) as vertex in the drawing. And although 88% characterized the function $y = -10x^2 + 2$ completely correctly, three of them sketched the graph incorrectly.

3.4 Use of built in Computer-support fosters independent learning.
A special value of the computer use can be seen in the offering students the opportunity to work independently. The study showed that the PC actually made the students in most case to try longer. Even if the task could not be solved satisfyingly a teacher was rarely called or not at all. Those students obviously saw the computer as a medium for independent working. The request of the students for more opportunities of support directly on the computer underlines this. But it was also observed that in some cases, students – maybe out of habit - very quickly called for a teacher

In the task collection the task sequences "cylinder" and "raising to a power" offered direct computer support. Beside the permanent sight on the basic formulas in the upper table area the opportunity for direct support was given as well. The texts and examples, which are recallable in the support system basically, address the comprehension. In the support of the task 3 raised to the negative 2 = it is not instantly mentioned that the power is equal to the term $1/3^2$. 3 to the power of -2 results e.g. from a fraction with the numerator 3 to the power of 2 and the denominator 3 to the power of (2-4), so it is 3 to the power of -2.

The task sequence for the topic cylinder offers three dissimilar sorts of facilities. Beside the support call and the permanent visible formulas there is also the opportunity for the experimental checking of the variation of the cylinder variables and of the drawing.

Even if it can be detected, that at an average 7,0 tasks of the cylinder file were solved correctly at first go and that the students, who used the support, solved at an average 7,3 tasks correctly at first go, it is not possible to correlate how the using of support is affecting the result positively. It is however much more significant how often such supports were used, as this is an important condition for independent learning. Remarkably enough, direct support opportunity was used only very little. Only 6 of the 14 students who chose the tasks for raising to a power used the support system. In the task sequence cylinder just 3 of 26 students chose the support system. In the subject area raising to a power, on an average, every student used the support only once although on an average 9 mistakes were made at first go. The reasons are multifaceted. It is rather unlikely that the support texts are too complicated as many students did not retrieve the texts at all. It was not tested if negative preexperiences with support offerings existed from other software systems. From the conversation with the students, we discerned that the reason was instead the ambition of being able to solve the tasks without using any support. Maybe the influence of prior lesson structures is the reason why using support could be interpreted as the confession of weakness. Possibly the students count mainly on the direct support of the teacher. This could partly be observed by the student employees. Therefore it follows that in the school setting students are offered few opportunities for independent learning. In addition none or only a few preexperiences existed with similar tasks. Since independent learning with Tablet PC's demanded the proper and optimal use of support, it is conceivable students could have avoided many mistakes by using the visible rules in the upper table area. This built in support was largely unused. Maybe the computer is not familiar enough as an information medium.

In the work sheet for quadratic functions, the computer was to be used to as a control medium – as support for recognizing mistakes. It was explicitly intended that students use the software for the comparison of their solutions, for reflection over mistakes, and to note this on the work sheet after the tasks on the sheet were completed. The observation of the lesson showed that the students used this control function. The results of incorrect graphical delineation described in 3.3 suggest that the computer is used as a control medium and not as a "cheating medium".

3.5 Mathematical activities and competences
The observation of the lessons, the notes and video recordings showed that nearly all the students were continuously motivated to work on the math activity by working on the Tablet-PC. Often the solution was discussed before it was entered. Error messages activated particular students to find the correct solution independently. The observation of the lessons relate to the following activities:

Observing and realizing mathematical connections

Numerous international comparative studies have consistently shown that algorithms and routine calculations are over emphasized in German math lessons, and less emphasis is placed on student discovery of concepts and connections. In the guiding principle for the competence acquirement in maths the curriculum in Baden-Württemberg for example points out (curriculum 2004, p. 60): the connections between mathematics and reality becomes more important through the use of modern media, which allow modelling of original situations Under the guiding principle of number, one reads: students should realize and describe connections, orders and structures. The task sequence prepared by us considers both aspects according to the formulated educational standards. On the one hand calculations have to be carried out and formulas have to be evaluated, but on the other hand relations have to be realized. This relational aspect is also addressed in our research. The results of this research are reported next.

Example 1:
The task sequences for plane geometry allow the dynamic changing of the basic combination. In the "Pythagoras task" for example the „Pythagoras figure" - consisting of the triangle ABC and the squares of side - can be changed by drawing at the triangle points A, B and C. At the same time the surface area of the squares is calculated. Other files of the task collection allow changes as well. For example through the input of new files as for instance radius and height in the cylinder file. Other changes arise from the progression of modified terms as in the raising to a power file and the work sheet for the quadratic function.

Particular observations:
The changing of the basic combination can be ascribed to specific geometrical coherences.
This central and basic character of dynamical geometry systems was observed in the project, too. "Pythagoras only works at 90 degree." Or "If stretch factor z_1 is doubled, s_2 is doubled as well" are appropriate observations. How far reflection with hindsight, conversations about the observation have to occur was already discussed elsewhere (cp. Beckmann 2001).

Example 2:
In the cylinder and powers file the calculations as well as the relational aspect are considered. In the cylinder tasks the recognition of connection especially within the tasks (1e to i) is picked out as a central theme as well as the recognition of an inexistent connection in task 1j. In the power tasks the recognition of connections is not only picked out as a central theme between the terms within the task but as well between consecutive tasks.

Example 3:
In the tasks 1 and 2 of the plane geometry file (dilation 01 and 02) dynamic changing of the basic combination is explicitly asked for. By drawing at the point in the box the position and length of the line segment is changed.

Particular observations:
The moving of a point out of the basic combination is irritating to some extent. Some of the students tried to move the figure directly, while others did not understand the connection between Z1 and the drawing.

With the dynamic changes the leisure for casual observation was partly missing. Direct manipulation and slow observation of the changing would have immediately explained the connection between Z1 and the construction. The observation could have been more complete to some extent. Example of the answers of task 1b: "when Z1 is doubled the line segment s2 is getting bigger."

Altogether the students tried to solve the tasks quickly. In the following example (from video observation) the observation took just 13,4 seconds, while the written work took 37,7 seconds.

The following vignette reveals the communication process at particular time durations.
Task plane geometry, task part 2b: what happens when Z1 gets negative?
(7 seconds)
Student A (moves mouse pointer towards slider in the box): Z1
Student B: yes, up there
A (changes the figure of the theorem on intersecting lines by changing of the stretch factor)
A (draws; stretch factor gets negative): yes, great, it is in the center (sounds unsatisfied)
(6,4 seconds)
B: It is behind the centre
A (moves forward, back again, to and fro)
B: Hey! (awaits reaction of A)
A: Aha
B: Please write it down
(11 seconds)
A (recalls text support to fill in their observation): What shall I write? Is it 2b?
Silence
(26,7 seconds)
B: It is on, the point, what's that, the line segment, what's that?
B: The line segments Z1 is behind the center then
A (types into the description field): the line segment is behind center.

The tasks were disposed in a way that the machining was concentrated on maths and not on the software. One of the two classes worked with the dynamic geometry system for the first time in the project. The construction background was unfamiliar. In two groups of the class, understanding difficulties appeared. They did not know why the line segment s1 did not change when Z1 was modified. This would be unproblematic in one's own construction and the appropriate period of vocational adjustment would not have been necessary.

Example 4:

One particular task within quadratic functions deals with the observation of connections due to discrete, „static" changes. Example: draw the graphs in each case and observe: $y = x^2$, $y = 2x^2$, $y = 3x^2$
The students partly did not acquire all aspects in the example above. Some of them only looked for similarities and not at differences and noted: [graphs are] "always upwards opened, vertex is on the zero-point."
The observation on the computer depends on the chosen scaling. How far the observations on the computer really lead to new perceptions partly depends on the chosen delineation/ scaling as well. A group, for example noted for the task given above: "Nearly equally slim".

Experimental working, developing and using strategies

The development of problem solving abilities is stated as in the educational standards as an important mathematical competence. The computer offers particular chances as it can activate experimental proceeding and the development of strategies. Our research indicates that this was accomplished with the use of Tablet PC's.
Example 1:
In the second table of the cylinder file (see figure in appendix) the changing of the cylinder variable through appropriate input effects a direct change of the dependent variable like surface and volume. In task 2 this is utilized for the experimental ascertaining of the minimal surface (material consumption) of a cylinder (tin).

Particular observation:
The discrete variation of variables for the solving of a task can be a contribution for the problem solving ability as functionings are systematically listed. The following student dialogues for judging of the least purchases in the task sequence cylinder suggest this:

Dialog 1:	Dialog 2:
student C: We could go on like this for hours.	student E: 5,9
student D: No, try 4,1.	student F: Try 5,2
student C: And now?	student F: Try 6
student D: Type in 5,0 please .... You see	student F: Try 5,5
	student E: It is already less

Other students developed an interesting software specific strategy. With it they also managed a very clearly arranged solution. They copied all formulas into 11 consecutive lines and increased the radius gradually by 0,1 from 5,0 to 6,0. In this connection the radius with the minimal surface 553,6 can be read clearly (cp. table 6).

		$h = \dfrac{V}{\pi\, r^2}$			$O{=}2^*G{+}M$	
Radius	Durchmesser	Höhe	Grundfläche	Mantel	Oberfläche	Volumen
50	100	12,7	78,5	400,0	557,1	1000,0
51	102	12,2	81,7	392,2	555,6	1000,0
52	104	11,8	84,9	384,6	554,5	1000,0
53	106	11,3	88,2	377,4	553,9	1000,0
54	108	10,9	91,6	370,4	553,6	1000,0
55	110	10,5	95,0	363,6	553,7	1000,0

bei dem radius 5,4 braucht man am wenigsten blech

Table 6
Solution of the students of the cylinder file: experimental proceeding with strategies

That the computer provides particular opportunities for experimentation is impressively shown by the solution of the students above (table 6), by the particular opportunities for making connections with dynamic geometry software (cp. plane geometry file) or by the depiction of dissimilar graphs and functional equations (cp. the work sheet of quadratic equation).

Example 2:
In the task group quadratic function, one aspect of the task was for students to observe the effect of the coefficients on the graphs. Students had to type in different functional equations, such as $y = 2x^2$, $y = 0{,}5x^2$, $y = -4x^2$ or $y = x^2 -3$ and consider their graphs.

Particular observation:
The observation of the effect of changes on the computer led to clever and strategically more advantageous work forms. After our observation many students typed in all given equations at the same time. Therefore the screen was abuzz with graphs which led to solving and to new systematic input and to observation.

Example 3:
The developing of coherences and connections through variation can occur through experimental proceeding. This demands a minimum amount of systematization. Beside the tasks in plane geometry, the second task of the cylinder file is aimed at this purpose. The radius for a 1l unit tin with the minimal usage of a steel sheet is to be found out to one significant decimal place. Therefore different radius values are typed in and the change of the surface is researched with the support of a table which is provided with appropriate formulas.

Particular observations:
It seemed as if the students had very few prior experiences in experimentation. Only 9 of the 26 tasks given above were correct.

Working with formulas and terms
The relational aspect as well as the formal aspect and the ability to use formal knowledge textually belong to the „mathematical literacy". The task collection considers both. The formal aspect is accommodated in the new

educational standards as well. Again the curriculum of Baden-Württemberg is quoted here under the process standard modelling: Students are able to accomplish solving equations with the computer when asked (curriculum 2004, p. 66).

Example 1:
The file sequence cylinder and rising to a power contain computational and relational tasks.

Particular observation: the computational aspect was mastered much better by the students (cp. above and table 2 and 4).

Example 2:
In the practical problems file, connections between the variables were demanded by formulas. The spreadsheet offered novel opportunities for simple entry, quick calculation and discerning connections among variables.

Particular observation:
The computer was used only to some extent for the entry of formulas. In task 2 (purchase at wholesale) for instance the solution was not typed in over a formula but directly into the computer in 5 of 13 cases.
Formulas very often were typed in incorrectly into the computer. The tabular entry of the variables and formulas – which was meant as support - could be the reason for this. For instance as an already typed in variable, the interest rate, does not occur in the formula any more (table 9). In the traditional working with the exercise book this mistake probably could have been avoided. But it could also have been the case that due to the experiences with spreadsheets with already integrated formulas the students consciously set the new input aside. Finally, notional uncertainty and the missing knowledge of formulas could have played a role as well.

	A	B	C	D
1				
2				
13	Zahlungsbedingung	Grundwert	Zinssatz	Zinswert
14	bar	1100	3%	(3/100)*1100
15	6 Raten	1100	0,7%	0,7*6
16	12 Raten	1100	0,7%	0,7*12

	A	B	C	D
1				
2				
13	Zahlungsbedingung	Grundwert	Zinssatz	Zinswert
14	bar	1100	3%	B14/100
15	6 Raten	1100	0,7%*6	B15/100

Table 7

Solutions of students to task 4 which deals with the calculation and the comparison of the costs of a mountain bike if it is paid in cash or by installments.

Example 3:
The work sheet for quadratic functions required a engagement with functional equations. In particular the connections between graphs and equation have to be observed. Therefore specific characteristics of functional

equations are developed by one's own observation at the computer as for example the meaning of a and b in ax² + b for the appearance of the parabola.

Particular observation:
Terms were only cursorily considered by the students. In some instances students did not pay attention to features of an equation. For instance, some mistook the linear equation y = 4x + 2 as an upwards opened parabola.

Working with notions
Human communication is based upon notions. Thereby maths has developed its own language with particular technical terms. Particular notions facilitate communication and lead to explicit understanding.

Example 1:
The use of notions occurred especially in the task group practical problems with per cent.

Particular observations:
The notional lack of knowledge complicated the solution. The observations and the video recordings showed that students obviously had knowledge deficits in finance-mathematical notions. Here they explicitly wished for more explanations with support buttons. The following dialogue on task 5 of two students S1 and S2 shows how a notional lack of knowledge affects problem solving behaviour negatively.

subscription price	business expenses	assets	added value tax	final prize
153		95		248

S1:   ... business expenses...
S2:   This is namely this: look, business expenses, business expenses, so it's 16 % here and 16 % here (points at the subscription price and assets filled out by themselves, cp. Table above)
S1:   yes
S2:   and you have to calculate the 16 %out of it I think, 153 plus 16 % and plus 16 % as well
S1:   here .. wait a second ..
S1:   So, here we have 100 plus 16 %, as it is the 0,16 %, so here it has to be ... 0,16 % as well ..
S2:   The per cent is correct
S1:   ... you have to add 16 % to the added value tax here (points at the crevice added value tax)
S2:   No, no that doesn't work, you can't type that in.

Example 2:
In one part of the task sequence cylinder, students have to state which radius determines the least amount of sheet steel for a 1-liter-tin. In the already prepared table the measures for surface, nappe and volume are automatically calculated when the radius is typed in. For the solution of the task the essential variable of the surface has to be observed.

Particular observation:

The notional lacks of knowledge complicated the result. In the task given above for example the students were uncertain if they had to examine the nappe or the surface. Citation of a student: Now, how much sheet steel is needed? That's the nappe. 2000, it is the nappe. Surface.

Graphical describing and changing between dissimilar display formats

An essential competence, which plays an important role in everyday life, is the graphical description of coherences and the understanding of diagrams. In the curriculum much of the content relies on a firm understanding of the notion of a function. Numerous empirical as well as international comparative studies have shown that students have insufficient abilities in this area (German PISA- syndicate 2001, PISA- syndicate Germany 2004). Often the basic competence for the change between the notations is missing in particular. In the task collection this aspect is considered in subtasks.

Example 1:
The change between term/equation and graph is the basic topic on the work sheet quadratic function. In the first task part the connections between functional equations and appearance of the graph are picked out as a central theme. The following task deals with connections between functional equations and number of zeros or intercept points. Because of the few students, who worked on the latter, we only report on the first part.

Particular observation:
The students need more experiences with graphs.
The computer can be seen as an opportunity which motivates precise observations. The study shows graphs are examined only very superficially by a lot of students. The aforementioned particular comment of a student "nearly equal slim" for the characterization of the graphs of x², 2x², 3x² mentioned above confirms this. At the same time it was observed in a group of students that they had difficulties with checking their solutions. After the input of the functional equation to the quoted graphs in task 3 they found favour with they were uncertain in the beginning if the graphs on the screen and work sheet corresponded to each other. The graphical representation of a function for which the equation, the features and the vertex were given did not work in many cases (table 8).

task part/ functional equation	proportion of the false devolved graphs	reason
ab) y = -10x² + 2	50 %	upwards opened, reputedly the figure in the window is not possible
c) y = (x-3)² -2	38 %	vertex at (3/0),

Table 8

Proportion of incorrect solutions in task 2, quadratic function and reason

Constructing

In conjunction with independent problem-solving, constructing plausible solutions via the use of mathematical constructions is considered as an important competence. In dynamic geometry systems, constructions can lead to essential detections. As the tasks of the collection are predominantly designed for direct use in class without previous knowledge, this activity was largely set aside and the constructions were given. Only in task 4 (Limacon) of the plane geometry file students were explicitly required to perform a construction.

Particular observation:
Constructions with dynamic geometry software resulted in the highest error ratio. Overall just 28% of the handed in solutions were correct in task 4. In addition, many students mentioned that they would have liked a deeper introduction to the program.

The students tried to avoid a construction in the program in the same way as they used the set square for drawings in the traditional lessons. Unlike to drawings on the paper this has serious consequences in the exposure to dynamic geometry systems. The coordinates of points can be changed at any time. If the drawing was not constructed properly, the whole drawing would not be correct any more after the moving of a point. The necessary insight in constructing properly could be arranged more effectively on the computer as they would come from the students themselves. This becomes clear at the latest, when the drawing is changed and therefore is not correct any more. In this study not all of the students achieved this level of understanding – probably because of the invested effort for the understanding of the technical process of the construction. The following vignette from a dialogue between student S2 who helps student S1 shows that S1 was glad to have a picture after long construction efforts which was in some degree correct:

S1:   Now I've got the hang of it, I've got it.
S2:   circle around it, oh well (turns head away), you've got it (critical undertone). But this gotta be on the line. That's not right-angled.
S1   (agrees at first grudgingly): Hmm (then exhaling): Oh well, a little bit

Summary and competences

This project studied how individual learning occurs in a computer mediated environment, given the call of the new Educational Standards for incorporating the use of technology in the mathematics curriculum. In particular we analysed the role of carefully designed tasks for facilitating independent learning on Tablet PC's. The study took place in two 9th classes of Ostalbkreis secondary school. The computer use was implemented by portable Tablet-PCs, which were introduced as an exercise and repetition medium for the mathematical subject matter at the end of the school year. In order to keep the setting as naturalistic as possible, the tasks were conceived in a way that they did not demand any additional preparation for the teachers and could be introduced directly. But one consequence of this approach was that students were not introduced sufficiently to the programs as well. We summarize the main findings of this study.

The Tablet-PCs are suitable for the spontaneous use in maths lessons (catchword: media literacy). For the uncomplicated use in everyday life in class, a suitable trolley should stand by in school. The handling with the pen-mode was unproblematic and was used easily by many students.

The experiences have showed that the Tablet-PCs with the given tasks are suitable as an exercise and repetition medium (catchword: achievement of the educational standards). The students worked intensely and stayed committed to the tasks.

The chosen tasks were in their composition a suitable material for independent working of the students as well (catchword: individual learning). Routine/procedural computer tasks were favoured more by the students. However, the students viewed the diverse tasks introduced via worksheets and computer as positive, too. Even if the tasks can be used in the present form, we recommend a solid introduction to the handling of dynamic geometry systems, more exposure to spreadsheets, and stronger encouragement for the use of built in support as a strategy for independent working. The construction of one's own can also be used as a strategy for the solving of problems or geometrical detections (catchword: competences). Depending on the view of the students, the screen pages to the exercise tasks should not contain too much text and not too many task parts. The tasks should be well structured and numbered and support buttons and basic definitions should be available. It also should be possible to check the solutions directly on the computer.

Tasks which are particularly suitable are those which lead to the recognition of connections through the changing of basic combinations (input values, point in the draw mode) (catchword: competences). For this the computer offers good opportunities. The study showed that during the machining of the tasks, mathematical coherences were detected and strategies for solution or clever procedures were developed partly. Computational tasks were normally solved successfully.

On the other hand, it could be recognized that some students could not use the computer sufficiently as a medium for independent working or as a support and control medium yet (catchword: individual learning). The computer work was often carried out without delight, patience and accuracy which led to disadvantage during the solving of the tasks. This phenomenon was particularly observed with tasks in which no direct checking was offered. This appears e.g. when the observation of functional graphs and terms is not exact,

when it seems to be okay for the students if the geometrical construction is not totally correct or when the data in a graph are not summarized in an adequate way. The tendency of solving a task quickly is as well shown in the time given above of only 13,4 seconds for a geometrical observation but it also shows that a lot of students did not read the text closely until their attempt in the task failed.

The essential results suggest that students should be encouraged to slow down a bit when using the computer. Probably this is the most difficult task as the students of today are mostly familiar with the use of computer as a game playing device, in which quick reactions are the norm. Integrated facilities and opportunities for checking the solutions were deemed as essential features by the students to facilitate the use of the Tablet- PC's on the various tasks. The computer offers better conditions to achieve this as opposed to the use of paper and pencil for such tasks.

Conclusion

The use of Tablet-PCs as exercise and repletion medium in the chosen form creates new learning opportunities for mathematics lessons. The Tablet-PCs and the tasks create problem-solving experiences, facilitate conceptual knowledge, the recognition of errors in some instances, formation of connections and enhance the possibility for students' acquiring the competences outlined in the educational standards.

References

Beckmann, A. (1996). *Propaedeutic curve sketching on the computer.* In: mathematic school (Propädeutische Kurvendiskussion am Computer. In: Math. Schule) 34 (1996) 1, S. 40-50

Beckmann, A. (1999*). The functual notion as subject at the beginning of maths lessons in the biennial full-time vocational school (Der Funktionsbegriff als Unterrichtsgegenstand zu Beginn des Mathematikunterrichts in der zweijährigen Höheren Berufsfachschule),* in: JMD 20 (1999) 4, S. 274-299

Beckmann, A. (2001). *Problems in learning how to prove – DGS as solution? (Probleme beim Beweisenlernen – DGS als Lösung?),* in: Elschenbroich, H.-J., Gawlick, Th., Henn, H.-W. (Hg.): drawings – figure – drwa figure (Zeichnung – Figur – Zugfigur), Hildesheim, Berlin (Franzbecker) 2001, S. 21-30

Beckmann, A. (2003). *Interdisciplinary maths lessons – part 4: maths lessons in cooperation with computer science* (Fächerübergreifender Mathematikunterricht – Teil 4: Mathematikunterricht in Kooperation mit Informatik), Hildesheim, Berlin (Franzbecker) 2003

Bescherer, Chr. (2005). *A short story of the enducational standards in maths lessons (Eine kurze Geschichte der Bildungsstandards in Mathematik),* in: Engel. J. et al (Hg.): structuring – modeling - communicating (Strukturieren - Modellieren – Kommunizieren), Hildesheim, Berlin (Franzbecker) 2005

Brockmann, B. (2002). *The use of the computer is in flux. The try of a longitudinal section, in. contributions for the maths lessons (Computereinsatz im Wandel. Versuch eines Längsschnitts, in: Beiträge zum Mathematikunterricht)* 2003, Hildesheim, Berlin (Franzbecker) 2003, S. 149-152

Bildungsplan (2004), curriculum for secondary level (Bildungsplan für die Realschule), Baden-Württemberg, Ministerium für Kultus, Jugend und Sport, Stuttgart (Neckar-Verlag) 2004

Deutsches PISA-Konsortium (Hg.) (2001). PISA 2000, *Basic competences for students in international comparison (Basiskompetenzen von Schülerinnen und Schülern im internationalen Vergleich),* Leske+ Budrich, Opladen 2001

Drabe, M. (2002). The integration of media in school (Medienintegration in der Schule), in: Herget, Sommer, Weigand, Weth (Hg.): media bandy mathematics (Medien verbreiten Mathematik), Hildesheim, Berlin (Franzbecker) 2004

Elschenbroich, H.-J. (2001) *visual teaching and learning, in: contributions for the maths lessons (Visuelles Lehren und Lernen,* in: Beiträge zum Mathematikunterricht), S. 169-172, Hildesheim, Berlin (Franzbecker) 2001

Elschenbroich, H.J., Seebach, G. (versch. Jahre). *Detect dynamic geometry. Electronic work sheets with Euklid-Dynageo and Cabri II (Dynamisch Geometrie entdecken. Elektronische Arbeitsblätter mit Euklid-Dynageo und Cabri II).* CD, CoTec, Rosenheim, 2000 bis 2005

Gieding, M. (2003*). The didactics of spreadsheet!? 25 years spreadsheet and nearly no perceptions in: contributions for the maths lessons (Didaktik der Tabellenkalkulation!?. 25 Jahre Tabellenkalkulation und kaum didaktische Erkenntnisse,* in: Beiträge zum Mathematikunterricht) 2003, (Franzbecker), Hildesheim, Berlin 2003, S. 237-240

Healy, L., Hoyles, C. (2001). *Software tools for geometrical problem solving: Potential and pitfalls.* In: International Journal of Computers for Mathematical Learning 6, Kluwer Academic Publishers 2001, S. 253-256

Heintz, G. (2003*). The use of DGS considering as example of Cinderella teaching and learning programs for maths lessons (Einsatz von DGS am Beispiel von Cinderella Lehr-und Lernprogramme fuer den Mathematikunterricht),* in: Bender, P., Herget, W., Weigand, H.-G., Weth, Th. (Hg.): Proceedings 20. Arbeitstagung des Arbeitskreises 'Mathematikunterricht und Informatik' in der GDM e.V., Soest, (Franzbecker), 2003 S. 71-78

Heugl, H. (1999). *Computer-algebra-system - the Promised Land of maths lessons? (Computeralgebrasysteme – das gelobte Land des Mathematikunterrichts?)* In: Kadunz, G. et al. (Hg.): mathematical education and new technologies (Mathematische Bildung und neue Technologien), Stuttgart, Leipzig (Teubner), S. 127-146

Hoelzl, R. (1999). *Qualitative lesson studies fort he use of dynamic geometry software (Qualitative Unterrichtsstudien zur Verwendung dynamischer Geometrie-Software),* Augsburger mathematisch-naturwissenschaftliche Schriften v. 32, (Wissner) Augsburg 1999

Hischer, H. (2002) *maths lessons and new media (Mathematikunterricht und Neue Medien),* Hildesheim, Berlin (Franzbecker) 2002

Hole, V. (1998). *Successful maths lessons with the computer, methodic and didactic basics in secundary level (Erfolgreicher Mathematikunterricht mit dem Computer, Methodische und didaktische Grundfragen in der Sekundarstufe I),* Donauwörth (Auer Verlag) 1998

Hole, V. (2002). *Neue Ansätze zum Programmieren in der Sekundarstufe I,* in: Abele, A., Selter, Chr.: maths lessons between tradition and innovation (Mathematikunterricht zwischen Tradition und Innovation), Weinheim (Beltz Verlag) 2002, S. 285-301

Malle, G. (2000*). Two aspects of functions: classification and covariation (Zwei Aspekte von Funktionen: Zuordnung und Kovariation),* in: mathematik lehren (2000) 103, S. 8-11

Kautschitsch, H. (2001). *DGS supported assuming and proving (DGS-unterstütztes Vermuten und Beweisen).* In: Elschenborich, H.-J., Gawlick, Th., Henn, H.-W. (Hg.):

Proceedings, Oberwolfach 2000, (Franzbecker) Hildesheim, Berlin 2001, S. 113-122

Knoche, N., Lind, D., Blum, W. et al (2002). *The Pisa study, some results and analysis (Die Pisa-2000-Studie, einige Ergebnisse und Analysen),* in: JMD 23 (2002) 3-4, S. 159-202

Laborde, C. (2001). *Integration of technology in the design of geometry tasks with cabri-geometry.* In: International Journal of Computers for Mathematical Learning 6: 283-317, Kluwer Academic Publishers 2001

NCTM - National Council of teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, Virginia USA 2000

PISA-Konsortium Deutschland (Hg.) (2004*). PISA 2003 – educational background of teenagers in Germany, results of the second international comparison (Der Bildungsstand der Jugendlichen in Deutschland, Ergebnisse des zweiten internationalen Vergleichs),* Waxmann, Münster, New York, München, Berlin 2004

Schaumburg, H., Issing, L.J. (Hg.) (2002). *Lernen mit Laptops – Ergebnisse einer Evaluationsstudie,* (Verlag Bertesmann Stiftung) Gütersloh 2002

Sinclair, M.: (2004*) Working with Accurate Representations: The case of Preconstructed Dynamic Geometry Sketches.* Journal of Computers in Mathematics and Science Teaching (2004) 23 (2), 191-208

Straesser, R. (2002). *Research on Dynamic Geometry Software (DGS) – an introduction.* ZDM, 2002, v. 34 (3)S. 65

Weigand, H.-G., Weth, Th. (2002). *The computer in maths lessons. New paths to old targets (Computer im Mathematikunterricht. Neue Wege zu alten Zielen),* Heidelberg, Berlin (Spektrum- Akademischer Verlag) 2002

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Appendix

The tasks (with examples)
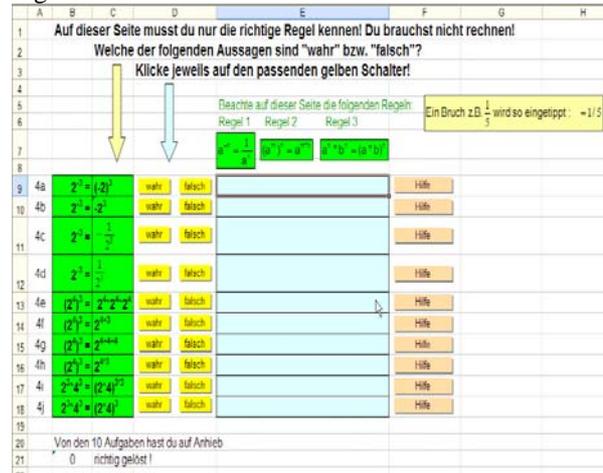Cylinder file
Figure 1.

Figure2.

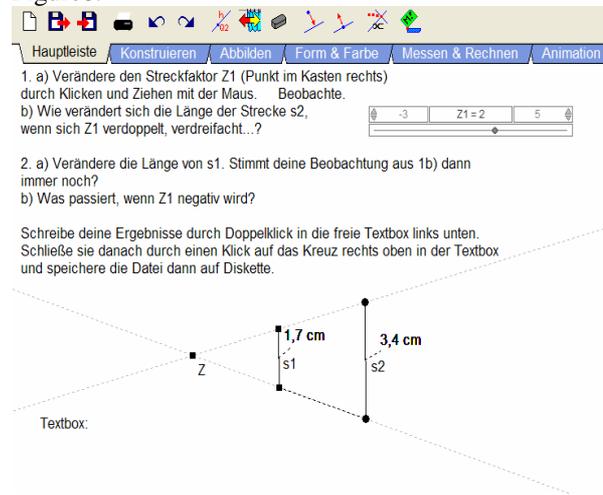Figure 3.

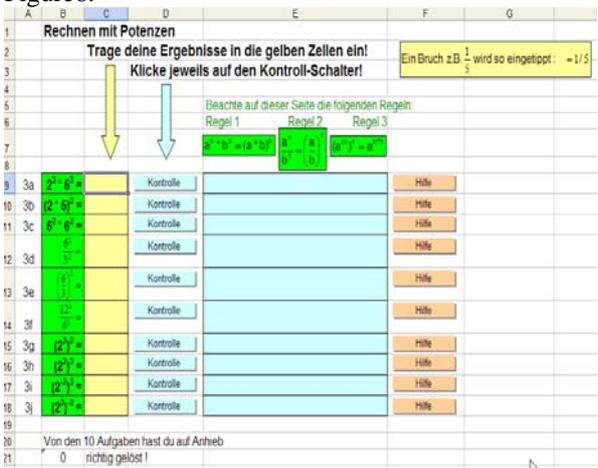Figure 4.

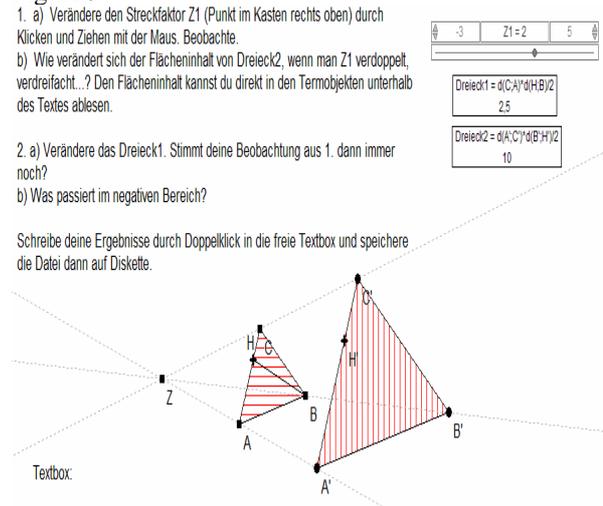Figure 5.

Figure6.

Figure7.

Figure8.

Figure 9.

Figure10.

1. In den Rechtecken (Termobjekte) siehst du Berechnungen. Was wird jeweils ausgerechnet? Schreibe deine Beobachtungen in die Textbox.
2. Wenn du an den Punkten A,B und C ziehst, verändert sich der Winkel Gamma. Gilt deine Beobachtung dann immer noch? Schreibe deine Beobachtungen wieder in die Textbox.
3. Suche weitere Winkel, für die der Satz des Pythagoras gilt.
Speichere die Datei auf Diskette.

d(A;C)^2	d(B;C)^2	d(A;B)^2
12,3	9,1	21,4

Textbox:

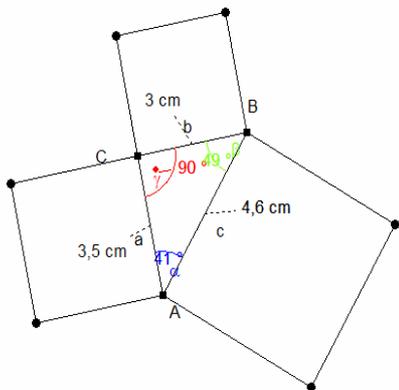Figure11.

1. Berechne die Strecken r1, r2 und r3 auf einem Blatt Papier.
2. Was fällt dir an der Konstruktion auf?
3. Versuche die Schnecke mit dem Computer weiter zu konstruieren.
(Hilfe: Klicke auf "Konstruieren" und wähle ein Symbol für die Zeichnung aus. Wenn du mit der Maus über die Symbole gehst, werden sie dir erklärt. Überlege: Wie würdest du es mit Zirkel und Lineal machen?)
Berechne die jeweils neu entstandene Außenkante. Speichere die Datei auf deiner Diskette.
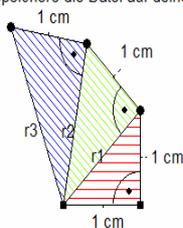

Figure 12.
Arbeitsblatt Quadratische Funktion/ Gleichung

1. Zeichne jeweils die Graphen und beobachte.
Hinweis: Bevor du eine neue Funktionsgleichung eingibst, guck dir erst den Graphen an.

a) $y = x^2$ $y = 2x^2$ $y = 3x^2$
b) $y = 0,5 x^2$ $y = 0,25 x^2$ $y = 0,1 x^2$
c) $y = -x^2$ $y = -4x^2$
d) $y = x^2+4$ $y = x^2-3$

Beantworte: Welcher Zusammenhang besteht zwischen der Gleichung und dem zugehörigen Graphen ?
Beobachtung zu a): ...
2. Unten siehst du das Bild der Funktion $y = x^2$ (Bild der Normalparabel)
Kreuze an: Ist die Funktion $y = -10x^2 + 2$
auch nach oben geöffnet?
weiter geöffnet als die Normalparabel?
Liegt der Scheitelpunkt „höher" als der der Normalparabel?
Skizziere das Bild der Funktion $y = -10x^2 +2$ in obige Abbildung.
Kreuze an: Ist die Funktion $y = (x-3)^2 - 2$
auch nach oben geöffnet?
weiter geöffnet als die Normalparabel?

Gib den Scheitelpunkt an: S(/)
Skizziere das Bild der Funktion $y = (x-3)^2 - 2$ in die obige Abbildung.
Überprüfe nun deine Antworten mit dem Programm MATHEASS.
Welche Fehler hast du gemacht? Beschreibe.
3. Beschreibe, in welche Richtung musst du die Normalparabel verschieben, um auf die Bilder der folgenden Funktion zu kommen:

$y = (x-8)^2 + 5$ $y = (x+3,4)^2 - 1$ $y = (x-1,2)^2$

4. a) Kreuze an : Welche Gleichung gehört zu dem Graphen (linkes Bild)
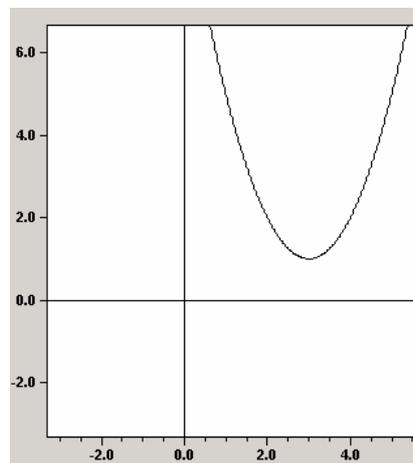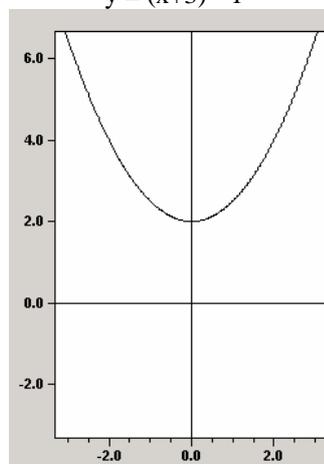
$y = 4x^2 + 2$ $y = 0,5x^2 + \sqrt{4}$ $y = -0,5x^2 + 2$

b) Kreuze an : Welche Gleichung gehört zu dem Graphen (rechtes Bild).

$y = (x-3)^2 + 1$ $y = (x+3)^2 + 1$
$y = (x+3)^2 -1$

c) Zeichne alle sechs Graphen mit MATHEASS, vergleiche jeweils das Bild mit der Funktionsgleichung und überprüfe.
5. Prüfe durch Rechnung, ob die Gleichung nur eine, keine oder zwei Lösungen hat. … Zeichne jeweils den Graphen…. Welcher Zusammenhang besteht zwischen der Anzahl der Lösungen und dem Schaubild?
6. Das Bild der Funktionsgleichung $y = 4x + 2$ ist
eine nach unten geöffnete Parabel
eine nach oben geöffnete Parabel
keine Parabel, sondern eine_____
Berechne die Schnittpunkte von $y = 4x + 2$ und $y = 2x^2$.

Wie viele Schnittpunkte hast du erhalten?
Überprüfe dein Ergebnis am Schaubild.

Figure 13.
Zwei Beispielprogramme aus der Datei zum Potenzieren:
Drückt man bei Aufgabe 1b auf den Hilfebutton, so wird
das folgende Programm gestartet:

```
Sub a1bh()
i = 10: MsgBox "Rechne 3*3": Cells(i, 9) = Cells(i, 9) &
"h"
End Sub
```

Beim Drücken des Hilfebuttons erscheint die Message-
Box mit der Mitteilung "Rechne 3*3". Ferner erfolgt in
der Zeile i=10 und Spalte I (9.Spalte) der Eintrag h, der
an den bestehenden Eintrag angehängt wird und
protokolliert, dass der Hilfebutton gedrückt wurde. Die
Breite der Spalten I, J und K ist auf 0 gesetzt, damit die
Spalteneinträge von den Schülerinnen und Schüler nicht
direkt eingesehen werden können.

Drückt man bei Aufgabe 1b auf den Kontrollbutton, so
wird das folgende Programm gestartet:

```
Sub a1bk()
i = 10
If Cells(i, 3) = Cells(i, 11) Then
   Cells(i, 5) = "richtig!": Cells(i, 9) = Cells(i, 9) & "1"
 Else
   Cells(i, 5) = "Leider falsch! Versuche es nochmals!"
   Cells(i, 9) = Cells(i, 9) & "0"
End If
Cells(i, 10) = Left(Cells(i, 9), 1)
End Sub
```

Hier wird geprüft, ob der Schülereintrag von Zelle C10
(3. Spalte, 10. Zeile) mit dem vorgegebenen Inhalt der
Zelle K10 übereinstimmt. Im positiven Fall erscheint in
E10 der Kommentar „richtig!" und in I10 wird der
Eintrag „1" angefügt. Andernfalls steht in E10 der
Kommentar „Leider falsch! Versuche es nochmals!" und
in I10 wird der Eintrag „0" angefügt. In J10 wird nur das
erste Zeichen von I10 übernommen. So kann über die
Spalte J durch Summieren schnell ermittelt werden, wie
viele Aufgaben auf Anhieb richtig gelöst sind.