Appropriate Problems for Learning and for Performing – an Issue for Teacher Training

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Abstract: Selecting, modifying or creating appropriate problems for mathematics class has become an activity of increasing importance in the professional development of German mathematics teachers. But rather than asking in general: "What is a good problem?" there should be a stronger emphasis on considering the specific goal of a problem, e.g.: "What are the ingredients that make a problem appropriate for initiating a learning process" or "What are the characteristics that make a problem appropriate for its use in a central test?" We propose a guiding scheme for teachers that turns out to be especially helpful, since the newly introduced orientation on outcome standards a) leads to a critical predominance of test items and b) expects teachers to design adequate problems for specific learning processes (e.g. problem solving, reasoning and modelling activities).

Kurzreferat: Die Arbeit mit Aufgaben, vor allem das Einschätzen, Auswählen und Entwickeln von Aufgaben, ist in Deutschland aktuell ein zentraler Bereich der Professionalisierung von Mathematiklehrerinnen und -lehrern. Anstelle der allgemeinen Frage „Was ist eine gute Aufgabe?" muss dabei zunächst geklärt werden, welche Ziele mit dem Einsatz einer Aufgabe verfolgt werden. Die so schärfte Fragestellung könnte dann z. B. lauten: „Was macht eine Aufgabe zu einer guten Aufgaben für entdeckendes Lernen?" In diesem Beitrag wird ein Modell für das Arbeiten mit Aufgaben vorgestellt, das besonders die neuen Anforderungen im Blick hat, denen sich Lehrerinnen und Lehrer seit der Einführung von am „Outcome" der Lernprozesse orientierten Bildungsstandards und neuen Lehrpläne ausgesetzt sehen. Während diese Standardsetzungen durch Leistungsaufgaben konkretisiert werden, sind Lehrerinnen und Lehrer mehr als je zuvor gefordert, die neu gewonnenen Freiräume bei der Gestaltung der Lernprozesse mit geeigneten Aufgaben zum Lernen zu füllen und dabei besonders die typischen mathematischen Prozesse (Argumentieren, Begriffsbilden, Modellieren und Problemlösen) zu berücksichtigen.

ZDM-Classification: B50, C30, D50, D60

1. The Role of Mathematical Problems for the Development of Mathematics Teaching

Though problems do not constitute the entire teaching and learning process they can be regarded as the "elementary particles" of designing a learning environment:

1) Considered as "seatwork" they are the "seeds of crystallization for self-regulated student's activities" (J. Neubrand 2002).
2) They serve for assessing student's performance in class tests and central tests.
3) Inside schools and in educational policy as well they constitute the medium of communication about performance expectations.
4) Finally they are used as a key element in pilot projects for improving mathematics teaching (e.g. "Increasing the Efficiency of Mathematics and Science Teaching" abbreviated SINUS, 1998-2003, 2003-2007)

Especially the third aspect, the use as a "medium of communication about performance expectations" is gaining increasing importance since the German education system is undergoing a paradigm shift towards *outcome orientation*, introducing outcome standards and central assessment (cf. Klieme u. a. 2003, Büchter, Leuders & Bruder 2005). Thus an essential aspect of this reform in steering the education system is to increase autonomy and accountability of schools on their way to reach appointed goals and to decrease the amount of regulations. As a consequence recent curricula abstain from giving detailed prescriptions with regard to designing learning arrangements (Blum et al. 2005, Leuders, Barzel & Hußmann 2005) and instead only give a catalogue of expected competencies. In this respect they deviate from recommendations (Klieme et al. 2003) and highly esteemed models (NCTM 2000) and put an even stronger emphasis on the outcome.

For pre-service and in-service teacher training this situation creates new challenges: Teachers need support when their professional tasks now include the creation of learning environments which are in compliance with performance expectations.

Some of the imminent "shortcut" interpretations that could result from the insufficiently supported implementation of an outcome-based system are among the following:

- Teachers tend to re-interpret only the content-related expectations in central tests as an implicit content curriculum while neglecting the process-related aspects of the expectations such as the problem solving, reasoning or modelling competencies

Example: The published PISA items that have been widely taken notice of in Germany already have an impact on creating problems for learning environments, e.g. the item "Antarctica":

CONTINENT AREA

Below is a map of Antarctica.

Question 4: CONTINENT AREA

Estimate the area of Antarctica using the map scale.

Show your working out and explain how you made your estimate. (You can draw over the map if it helps you with your estimation)

Fig. 1: Problem "Antarctica"

Extract from a recent textbook for grade 6 (Lambacher/Schweizer 2005, p.123):

Example 3: Estimating areas
Estimate the area of fig.2

Solution:
Draw a rectangle that has roughly the same area as the figure.

Measured length of the sides:
45m and 20mm
Area: $45\text{mm}\cdot 20\text{mm} = 900\text{mm}^2$
$= 9\text{cm}^2$.
The figure has an area of about 9cm^2.

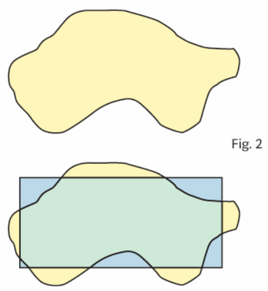


Fig. 2: Problem "Estimating areas"

In the same textbook we find further explanations, examples and problems on pp. 116, 117, 124, 125, 126, 127, 134, 138, 139 and 203. This is a "direct translation" of a detected deficiency of German students (solution frequency 40%) into the design of a learning environment. Nevertheless is has been overlooked that the PISA items not only detects a content-related gap in the curriculum (measuring areas with curved borders) but also a process-related one (combining basic knowledge and skills in an unfamiliar situation).

– Teachers use tasks from tests or model tasks from standards (which are meant to illustrate performance expectations) for a superficial training of calculation skills, thus denying the "competence" aspect of performance standards and neglecting the sustainable development of mental concepts.

Example: A central mathematics test in 2004 (Büchter/Leuders 2005b) revealed deficiencies in basic skill such as in ordering numbers:

Ordering numbers

a) Tick the smallest number

❐ 0,250	❐ 0,3753	❐ 0,625	❐ 0,125	❐ 0,51	❐ 0,7

b) Tick the biggest number

❐ 0,109	❐ 0,19	❐ 0,91	❐ 0,901	❐ 0,091	❐ 0,019

c) Order the numbers by size, beginning with the smallest

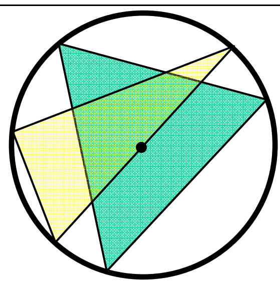
–0,1 ; + 0,01; –10; +0,1; –1

Fig. 3: Problem "Ordering numbers"

Many teachers who didn't expect the results (solution frequencies of about 60%) reacted by 'practising' the ordering of numbers as a skill instead of considering them as an indicator for missing basic concepts. It is not the skill represented by the single problem that has to be practised but the competence of representing and interpreting numbers. Thus it is necessary to create or use problems that help students build (or re-build) basic ideas and concepts (e.g. like those described by vom Hofe 1998)

– Teachers neglect open ended tasks – which are for technical reasons underrepresented in central tests – and thus miss to give students opportunities to develop and extend their competencies on individual learning paths

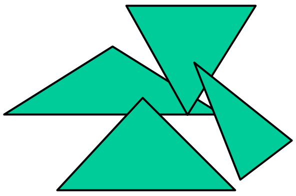
Example: For the purpose of a test it may be sufficient to pose a problem that shows if students are able to apply their knowledge of the "Thales configuration" to reason about the size of angles.



Where would you suppose a right angle? Why?

Fig. 4: "Theorem of Thales" – problem for performing

On the other hand, it needs quite another type of problem to give students the opportunity to (really) discover the phenomenon and to connect it to a broad individual experience with such configurations. Such a problem could look like this (or alternatively the problem in Büchter/Leuders 2005b, p.326)



Try to put (two or three) of the triangles together to get new triangles. When does it work? What do the given triangles have in common?
Cut out the new triangles and group them. Examine the angles and state several conjectures. Give reasons, if possible.

Fig. 5: "Theorem of Thales" – problem for learning

Problems like this are only rarely to be encountered in a central test for several reasons: They are not economical with respect to evaluation, since a large amount of students productions result, they give poorly interrater-reliabilities mostly due to lack of objectivity of the evaluation and they do not fit into a test situation that is restricted with respect to time and available material.

Concluding one can say that, to avoid such misinterpretations in an outcome-oriented use of mathematical problems, teachers in service or in training need guidance in dealing appropriately with them.

2. Goals and characteristics of problems for learning and for performing

In this section we present a framework for the use in teacher training and in everyday practice which consists of criteria and methods for selecting, modifying or creating mathematical problems with regard to their respective goal.

One of the central categories that has proved useful in this respect is the differentiation between *problems for learning and problems for performing*.

The examples in the preceding section show, that the misunderstandings mostly result from the lack of differentiation between the specific goals that have led to an optimised form of a problem. Problems that have been constructed to effectively measure performance follow a different design than those that are meant to initiate rich learning processes.

Consequently teachers should be expected to deal with mathematical problems they encounter in curricula, central tests and textbooks in the following way: They should reflect the goals and the characteristics of a problem and then systematically modify and optimise the task so that it becomes appropriate for its respective use.

Condensing (and somewhat simplifying) these arguments into a dichotomous structure the following table resumes.

Problems for learning	*Problems for performing*
are designed considering openness, multiple solutions, divergence, processes,	are designed considering evident results (for the purpose of assessment), convergence, products
take errors as a starting point for further learning processes	tend to demand avoiding errors
make possible or even promote cooperation and communication	tend to focus on individual performance
emphasise the (cognitive) activity of student's – even if not manifest	emphasise the performance students show

Tab. 1: Problems for learning an problems for performing

This is not meant to be seen as a strict division between two incompatible and completely disjoint types of problems. There are, for example, many test items that can be used in class to initiate learning processes (the "Continent Area" problem can be considered as an example). Nevertheless it has to be asked whether a problem can be optimised with respect to its intended use.

3. More specifically: Different goals – different characteristics

As a guideline in teacher training courses we try to emphasize the described difference between learning and performing when dealing with the quality of teaching in general and the quality of problems in particular. Moreover also *within* these areas we also specify different uses of a problem. The (pragmatically chosen) *areas of use* are (cf. Büchter & Leuders 2005, p.144ff):

Problems for learning can be designed for:
- Exploring, Discovering, Inventing
- Collecting, Reorganizing, Systematising
- Training, Connecting, Reconstructing

Problems for performing can be designed for:
- Applying ("experiencing competence")
- (Self)Diagnosing
- Assessing performance

The second category is intentionally named "performing" instead of "assessing" since there are also situations (and appropriate problems, such as for example many *Fermi problems*), in which students are given the opportunity to experience their competence without the necessity of an external assessment or a deliberate self assessment. "Learning" and "performing" both characterise students' activities in contrast to "assessment" which is mainly associated with teachers.

For each of these uses paradigmatic examples and their characteristics are discussed with teachers and heuristics for constructing specific problems are introduced and applied. While such classification schemes for problems have been suggested by several authors (cf. J. Neubrand 2002, Neubrand u.a. 2002, KMK 2003) mainly for scientific use in constructing tests or evaluating teaching methods, we emphasize the use of the categories in the professional development of teachers. Thus we are less interested in rating reliability than in comprehensibility and usability for the everyday work of a teacher.

To judge the appropriateness of a problem for a specific use we use paradigmatic examples on the one hand and criteria on the other hand. Also the list of these criteria has deliberately been kept simple (cf. Büchter & Leuders 2005, p74ff).

- The *openness* of a problem describes the extent of freedom students have in working with the problem or – seen from a different perspective – the extent of divergence that is to be expected. With respect to openness, teachers can use several simple techniques of opening problems, such as "reversing a problem" or "leaving out information" (cf. Bruder 2000, Leuders 2001).
- The *accessibility* of a problem refers to the feature that it allows students to work with the problem on different levels of abstraction and with different approaches. Techniques to reach such a "differential accessibility" are e.g.: offering students different aspects or parts of a problem that they can individually choose or even better: offering a single problem that can be dealt with on different levels of abstraction – so called "naturally differentiating problems".
- The *authenticity* of a problem does not – as one could presume – refer to the authenticity of the *context* but on the quality of the mathematical processes that are initiated by the task[1]. This concept of authenticity will be elaborated further in the following section.

In a given problem every single of these three characteristics can be realized to a smaller or to a larger extent. Depending on the use of a problem these realizations can be found appropriate or inappropriate.

We explain this statement by referring to the aspect of "openness": A completely closed problem ("What is the area of a given triangle?") can be opened by reversing it ("Draw a triangle with the area of 20cm^2"). Now the problem's "moderate openness" allows for using it in a test, to find out, whether a student cannot only apply a formula but also whether he or she can flexibly work with it and knows its implications. Such a reversion can also lead to further, enriched problems ("Draw some triangles with the area of 20cm^2" or "Find as many triangles with the area 20cm^2 and with a height of integer value").

Yet, when a problem is too open (like some of the examples above) this can result in difficulties when evaluating the performance of students in a central test.

On the other hand, the problem of drawing the triangle is not yet open enough to represent authentic mathematical processes. In the following section we explain our concept of authenticity.

[1] Even problems that use pseudo-contexts can lead to authentic problem solving or reasoning. The role of the context then is not to give the (false) picture of a mathematical application but to make the problem more accessible to students. In these problems "the real world should help to understand mathematics" and the potentially artificial context does not pretend that "mathematics helps to understand the real world" – as Jahnke (2005) puts it.

4. Authentic mathematical processes

When looking at the work of mathematicians on the one hand and on the mathematical work of students (beyond training elementary calculation skills) on the other hand one realizes that the processes of "doing mathematics" are rather similar than they are different. The professional's approach to mathematics may be more abstract but there are many similarities between the epistemological processes that constitute the research process and the (desired) learning processes of a student. This similarity becomes manifest from the perspective of so called "mathematical processes", some of which are:

- *modelling*: describing the world with mathematical concepts,
- *reasoning*: discovering logical connections and finding reasons,
- *problem solving*: stating and solving problems by using mathematical strategies like "examining examples" or "drawing diagrams",
- *concept building*: inventing and elaborating mathematical concepts.

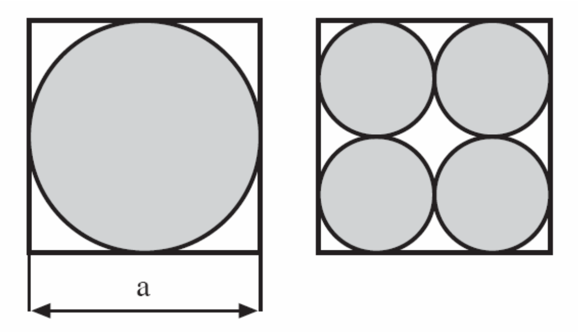
It is for this analogy that many models for standards for mathematics education like the NCTM standards (NCTM 2000) can use the same categories from pre-Kindergarten through to university education. Also many authors see the universal characterisation of mathematics as "exploring patterns" (Devlin 2000, Stewart 2001)

When judging or designing problems for students one can ask: To what extent does a problem allow or even initiate such mathematical processes? This perspective can be elaborated when referring to the specific processes:

a) *Authentic modelling* in school then means that students not only translate word problems into mathematical formulae and solve them but that they have the opportunity to actively work in all steps of the modelling cycle as suggested e.g. by Pollak (1979), Schupp (1998) or Blum (1996). They should find and use models to mathematize real word problems, they should interpret models, they should judge their appropriateness and suggest modifications etc.

b) Part of *authentic problem solving* is that students not only solve given problems but also find and express own problems. Such situations are almost never encountered in assessment situations but several ideas for realising problem stating can be found like the principle of "problem variation" (Schupp 2002). Such problem can be created by teachers e.g. by opening problems from traditional textbooks:

Example: A problem from a recent textbook (Lambacher/Schweizer 2000, Gymnasium, NRW, grade 10, p. 93) looks like this:



a) From a quadratic panel with side length *a* one circular disc is cut out. What percentage is the waste?

b) From a quadratic panel with side length *a* four circular discs (resp. 9, 16,..) are cut out. What percentage is the waste? What do you notice?

Fig. 6: Problem "Discs in a quadratic panel"

This closed task can be modified to give students the opportunity to find and solve their own problems in an authentic way:

Form a quadratic panel cut out circular discs each with the same diameter. The waste area should be as small as possible.

When students are familiar with the technique of varying mathematical problems the problem can be made even more authentic by arising the mathematical question:

What other problems could one pose?

Students then can find problems like these: "What happens if the diameter can be chosen different for each circle? What happens if we take a rectangular or circular panel? Can we also cut out other forms? What if we want to optimise the circumference?"

This example also shows that openness of a task is decisive trait of authenticity. On the other hand this openness allows for different approaches on different levels without making the problem inaccessible to lower performing students.

c) Quite analogously an important criterion for *authentic reasoning* is that students have the opportunity to find and state conjectures on their own. Appropriate problems have to offer a broad spectrum of possible connections to discover and do not confine students to follow a single "planned discovery". A problem that possesses such an openness with respect to students' discoveries is the "Theorem of Thales" (figures 4 & 5) or the following problem:

Fold a sheet of paper once, twice or three times. Then find as many relations between the angles of the unfold sheet of paper.

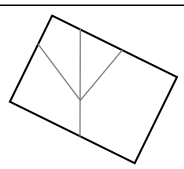


Fig. 7: Problem "Folding a paper"

Though appropriate problems can sometimes be created by opening traditional tasks, it is not so easy to find such problems since they often have a radically different character like the problems in the "open-ended approach" (Becker/Shimada 1997).

The example also shows that the authenticity with respect to reasoning correlates with the extent of differential *accessibility* that a problem allows.

When comparing the briefly described processes one can make out an important common feature of authenticity. The processes all can be described by similar "hermeneutic spirals" that can be run through several times during a learning process. Similar spirals can be drawn for "modelling", "concept building" and "problem solving" (cf. Büchter & Leuders 2005, p.76ff).

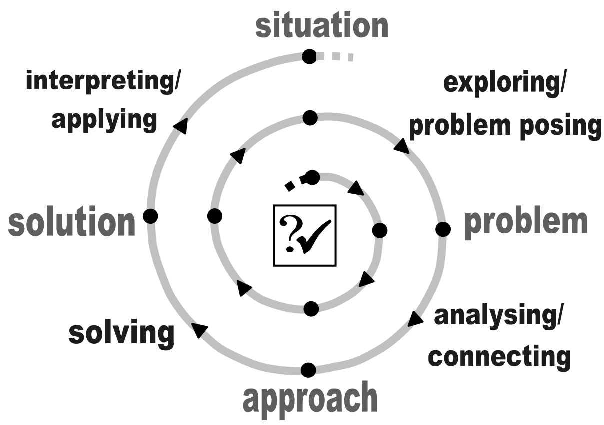


Fig. 8: Spiral "Authentic reasoning

Mathematical problems for performing often concentrate on certain parts of the spiral. Competencies are being assessed by problems in which students have to perform activities that constitute partial steps, often they begin on the right or lower side of the spiral and end after a half or a quarter revolution and almost never go around once. Often test items present a model that should be explained. This restriction is due to the economy of evaluation but also – positively speaking – on the intention to focus on certain competencies.

In contrast, problems for learning can include many partial processes, especially the divergent processes at the upper right side of the spiral.

Concluding this section we refer to the contingency of the choice and of the interpretation of the briefly described "process categories". In fact they overlap and are closely connected. Nevertheless, for supporting teachers in in-service training it is important not to discuss sophisticated differences in scientific terminology but to offer practical terms they can work with. In this sense we chose to make the following differences:

1) *Difference between modelling and problem solving* While in frameworks for large-scale assessments the concept of modelling is frequently considered as a psychological category (PISA, 2000), the terms of "problem solving" and "modelling" more or less coincide. Modelling can be seen as the general term, in-

cluding processes of finding a mathematical representation, solving the mathematical problem and interpreting the solution also in purely mathematical situations. This concept is coherent and plausible but difficult to communicate to teachers. Therefore we decided to draw an analytic line between problem solving as the application and reflection of mathematical strategies and modelling as the use of mathematics to describe real world situations.

2) *Difference between reasoning and problem solving* Reasoning as the activity of finding (logical) reasons that connect mathematical statements can also be seen as problem solving process. In fact, many mathematical problems seem to have this character (cf. e.g. Polya 1945). Still there is also another approach to doing mathematics that is less focussing on finding logical connections and give rigorous reasons than on giving a solution to an open situation and reaching a result. A mathematical problem can be solved even without looking for insight into the deeper connections – a frequent approach in almost all disciplines that apply mathematics and also in many problems in mathematics classes. To see the difference between problems focussing on the result (problem solving) and those focussing on the logical connection (reasoning) has also proven useful in examining the characteristic of problems for mathematics instruction – if one abstains from trying to divide between these types too strictly.

Also the other processes overlap or interrelate in a specific way: One can, for example, think of modelling and problem solving on the basis of concept building as Freudenthal (1976) suggests.

Before presenting some models for using the presented categorization in teacher training we give a graphic summary:

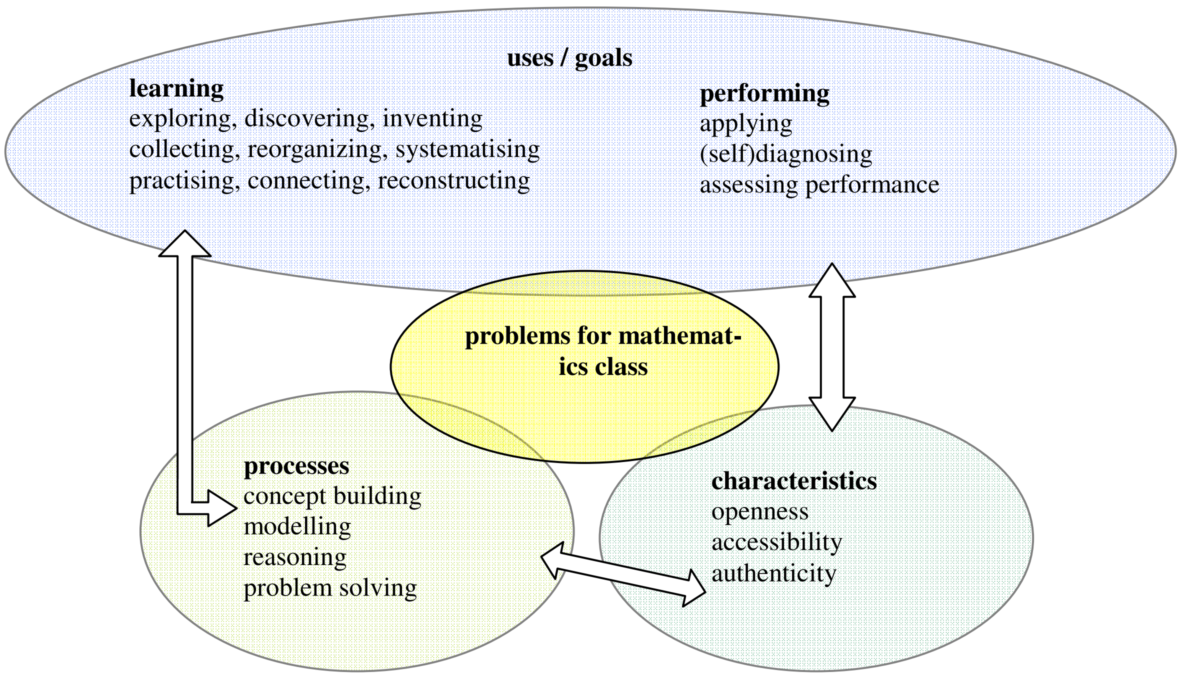


Fig. 9: Model for working with tasks

3. Examples for „problem-related" course models in teacher training

In this section we want to briefly describe some models of pre-service and in-service teacher training courses and reflect the experiences we made.

In teacher education for middle school teachers at university level we offer different kinds of problem-solving workshops. During those courses that run 2 hours a week for about 10-12 weeks, students work on given mathematical problems. These problems are comparatively open to re-interpretation and variations and students are explicitly invited to do so. Students are expected to work on the problems during workshop – where they can exchange ideas – and at home and write down all their ideas in a journal. They are also asked to read some comprehensive papers on mathematical processes such as problem solving or reasoning that mainly stem from didactic research and development. Students are expected to reflect their personal engagement and their understanding of the concepts of these texts by including their reflections into the journals. The goal of the course is mainly to connect a reflected concept building concerning ideas of problem solving, openness etc. that are important for a teachers profession with an active experience of doing mathematics.

Another course concept is based on active working with mathematical problems. Students learn to judge and construct problems with regard to the successively introduced categories described in this article. Analysing critically and modifying problems from recent textbooks is a crucial part of this course. In the end students are certified for delivering a number of individually created and reflected problems.

Another model for university teacher training that works with problems is described by Bruder in this issue.

For in-service teacher training it is necessary to take account of the necessities of teachers that come from their daily practice in the exceptional situation of a training course. They expect a high level of usability concerning the subject matter of the course. Taken this expectation into consideration we offer a programme during which teachers are asked to optimise existing problems from their current classes with respect to their uses. This means for example that they are asked to create appropriate open learning environments beginning with a closed problem in a class test or a central test that expresses a certain expectation. In reflecting these processes "from learning to performing" and the way back participants gain more insight into their capabilities (and responsibilities) by using problems from different sources like central tests or textbooks.

4. Concluding remarks

We consider the depicted interplay between uses (as to processes, learning/performing) and characteristics (openness, differentiation, authenticity) as a helpful frame for working with teachers on mathematical problems. One of the pre-eminent tasks of teachers in times of standard-orientation is to retransform problems for performing back into problems for learning - or as Heinrich Roth has put it (1957):

> *„How can I transform the subject-matter that resulted as an answer to a question back into a question? And conversely: How can I sustain the original question posing of the child? […] All methodical art is encompasses in the ability to transform dead facts into the living activities, they have originally sprung from [...].“* (Roth 1957)

Working with mathematical problems in teacher training cannot substitute all approaches to improve mathematics teaching, but can be considered pivotal in the following sense:

> „I am sure, many of you have encountered, when reading articles on mathematics teaching, the following experience: At first you were astonished how much the approach you were reading about reflected your own ideas, but when the first concrete examples were given, the discrepancies became more than perceptible." (van den Heuvel-Panhuizen 2003).

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