

“Open learning? Computeralgebra?... No time left for that...”¹

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Abstract: Nowadays mathematics teachers have to deal with two challenges concerning their classroom-arrangements: include new teaching methods and integrate computers. The title expresses the fear of many teachers when following those trends, that realizing both makes curricular prescriptions even more difficult to achieve. In contrast to this other teachers perceive those trends not as an impediment, but as a special opportunity to achieve aims in terms of contents and processes. It was intended to investigate the question whether impediment or opportunity by a research project at the University of Duisburg-Essen. Teaching material was developed to introduce investigating polynomial functions in an open classroom-arrangement integrating CAS.

According to the multi-faceted arrangement a complementary research design was chosen which collects qualitative and quantitative data. The qualitative part is an interpretive study based on video tapes. The quantitative part is an experimental large-scale study. The material was used in 45 classes (about 1200 students) from different schools in order to check if general conclusions can be drawn. The large-scale study also includes a post-survey and a comparative post-test. To understand the aims of the project it is necessary to grasp the idea of the material. Therefore chapter 1 points out the main ideas of the material, chapter 2 explains the focus of the research project and in chapter 3 you will find first results.

Kurzreferat: Heutzutage müssen Lehrpersonen im Mathematikunterricht sich mit zwei Herausforderungen bei der Unterrichtsgestaltung auseinandersetzen: das Einbeziehen neuer Unterrichtsmethoden und das Integrieren neuer Technologie. Der Titel drückt die Befürchtung vieler Lehrpersonen aus, dass der Unterrichtsstoff noch schwerer zu bewältigen ist, wenn man diesen Trends folgt. Im Gegensatz dazu steht die Erfahrung anderer Lehrpersonen, dass das Befolgen dieser Trends kein Hindernis sondern durchaus Chance sein kann inhalts- und prozessbezogene Ziele gleichermaßen zu erreichen. Die Frage ob Hindernis oder Chance sollte im Rahmen eines Forschungsprojektes an der Universität Duisburg-Essen untersucht werden und führte zur Entwicklung einer Lernwerkstatt zur Untersuchung ganzzahliger Funktionen mit integriertem Einsatz von Computeralgebra (CAS).

Entsprechend der Vielschichtigkeit des Unterrichtsmaterials wurde auch das Forschungsdesign vielfältig gewählt – eine Mischung aus qualitativen und quantitativen Untersuchungen. Der qualitative Teil bestand aus interpretativen Studien auf der Basis von Videoaufzeichnungen. Der quantitative Teil mit einem Abschlussfragebogen und einem vergleichenden Abschlusstest ist eine experimentelle Studie über den Einsatz des Materials in 45 Klassen (mit ca. 1200 Schüler/innen), um damit auch die Generalisierbarkeit zu untersuchen. Im Folgenden wird zunächst das Material vorgestellt (Kapitel 1), bevor das Forschungsdesign (Kapitel 2) und erste Ergebnisse (Kapitel 3) beschrieben werden.

ZDM-Classification: C70, D44, I44, U64, U74

¹ Modified version of the keynote lecture during the 39. Tagung für Didaktik der Mathematik, Bielefeld, 28.2.- 4.3.2005

1. The teaching material

At first the material will be presented to give an impression of what it looks like. Afterwards some theoretical aspects will be discussed concerning the topic, the classroom-organisation and the use of technology.

1.1 The material

The material is meant to be used as an introduction into the aspects of the investigation of polynomial functions with aspects of differentiation (like slope, zeroes, extrema, inflection point). Students of 11th grade (about 17 years old) receive a work folder on paper (Barzel / Fröhlich/ Stachniss-Carp 2003) with a set of worksheets (called “modules”) which they deal with independently in groups of 4-6 students for about 6 weeks. Supplementary material concerning individual stations is laid out in the classroom. The whole organisation of the teaching is like a work-place or a circle with different ways and possibilities of approaching the topic. This kind of classroom-arrangement is called a “Lernwerkstatt” (translated: learning workshop).

The only previous knowledge the students must have is the idea of derivation. It is possible to proceed through the learning workshop in different ways. Different ways of learning are also usually possible within the modules. There is no sequential arrangement of tasks in this case and the suggestions are given as mindmaps instead (an example of one module is given in fig. 1). At several points in the learning workshop, a comparison of the different types of representation of a function (graph, term, table, situation) is taken as the theme and the advantages and disadvantages are discussed. One example is shown in figure 1.

A variety of different types of tasks are involved in the material to evoke different kinds of student activities, for example:

- tasks which demand open ended approaches (Becker/Shimada 1997, Herget 2000),
- tasks which stimulate discussions between the students,
- tasks which initiate flexibility between the different representations in different modules to address different learning types (Herget/ Jahnke2001) (compare module E, fig.1),
- tasks which integrate students’ own experiences and experiments (Barzel 2002)

The following types of tasks should give an impression of this variety of the tasks:

Giving functions with concrete analysis assignments: This is performed for example in module “E” (compare fig. 1). Three different functions are given, one as a graph, one as a table and one as a formula. Without further previous knowledge concerning extreme points and their properties students have to recognise the properties by analysing the three examples. In module “L” (higher derivatives) graphs of a function and its derivatives are analysed and connections between the

grade of the polynomials and the maximal number of zeroes, extremas, points of inflection are recorded in structured form in a table.

Discussion of statements: If you concern yourself critically with a predetermined statement, you reflect on and interlink knowledge already acquired in order to arrive at an appropriate assessment. Therefore, assessment of the statement " $f'(x_0) = 0 \Leftrightarrow$ An extrema exists in x_0 " results in development of an arithmetical method for determination of local extreme values (module "E" – see fig.1).

Text analysis: In module L (higher derivatives) information concerning higher derivatives is given; the module K (curvature) requires research on the topic of "point of inflexion". By means of structuring and separation of important and unimportant aspects, dealing intelligently with mathematical texts is practised.

Fig 1: One module as an example: Module E - Extrema

Experimental experience: "Derivative graph walking" is a module that encourages trials with a sonic motion detector (CBR – "Computer Based Ranger coming along with a TI-calculator). Movements are recorded indirectly as a time-distance or time-speed diagram. A graph of a derivative is produced in this manner by one's own walking. This type of graph is analysed and conversely, predetermined graphs are followed and matched. Cognitive discussions are linked as a result with concrete experience, in order to facilitate comprehension of the new contents.

An overall reflection of the workshop is finally performed by preparing posters for a final presentation.

To give teachers an idea of how to use the material in their classroom teaching, an introductory booklet serves as a guideline with main ideas and recommendations for realising the workshop in their own teaching. The booklet contains as well additional material for laying out in the classroom. One recommendation concerns the documentation of the learning process in a kind of journal, in which you cannot only find tasks and results but also ideas, individual examples, meanderings, highlights etc. (Hußmann 2003, Ruf/ Gallin 1998). In order to evaluate the learning process the teacher has several possibilities: s/he can assess the group work by judging the individual student's participation and engagement during the group work and the way of presenting results (by poster or other visualisation) and apart from that the teacher can check the journals of the individual student and of course the results of the final written test.

1.2. Theoretical aspects concerning the topic

As the teaching material was supposed to convince the "average" teacher to think about a change of his/her teaching, the creation of the teaching material was firstly a matter of clarifying the question of the topics based on which the example was to be set. The following reasons led to the choice of the topic "investigating polynomial functions":

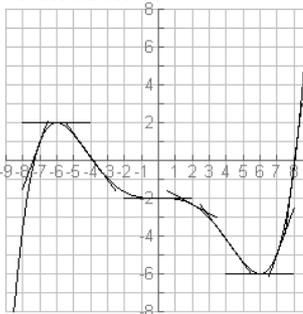
- It is a mandatory topic and not an additional teaching topic.
- It is a topic on higher secondary level, where teachers see usually a more appropriate place for computeralgebra than in lower secondary level.
- The topic "investigating a function" is quite often taught as a fixed procedure which has to be done in certain pre-determined steps with drawing the graph of the function at the end. This scheme provides much opportunity for criticism and is above all perceived as unsatisfactory by the teachers themselves. The core of the criticism in addition to the lack of satisfaction lies in the fact that the underlying mathematics is not understood by the students. Instead they often blindly follow a certain scheme and use formulas.
- "Functions" as a mathematical topic is a wonderful example of showing the benefits of involving CAS into the learning and teaching process. CAS offers the

You can see three functions in different representations. Determine the local extrema and try to define the concept "local extrema". What are the benefits and problems of the different representations?

Why is the adjective „local“ important?

„If the first derivative is 0, then there is a minimum!“ – Discuss this statement and correct it if necessary.

Function 1:



Function 3:

x	y
0	13,5
1	5,94
2	1
3	-1,69
4	-2,5
5	-1,81
6	0
7	2,56
8	5,5
9	8,44
10	11
11	12,81
12	13,5
13	12,69

Function 2:

$f(x) = x^2 + 2$
mit $-2 \leq x \leq 2$

Find a calculation to determine local extrema. Use this calculation for the functions given by the following equations.

Check by plotting the graphs.

$f(x) = \frac{1}{3}x^3 - x$ and $g(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$

possibility of representing a function in different parallel ways: as an equation, a table or a graph, thereby allowing an interactive alternation between these different mathematical representations. This “change of windows” (compare the “Window shuttle principle”, Heugl/ Klinger/ Lechner 1996) can be rendered useful for the learning process, since the different preferences of individual students can be addressed specifically. This can not only be of benefit in devising an investigation of functions, but also whenever functions are dealt with in a school context.

- The topic gives rise to many and diverse problem statements with which diverse activities in the learning process can be stimulated.

The central idea of the concept was to find a balance between instruction and construction, achieve an equilibrium between endeavouring to convey requested curricular contents and the desire for maximum opening up of the individual problem statements in order to stimulate many diverse and intensive debates on the part of the learners. It was intended to initiate a range of cognitive activities between receptive and creative activities.

What does this mean however in concrete terms within the context of the topic? Which activities are specifically meaningful and necessary in this case? An initial guideline for the realisation is first the description of “mathematical literacy” in the PISA framework (PISA 2000), the competence list in the educational standards of the KMK (2003) or the NCTM (2000) and likewise the hierarchical structuring of competences in the topic area “calculus” according to Tietze/ Wolpers/ Klika (1996). When creating new tasks for students these general categorisations have to be specified for the single topic area. Thus the following catalogue of activities has been developed. General, metacognitive competences have also been taken into account. The subdivision is in no way disjointed. It serves mainly as a guideline.

- *Receiving* activities: calculation, use of formulas, execution, listening, comprehension
- *Presentation* activities: mathematical representation (in term, graph, table and words), switching between different forms of presentation, e.g. visualisation
- *Analysis* activities: encoding of given texts and representations, interpreting, structuring
- *Reflection* activities: comparison, rethinking a solution method from modified standpoints, interlinking
- *Creation* activities: including new aspects, trying new viewpoints, devising and finding examples, systematising, generalising and investigating

1.3 Theoretical aspects concerning the classroom-arrangement

In order to be able to develop all these activities, it must be possible to verbalise and communicate freely. Hefendehl-Hebeker (2004) points out the characteristics of a rational dialogue in dialogic learning. Such a dialogue should be unbiased, informal and non-persuasive to ensure that learning constitutes an act of self-controlled formation of networks of knowledge

linked to one’s own existing knowledge and abilities. For this purpose however, a framework must be created in which ideas and thoughts can be exchanged in an “uncensored” manner. Students must learn to get involved with others and listen to their ideas in order to also comprehend strange and maybe unfamiliar trains of thought and be able to link the latter with their own. Connections are established, differences pointed out and difficulties elucidated in this manner. An organisational framework which provides order and orientation on the one hand and which gives learners the necessary latitude on the other hand can be helpful here. This is not possible with conventional teaching led from the front of the classroom with phases of work in pairs or groups that are limited in time, but requires a teaching method consistently focused on the student. An example of this method is a learning workshop. This method can be found in several German primary schools. In Switzerland you can also find it in secondary schools (Weber 1991). A learning workshop represents a place that – in the literal and figurative sense – allows a topic area to be dealt with in many different ways, while a joint product in common activity is developed. Historical models include for example the “laboratories” of Helen Parkhurst (1922). Pallasch (1997) points out four principles as characteristics of a learning workshop, which have also been the basis for creating this learning workshop:

- Principle of *participation*: All actors (students and teachers) should participate in forming the learning process.
- Principle of *structuring*: The work should be structured in a clear way to obtain an optimal transparency.
- Principle of *wholeness*: The idea is to present a topic as a “whole” by offering a big variety of different aspects and tasks.
- Principle of *balance*: The balance between a final product and the actual process describes the main idea of a learning workshop.

1.4 Theoretical aspects concerning CAS

CAS plays an important role during the learning process but it is not expressed explicitly in the material. The conceptual idea is to use the technology as a tool, which is always available for the students whether on a PC or on a calculator (like Voyage 200 or TI-89). It is the students’ and not the teachers’ decision whether or not, how often and when technology is used (Leuders/ Barzel/ Hußmann 2005). There are special assignments which require specifically the use of computer algebra and others where the solution is also possible without this technology.

The tasks where CAS is required or helpful, CAS is used for:

- generating examples,
- calculating (solving equations, systems of equations, determining derivatives and single values),
- checking calculations and ideas and
- visualising certain aspects. (compare Dörr/ Zangor 2000)

2. The research design to evaluate the material

Evaluating “real-life” lessons in a multi-faceted and long-term arrangement requires a clear focus of research and on the other side a complex research design.

2.1 The research question

The original idea of the whole project was to give teachers an example that pursuing curricular requests and using technology in an open classroom organisation are no contradictions but can be obtained both. The research however does not focus on the teachers’ but on the students’ view and their cognitive activities. The idea is to find out which activities are really promoted in such an open learning arrangement with an integrated use of technology. Thus the central questions of the study are: To what extent is this learning arrangement suitable for simultaneously pursuing aims both in terms of content and process? Which cognitive students’ activities are promoted by it? The list of cognitive activities in chapter 1.3 serves as a theoretical frame in this area.

2.2. The research design

A complementary research design was chosen which evaluated the teaching in a qualitative and a quantitative way. In the pilot phase, the teaching material was initially tested by six teachers, discussed with those teachers and afterwards further developed and published (Barzel/ Fröhlich/ Stachniss-Carp 2003). Furthermore, the questionnaires for the subsequent quantitative study also underwent piloting.

2.2.1 The qualitative part of the study

For the qualitative assessment, a class was monitored during the teaching with the learning workshop. Mainly the lessons were recorded on video, interviews with students and the teacher were conducted, students’ learning diaries were analysed and examination papers of the experimental class were studied in comparison to examination papers of parallel classes. Portions of the material were coded and analysed and assessed according to the guidelines of the Grounded Theory (Strauss/ Corbin 1996) – these include transcripts of teaching, students’ exercise books and the final examination paper of the experimental group (and the parallel classes as a reference group). In this process some lessons were chosen for analysing and interpreting. As an instrument for interpretation the epistemological triangle of Steinbring (2005, Bromme/ Steinbring 1990) has been used. In this model mathematical conceptualising and meaning is described as a mediation between signs or symbols and a suitable reference context, which is influenced by the mathematical knowledge and concepts as the third corner of the triangle. The whole process of conceptualisation is always a process which can only be described by series of triangles. For the specific purpose of this study the epistemological triangle as an instrument is supplemented by the perspective of cognitive activities. With this instrument the interactions between the students have been interpreted in a sense of meaning and in a sense of interacting in the communication process.

2.2.2 The quantitative part of the study

This part includes a post-survey (one for students and one for teachers) and a comparative post-test. For quantitative assessment, it was possible to win over a total of 45 teachers (about 1200 students) in the school year 2003/04 in order to work on the topic with the aid of the learning workshop. This was the basis for an experimental large-scale study to look across schools to support broad generalisation. Both teachers and students were asked to complete a post-survey and 578 students and 17 teachers gave feedback in this manner. Both questionnaires enquired among other aspects about the individual attitudes to the work in the learning workshop via statements that needed to be classified on a Likert scale.

The participating classes took part in a final test. This test consisted of two questions (question 1 - see fig.2), which were adopted from former central comparative examination papers in NRW (2002, 2001)² in order to select as the standard a requirement imposed from outside.

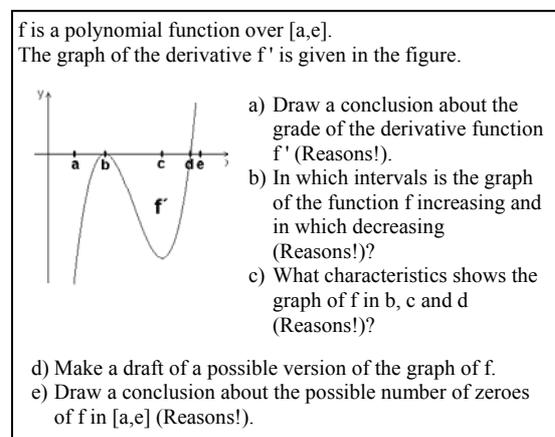


Fig 2: Question 1 of the post-test

3. Some results of the study

3.1 Results of the qualitative study

One of the interpretive studies is about a sequence, in which a group of students worked on a game. They have 13 little cards on the table (K1 to K13)– and every card shows a graph. “What belongs together?” is the question to find three sets of graphs for f , f' and f'' among the 13 function graphs. In the analysed section, the discussion of two female students is studied. Their different perspectives are apparent here. The first student mainly starts from a more global, dynamic way of looking at things, since she constantly moves the course and gradient of individual areas of the graph into a central position. The other shows a rather local-oriented approach, since she draws attention to points (extrema,

² It was respectively the 2nd problem of the comparative examination paper of 2001 and 2002, available at: www.brd.nrw.de/BezRegDdorf/hierarchie/lerntreffs/mathe/structure/sekundar2/vergleichsarbeiten.php

zeroes and certain values) in a focused manner in order to discover sets of graphs.

69 P: \ (points to K1)..the gradient is therefore in the negative range ... the ... is therefore, it falls and so.. y must be in the negative range for the 2nd derivative ... (P points to K2)

74 U: There is a point of inflexion at this point here (U points to K1, P to K2, approximately $x > 3$) and here is the extreme (U points to K2)

\ ...and is extreme again here (U+P point to K1) and here (U points to K2 average zero point) zero again (P changes from K2 to K1)

In the course of the discussion, both increasingly get involved in the viewpoint of the other and interlink both criteria in order to test, verify or negate the belonging together of graphs. This is increasingly successful, since their communication becomes more intensive and incorrect ideas are mutually corrected.

The experimental group (26 M + F students) and the three parallel classes from the same school (80 M + F students) took part in the same official central comparative examination paper in NRW (2003)³. All the students' solutions without the respective teacher's corrections were available to the group of researchers. In one problem, one equation of a function and graphs of four functions were predetermined and the task involved allocating the corresponding graph to the term. It was apparent while correcting this problem that the students in the experimental group, unlike the reference group, often began with the answer phrase and supplied the justification subsequently. These and other characteristics were consequently quantified in order to detect any possible trends. The impression was initially confirmed in this case: 69% of the students in the experimental group placed the answer phrase at the beginning in comparison to 21% of the remaining group. In contrast, the phrase was at the end of the explanations in 19% of the experimental group and in 65% of the other students. Examination of the other characteristics provided possible reasons for this phenomenon. The control group usually began to perform calculations on the basis of the equation of the function, e.g. determining zero points and extreme points. These calculations were geared to elements that the students knew as components of the curve discussion. In the course of the work process the students found criteria for relating graph and term. The working direction was predominately from term to graph in this case. In contrast, in the experimental group, the working direction was more frequently from graph to term or moved back and forth between the two representations.

3.2 Results of the quantitative study

3.2.1 The Post-surveys

³ This examination paper is to be found at: www.brd.nrw.de/BezRegDdorf/hierarchie/lerntreffs/mathe/struc-ture/sekundar2/vergleichsarbeiten.php

The Likert scales of the students' post-survey have been analysed statistically. A factor analysis lead to reduce the dimension of the datas to seven relevant factors (eigenvalue>1,3). Analysing the items in every factor leads to the following description of the factors:

- Positive attitude to self-regulated learning
- Feeling confident with the content
- Documentation of the learning process in a journal
- Positive attitude to use of technology
- Higher work expenditure.
- Positive attitude to the change of mathematical representations
- Further exercises

In every factor the highly correlated items were averaged to get an impression of the students' attitudes in every of the seven areas. All areas yielded clear attitudes by the students. For example figure 3 shows the result of the first factor concerning *positive attitude of self-regulated learning* and figure 4 the result of the second factor about feeling confident with the content. The evaluation scale runs from *very strongly disagree* (---) to *very strongly agree* (+++).

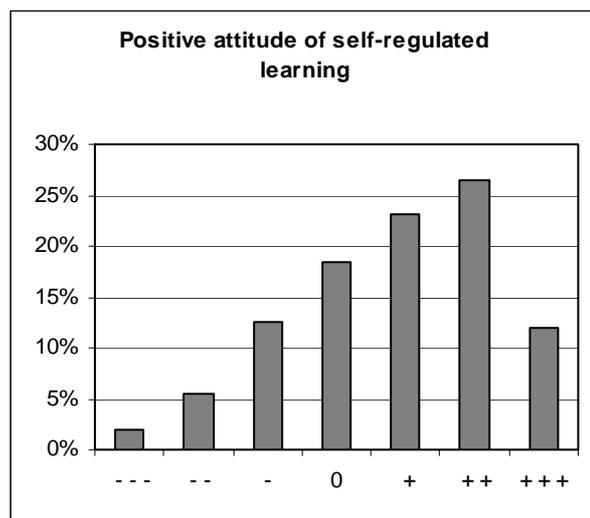


Fig. 3: Students' attitude: self-regulated learning

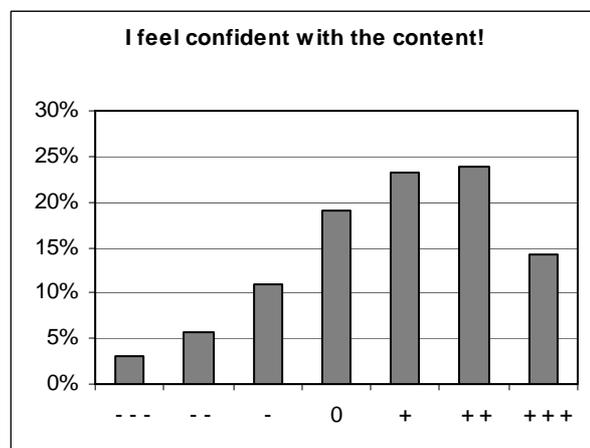


Fig. 4: Students feeling confident with the content

Many learners assess the expenditure in terms of work as high, but nevertheless regarded it as worthwhile.

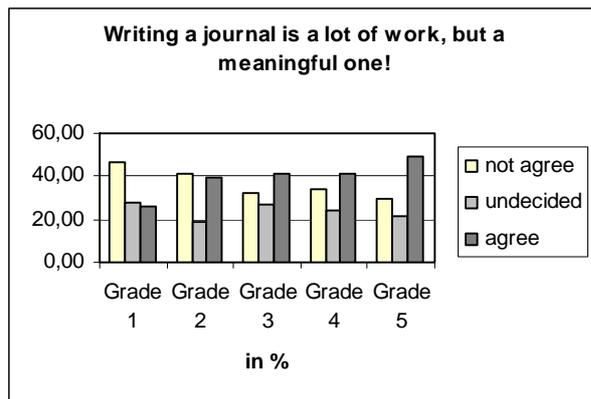


Fig. 5 Students' attitude towards the statement in relation to the last report mark

An interesting aspect in the frame of this factor is the relationship between this assessment of writing a journal and the last grade in mathematics on the last report. Figure 5 shows the students' attitude to the statement "Writing a journal is a lot of work, but a meaningful one!" in relation to the last grade. (In Germany 5 and 6 are the weakest marks and 1 is the best!).

A conspicuous aspect is the high proportion of agreement among those with grade 5, a very weak grade. It cannot be concluded from this however that this additional expenditure was actually used, but it can be concluded that a meaningful educational potential is attributed to this special challenge of the arrangement. Further investigation of the data has shown that especially the possibility of speaking informally about mathematics is highly appreciated by weaker students – that might be one reason for the positive attitude towards the learning arrangement as self-regulated learning.

A total of about 75% of the students assessed the use of computers as meaningful and useful, with a difference appearing here between boys and girls: 82% of the boys and 68% of the girls. The use of computers also tended to be more positively assessed by those who used a pocket calculator or handheld with CAS than by those who used CAS on a PC (fig. 6).

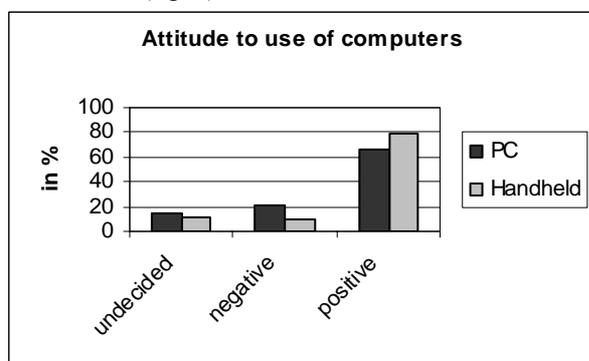


Fig. 6 Attitude to use of computers in relation to the type of CAS used (on PC or on handheld)

A possible reason for the different attitudes to the use of computers may be the individually constant availability.

Moreover, a handheld is outwardly less of a focus of attention and instead is "pulled out of the pocket" when needed and can therefore be more easily integrated in the learning process than a device that with respect to its size is more dominant and under certain circumstances is to be shared with others.

3.2.2 Comparative post-test

The participating classes took part in a final test with two questions (fig.2 shows question 1), which were adopted from former central comparative examination papers in NRW. Thereby a requirement imposed from outside was used as a standard. For comparison, only the averages of the number of points obtained achieved in the respective years in NRW were available. (12169 students (2nd problem) and 11364 students (1st problem)) The return of the final test amounted to 462 examination papers, corrected centrally at the university. The criteria centrally stipulated with the comparative examination papers were taken as a basis for the correction. Comparison of the mean results (in %) yields to figure 7 (Expert-group: 462 students; Reference-group: Question 1 – 12169 students & Question 2– 11364 students)

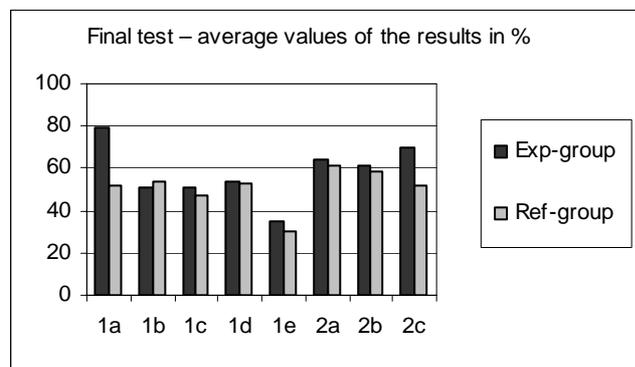


Fig. 7: Final test – average values of results in %

Since only average values from the reference group existed, reference was also made only to the average values from the experimental group for comparison. After weighing up all the influencing factors, it may be concluded from this result that the students in the experimental group satisfied the requirements of the central comparative examination papers at least as well as the students in the reference groups.

3.3 Final statement

The results selected should not obscure the fact that problems with an educational arrangement of this type might occur, for example when the necessary openness in teaching is new and unfamiliar for both the learner and the teacher. Such a form of teaching requires abilities in a special way, specifically on the part of the teacher, in order to be able to use the variety and diversity productively in teaching. Nevertheless, the results obtained to date with regard to the central question of the study allow a positive assessment of the learning workshop presented. The most diverse student activities are stimulated that are cause for hope that aims in terms of content and process are equally pursued in this manner.

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