Thinking wants to be Organized
Empirical Studies to the Complexity of Mathematical Thinking

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Abstract: Can the describable complexity of test problems concerning mathematical thinking and the empirical results of their dealing with be put into a relation? Can graded test problems be constructed which lead to results which can basically be predicted? Empirical studies give interesting and helpful answers which lead to didactically important consequences, just like the evaluation of the PISA results.

Key words: mathematical thinking, cognitive complexity, linguistically logical complexity, competences, standards of education.

Kurzreferat: Lassen sich beschreibbare Komplexität von Testaufgaben zum mathematischen Denken und empirische Resultate ihrer Bearbeitungen ins Verhältnis setzen? Lassen sich gestaffelte Testaufgaben konstruieren, die zu im Grunde vorhersagbaren Ergebnissen führen? Empirische Studien geben darauf interessante und hilfreiche Antworten, die ebenso wie Auswertungen von PISA-Befunden zu didaktisch bedeutsamen Folgerungen führen.

ZDM-Classification: C30

1. The Complexity of Mathematical Thinking

Learning at school calls for the acquisition of knowledge. Knowledge should be an intelligent and applicable, not an inactive and isolated one, because knowledge is supposed to develop into ability. It is just the same when learning mathematics. It is not only about acquiring knowledge, but also and especially about developing mathematical skills and mathematical thinking. Today when school and teaching lessons are rearranged this is represented by the concept “competence”.

The concept “competence” is even a keyword. In this context competences according to Weinert (2001, p. 27f) mean cognitive abilities and skills available for individuals or learnable by them in order to solve certain problems, as well as motivational, volitional and social readiness and abilities to be able to use the problem solutions successfully and responsibly in variable situations. Competences, whose profile of demands and expectations is expressed by educational standards (Sjuts 2004a), are specifically related to the subject and the results and can, in principle, be made operational by means of problems and test scales.

When converting this into test problems some questions for competence models arise: Which models describe competences adequately and in an empirically ascertainable way? Which demand and level areas can be identified? Can test problems be undoubtedly related to certain competence levels? Is every competence level within the models characterized by cognitive processes and actions in such a way that persons of this level can cope with them, but not persons of a lower level. Developed, scientifically everywhere accepted and empirically validated competence models are obviously only recognizable in attempts so far (Helmke & Hosenfeld 2004).

The study in hand wants to look into the following two questions. First: Can the describable complexity of test problems and empirical results of their dealing with be related? Second: Can graded test problems be constructed which lead to results that can basically be predicted? The basis for the study is the complexity concept which has been developed at the Institute of Cognitive Mathematics of the University of Osnabrueck (Cohors-Fresenborg & Sjuts & Sommer 2004). This cognition theoretically orientated concept has tried to make comprehensible several characteristics that fix the degree of difficulty. On the one hand this offers the possibility to judge cognitive and metacognitive performances (Cohors-Fresenborg & Sjuts 2001), on the other hand analysing instruments which – and this is new – leads to remarkable evaluations of PISA-results and didactically significant conclusions (Cohors-Fresenborg & Sjuts & Sommer 2004).

This article picks up the characteristics “linguistically logical complexity” and “cognitive complexity”, which can be outlined as follows (Cohors-Fresenborg & Sjuts & Sommer 2004):

The characteristic “linguistically logical complexity” includes demands on the identification and understanding of a problem text (which is formed by a logical structure and linguistical interconnection), before this is transferred to mathematical description and processing.

The characteristic “cognitive complexity” includes demands on the extent, intensity and complexity of thinking processes when solving a problem – especially when the simultaneity or the interconnection of thinking steps in the solving process have to be organized in an order that has to be followed.

2. Test Problems on Mathematical Thinking: Results Part A

2.1 Description of the Test Situation

At the beginning of the school year 2004/2005 the school system in the Federal State of Lower Saxony was in a special situation. The independent “Orientierungsstufe” (the period of years 5 and 6 at school during which pupils were selected to attend different schools) had been closed. The four years at primary school were (again) followed by the structure of three types of secondary school with different demands in performance (Gymnasium – highest level, Realschule – medium level, Hauptschule – lowest level), as is the case almost everywhere in Germany. Suddenly these secondary schools had to take on not only pupils of year 7, but additionally pupils of year 5 and 6.

This was also the case at the Ubbo-Emmius-Gymnasium in Leer. Pupils for year 5 came from 30 primary schools, for year 6 and 7 from the “Orientierungstufen” that had been closed. This offered the opportunity for an extensive test. The test was to show the level of competence of the individual age-groups (and individual classes) and – at the same time – to find pupils who were worth being considered for the participation in supportive measures for mathematical competitions and in the regional co-operation for the bursary for gifted pupils.

145 pupils (of 5 classes) of year 5, 174 pupils (of 6 classes) of year 6 and 139 pupils (of 5 classes) of year 7, i.e. 458 pupils altogether took part in the test. They had to work on problems which were the same for all of them, but also on problems (approximately one third) which were different, this means three-levelled problems. The following two chapters deal with some of those problems and with the results achieved.

2.2 Analysis of the Results of the Test Problems which were the same for all Pupils

240 Euros

240 Euros are to be divided among three sons in such a way that the middle son gets 20 Euros more than his older brother and 20 Euros less than his younger brother.

The eldest son gets ...., the middle son gets ...., the youngest son gets ....

Looking at the problem superficially it is a so-called "Textaufgabe". The correct answer is: The eldest son gets 60 Euros, the middle one 80 Euros, and the youngest one 100 Euros. 298 out of 458 pupils of all three age groups of years 5, 6 and 7 solved the problem, this corresponds to a success rate of 65.1 %.

A certain difficulty of this problem, which can also lead to mistakes in the answer, lies in the linguistically logical complexity. In this respect those cases have to be considered where test persons got the correct sum (240 Euros), but distributed it incorrectly (e.g. 20 Euros, 100 Euros, 120 Euros), or figures the sum of which was not 240 Euros. These two types of mistakes arose at rates of 30.3 % in year 5, 25.3 % in year 6 and 19.4 % in year 7.

The results can be interpreted as a lack of controlling measures on the part of the test persons. They do, however, also show that this lack of performance decreases from year to year.

Three-figure Numbers

How many three-figure numbers are there which can be formed by the three figures 1 and 2 and 3, whereby every figure may only occur exactly once.

There are .... of such numbers.

This problem requires the ability of combinatory thinking. 315 of the 458 test persons got the correct result (there are six of such numbers). The success rate is 68.8 %. With this problem it is not so much the linguistically logical complexity, but the cognitive one, which determines the probability of success. It is mainly about completely understanding the six combinations of the three figures. The monitoring of one's own thinking is required.

Rectangle

The rectangle with the sides of 8 cm and 10 cm has got an area of 80 cm^2 and a circumference of 36 cm.

Which statements are correct (Yes), and which are wrong (No)?

Every rectangle with a circumference of 36 cm has got an area of 80 cm^2. □ *Yes* □ *No*

Not every rectangle with an area of 80 cm^2 has got a circumference of 36 cm. □ *Yes* □ *No*

There are rectangles with a circumference of 36 cm which do not have an area of 80 cm^2. □ *Yes* □ *No*

The first statement is wrong, the other two are correct. 217 of 458 test persons marked the correct result (*No*) in the first statement; 209 marked *Yes* in the second statement; and 223 marked *Yes* in the third statement. 132 marked all three statements correctly. This results in a success rate of 28.8 %. A certain familiarity with the calculation of areas and circumferences is surely necessary. But apart from this the problem is also characterized by its linguistically logical complexity (*every, not every, there are ... , which do not*). It is therefore not surprising that the result rates to these statements hardly differ. This does also explain the total result of 28.8 %. Linguistically logical complexity is put in front of the actual dealing with the problem as an obstacle (Cohors-Fresenborg & Sjuts & Sommer 2004) and thus decreases the probability of solution. When assessing the results it has, of course, to be considered that all three parts of the problem have been answered correctly.

300 Marbles

Anna has got 300 marbles. She puts half of them into a bag. Then she puts one third of the marbles in her bag aside.

Martin has also got 300 marbles. He puts one third of them into a bag. He then puts one half of those marbles in his bag aside.

Mark the sentences with a cross which are correct!

□ *Anna has put more marbles aside than Martin.*

□ *Martin has put more marbles aside than Anna.*

□ *Anna has put as much marbles aside as Martin.*

The analysis of this problem can be compared to that of the previous problem. 90 test persons marked the first possibility, 110 the second one (which are both wrong), and 219 marked the third possibility, the correct one; 33 did not make any decision at all. This is a success rate of 47.8 %. It is not so much about fractions and the use of commutativity of multiplication (one half times one third is the same as one third times one half), but about the actual thinking process of a two-step thinking and calculating process, which is expressively allowed or even suggested by mentioning a figure (300 marbles). Obviously the two stages, and thus a certain cognitive complexity, prove to be an obstacle. The calculating processes themselves (a half of 300 is 150, a third of 150 is 50 – a third of 300 is 100, a half of 100 is 50) cannot take first place in serving as an explanation.

Sequences of Numbers

Continue the following sequences of numbers!

2	*5*	*8*	*11*	*14*	...	...	...
1	*3*	*7*	*15*	*31*	...	...	...
0	*5*	*15*	*30*	*50*	...	...	...

The pattern is clear for all three sequences of numbers. Divergent solutions, which could have been considered conclusive, did therefore not occur. The pattern of the first sequence is fixed by a constant distance between the elements of the sequence, that of the third one by an arithmetically increasing distance, that of the second one by a geometrically increasing distance. According to this sequence the solution success rates of 95.0 % for the first sequence of numbers, 61.6 % for the third one, and 37.6 % for the second one are clearly graded. Obviously the demands of the three sequences of numbers differentiate considerably according to their cognitive complexity.

The difficulties in working on the second sequence of numbers can be identified. First of all it can be assumed that the continuation of the pattern was widely realized. After all, 285 (out of 458, i.e. 62.2 %) of the test persons found the next element of the sequence (the figure 63) and noted it down correctly; 233 (out of 458, i.e. 50.9 %) found the next following element (the figure 127) and 172 (out of 458, i.e. 37.6 %) also the next element (the figure 255). It is remarkable how often the third figure, which had to be determined, was mentioned incorrectly, especially, what type of incorrect results were given. 172 of the 233 test persons who noted the first two following numbers correctly (namely the numbers 63 and 127), also put down the third correct following element. 61 of them, however, did not. The continuations offered were quite informative: eleven times the number 256, ten times the number 235, five times the number 254, five times the number 245, four times the number 155, three times the number 253, three times the number 265 as well as other numbers. How can this be explained? One possible way of calculating would be to add the figure gained from the sequence of differences (namely 128) to the figure 127 already obtained. As a result of mental arithmetic you can – without a thorough self-control – easily make a mistake in one figure. This is how you get numbers like 256, 235, 254, 245, 155, 253, 265. Even if you calculate differently, i.e. you double the element achieved and add 1, the calculation and thinking processes overlap. It is the cognitive complexity lying therein which makes success difficult and influences it.

It is striking that the success rates increase from age group to age group. The following table records the results:

	240 Euros	*Three-figure Numbers*	*Rectangle*	*300 Marbles*	*Sequences of Numbers*
year 5 (145 test persons)	54.5 %	59.3 %	21.4 %	34.5 %	54.7 %
year 6 (174 test persons)	61.5 %	69.6 %	21.3 %	48.9 %	64.1 %
year 7 (139 test persons)	80.1 %	77.7 %	46.0 %	60.4 %	75.3 %
years 5-7 (458 test persons)	65.1 %	68.8 %	28.8 %	47.8 %	64.5 %

First of all it is noticeable that the problems “*Three-figure Numbers*”, “*240 Euros*” and “*Sequences of Numbers*” were coped with best, the problem “*300 Marbles*” not so well and the problem “*Rectangle*” the least well. A first analysis was made when giving the results. Now attention has to be turned on the increase of performance from age group to age group, which can almost be observed throughout.

As mentioned before, the decrease in mistakes from age group to age group in the problem “*240 Euros*” had been ascribed to corresponding controlling measures. Linguistically logical complexity had been identified as the difficulty of this problem. In this sense it could be assumed that, over the years, mathematics teaching and school as a whole contribute to the understanding of language and text. This contribution seems to – if only implicitly – shape controlling mechanisms. This is how the increase in the success rate from age group to age group can be explained.

This increase from year 5 to 7 is also striking with the problem “*Three-figure Numbers*”. The monitoring of one’s own thinking process, given as an explanation above, obviously improves with teaching instructions and demands from year to year. It may furthermore be considered that the examples in mathematics lessons increase year by year where there is not only one solution to the problem, and thus completeness and full number of solutions develop with a concept.

With the problem “*Rectangle*” the increase in the success rate within the three age groups is different from the other problems. There is an obvious rise from year 6 to year 7. The explanation already given as regards the familiarity with areas and circumferences is possibly decisive.

A clear increase from age group to age group is noticeable in the problem “*300 Marbles*”. This does obviously also show that mathematics teaching and school as a whole seem to strengthen metacognition.

The problem “*Sequences of Numbers*” is suitable to state more precisely the attempts of analyses and explanations made so far. First of all the rise within the age groups is to be mentioned considering the total result of the problem. The explanations made above have already picked out the differences as a central theme. These differences become even more conspicuous when looking at the three sequences separately. The increase in the success rate from age group to age group is lowest in the first sequence (92.4 % – 94.8 % – 97.8 %), clearly higher in the third sequence (49.0 % – 58.0 % – 77.7 %) and highest in the second sequence (22.8 % – 39.7 % – 50.4 %). Here it can be expected that self-monitoring increases due to school and teaching in the course of the years as well.

	First Sequence	Second Sequence	Third Sequence
Year 5	92.4 %	22.8 %	49.0 %
Year 6	94.8 %	39.7 %	58.0 %
Year 7	97.8 %	50.4 %	77.7 %
Years 5-7	95.0 %	37.6 %	61.1 %

Increase in competence can be considered as a normal process of development at school and thus differences in competence can be translated into approximate times of development (school years) (Prenzel et al. 2004, p. 36). Information about average differences in competence among pupils of different school years can be converted into certain values on the usual ranges of competence grades.

But how can the competence be increased specifically and systematically? PISA-analyses have pointed to efficiency supportive possibilities in school and teaching lessons. According to that availability of pre-knowledge and learning strategies are considered to be promising influenceable significances to achieve efficiency improvements. Concerning the test problems above it should be convincing to achieve better results for these problems in particular, because the material demand is certainly not high. The result rates are explained by the cognitive and metacognitive demand, just like PISA: The "cognitive oriented analysis gives more further-reaching explanations than the one oriented to the subject matter." (Cohors-Fresenborg & Sjuts & Sommer 2004, p. 138)

It has to be separated between descriptive characteristics of problem difficulties and prescriptive statements about teaching and learning concepts for successful treatments of problems. PISA-analyses give well-founded criteria for the description and explanation of demands of problems, but no secured knowledge about learning mathematics successfully. There is need for more research.

3. Test Problems on Mathematical Thinking: Results Part B

The analysis of the problems first investigated the connection of the describable problem difficulty and the solution rate noticed. Now the second question starts to be interesting. How can problems be constructed to a difficulty level given – even with the demand to predict the success rate quite positively?

Theoretical approaches and empirical results can be found in the PISA-2000-analysis under the title "The Complexity of Thinking Processes and the Formalizing of Knowledge" (Cohors-Fresenborg & Sjuts & Sommer 2004). It showed the special importance of cognitive theoretical concepts. So it must be possible to construct problems, which are hardly different as regards the curriculum of subject matters and which can refer to the same content and have the same formulation, but which are so enriched that the demands on thinking, in particular on the organization of thinking, grow visibly and measurably (by the success rate).

Remember the problem *"Sequences of Numbers"*. To solve this problem cognitive processes were to be organized; several things were to be considered at the same time, especially the adherence to the pattern, which had to be recognized, neither forgotten as well as the remembrance of numbers, with the help of which calculating operations had to be made in a particular way and occasionally while processing provisional results.

The problem "S*equences of Numbers"* was to be worked on by all 458 test persons. It was made of three parts of problems without having a strict progression of difficulty, but the first part had the easiest demand, the second part had the hardest demand, and the third one the medium demand.

Two more problems shall follow now. They were to be worked on in the said test as well, but in a different way. To avoid a possible "work one's way up" from difficulty to difficulty within a particular problem context and to exclude familiarity with matching terms of problems, one third each of the 458 pupils got problems with a very different cognitive demand. The different cognitive complexity (CC) was mixed for the test persons in a random procedure.

Slab path (CC 1)

A garden path of a length of 9.00 m is to be covered with slabs. There are two kinds of slabs. They have a length of 1.00 m and 0.50 m. They are to be put down alternately.

Which slab do you have to begin with?

- □ *You have to begin with a slab of the length 1.00 m.*
- □ *You have to begin with a slab of the length 0.50 m.*
- □ *It doesn't matter which slab you begin with.*
- □ *It's not possible to cover the path alternately with these slabs.*

Slab path (CC 2)

A garden path of a length of 10.00 m is to be covered with slabs. There are two kinds of slabs. They have a length of 1.00 m and 0.50 m. They are to be put down alternately.

Which slab do you have to begin with?

- □ *You have to begin with a slab of the length 1.00 m.*
- □ *You have to begin with a slab of the length 0.50 m.*
- □ *It doesn't matter which slab you begin with.*
- □ *It's not possible to cover the path alternately with these slabs.*

<u>Slab path</u> *(CC 3)*

A garden path of a length of 10.20 m is to be covered with slabs. There are two kinds of slabs. They have a length of 0.60 m and 0.30 m. They are to be put down alternately.

Which slab do you have to begin with?

- □ *You have to begin with a slab of the length 0.60 m.*
- □ *You have to begin with a slab of the length 0.30 m.*
- □ *It doesn't matter which slab you begin with.*
- □ *It's not possible to cover the path alternately with these slabs.*

147 test persons of the 458 pupils had to deal with the problem with the cognitive complexity CC 1, 153 test persons the one with CC 2, and 158 test persons the one with CC 3.

Counting up the solutions of all three problems there is a solution rate of 36.2 %, which – compared to the results from the previous chapter – only increases slightly from age group to age group (33.8 % in year 5, 36.8 % in year 6, 38.1 % in year 7). However the difference in the success result between the versions CC 1, CC 2 and CC 3 is very clear. 89 (out of 147, i.e. 60.5 %) pupils solved the problem CC 1, 48 (out of 153, i.e. 31.4 %) the problem CC 2 and 29 (out of 158, i.e. 18.4 %) the problem CC 3.

Looking superficially it is just the changed numbers and solution variations which make the difference. Looking more thoroughly at this matter, it can be presumed that the decisive factors are again organization regulation and supervising of the cognitive operations. The one, who solves the problem, has to think and remember in his head, which kind of slab the path begins with, has to develop a calculation strategy at the same time, must not forget connecting steps which are important. Even if different solution ways are explicitly allowed, the cognitive efficiency does certainly increase from CC 1 via CC 2 to CC 3.

<u>Tub</u> *(CC 1)*

You need three minutes to fill a tub and six minutes to empty it.

How long does it take to fill the tub when it is plugged out?

Answer: ....

<u>Tub</u> *(CC 2)*

You need two minutes to fill a tub and three minutes to empty it.

How long does it take to fill the tub when it is plugged out?

Answer: ....

<u>Tub</u> *(CC 3)*

You need three minutes to fill a tub and five minutes to empty it.

How long does it take to fill the tub when it is plugged out?

Answer: ....

155 pupils had to deal with problem CC 1, 153 with problem CC 2 and 150 with problem CC 3.

The success rate altogether is 15.1 % (13.1 % in year 5, 13.8 % in year 6, 18.7 % in year 7). The age group increase is relatively small again. The results concerning the difficulty level have a noticeable difference. 43 (out of 155, i.e. 27.7 %) pupils solved the problem CC 1, 24 (out of 153, i.e. 15.7 %) the problem CC 2 and 2 (out of 150, i.e. 1.3 %) the problem CC 3.

The problem can be solved in different ways. Here is a short sketch, based on problem CC 2.

Solution alternative 1: From the problem text it can be taken that six minutes are needed to fill the tub three times and six minutes to empty it twice. So the tub is full (once) after six minutes. An equivalent consideration in a slightly simpler way also leads to the result in problem CC 1. Six minutes are needed to fill the tub twice and six minutes to empty it once. In this case it is full (once) after six minutes as well. There isn't such an obvious solution at problem CC 3. The equivalent consideration is far more difficult. You need 7.5 minutes to fill the tub twice and a half and 7.5 minutes to empty it once and a half. So the tub is full (once) after 7.5 minutes. Or modified: You need 15 minutes to fill the tub five times and 15 minutes to empty it three times. Therefore the tub is full twice after 15 minutes, so it is full (once) after 7.5 minutes.

Solution alternative 2: The problem text says the tub fills half and empties at a third after a minute, hence the tub is full to a sixth. So the tub is completely full after one minute. There are equivalent considerations for CC 1 and CC 3. In CC 1 the tub is full to a sixth after a minute, in CC 3 the tub is full to two fifteenth after a minute.

Solution alternative 3: Taking two minutes instead of one minute as a time period for the provisional result the tub fills once and empties for one third. Therefore it is actually full after six minutes. The equivalent way for the problem CC 1 is the following one: The tub fills once after three minutes and empties half at the same time. Therefore it is actually full after six minutes. An equivalent solution of problem CC 3 with regular fractions does not exist (see solution alternative 2).

It is obvious that the cognitive demand increases a lot (from CC 1 via CC 2 to CC 3). Looking for a solution it is not possible to overview right from the start whether the idea promises a success. The fact that CC 3 could do with a simplification of figures demands an additional amount of intellectual work. Metacognitive controlling is important, too, because several lines of thoughts are to be pursued and not to get lost.

The answers of the test persons at this problem give an additional result. The idea that the tub actually fills, even when it is plugged out, was missing quite often. 99 pupils (out of 458) gave kind of the following answer: *It never fills up. Very long. Infinitely long. Impossible. You can't fill the tub up when it is plugged out, so: eternal*. This phenomenon appeared at 24.1 % in year 5, at 23.0 % in year 6 and at 17.3 % in year 7. A particular (namely static) thinking obviously prevents to adequately understand the (dynamic) occurrence of two processes (here the one of flowing into the tub and the one of draining off). There is (again) a hint to "Achilles and the Turtle" concerning the famous paradox of Zenon (Sjuts 2002a).

Taking everything together, the clear differences in the results on the version CC 1, CC 2 and CC 3 – compared to the other problems – the relatively little increase from year 5 to year 6 and then to year 7 as well as the discovery that the calculation demand is not very high, lead to the conclusion that the main explanation for the success rate is within the area of cognitive complexity. The "mental operating system" is used differently concerning necessary activities of organizing and supervising one's own thinking process. And this already starts with reading the problem, while trying to get an idea of the problem and the cognitive tools, which seem to be promising for treating the problem.

Mathematics teachers are well familiar with the fact, that the cognitive complexity has a main importance. When constructing A- and B-versions of a class exam problem (half of the class each) differences must not concern the cognitive complexity, which might already happen due to different numbers in the problem which cause appropriate demands on the organization of thinking.

The following overviews are to emphasize again the connections between the success rate and the school year as well as between the success rate and the cognitive complexity.

	Slab path	*Tub*
Year 5	33.8 %	13.1 %
Year 6	36.8 %	13.8 %
Year 7	38.1 %	18.7 %
Years 5-7	36.2 %	15.1 %

	CC 1	CC 2	CC 3
Slab path	60.5 %	31.4 %	18.4 %
Tub	27.7 %	15.7 %	1.3 %

In all it is possible to identify characteristics of cognitive complexity, especially demands on extent, intensity and complexity of thinking processes, and to deduce clues which are suitable to construct problems of different demands at a comparable content base. This again allows conclusions on an arrangement of thorough and careful learning of mathematical thinking, in particular, when metacognition accompanies learning und understanding processes.

4. Mathematical Thinking – Overcoming instead of Avoidance

"Problems with a high demand are also suitable to achieve the general education which relates to mathematics. General education is in particular to learn something important that you wouldn't learn without school. Mathematical thinking belongs to it." (Sjuts 2003b, p. 75) One condition of the opportunity to learn mathematical thinking is a lesson atmosphere marked by both demand and confidence, that is characterized by a complementarily feature: Where mathematical thinking is allowed to unfold there it is able to be developed.

Another condition is the adjustment of mathematics lessons on competences. Educational standards have become outer expressions for that. How does a teaching arrangement look, where the mentioned competence development could take place in? Even here the educational standards give the answer. They refer to the active analysis of various mathematical contents, while students work on problems and projects with mathematical methods and while they read and write mathematical texts as well as they communicate about mathematical contents.

The test problems named above (with certain cuts) could have represented standards at the end of year 4 at primary school (Grundschule). Thus the lessons at primary school would have had to be aimed to that. This is also shown in the published problem examples in educational standards for primary school (KMK 2004). A special characteristic is to demand the writing of illustrating, explaining and founding texts from students. Educational standards are therefore not only an instrument to codify higher performance as the one to be demanded, but they are also helpful in the sense that obtaining and acquisition of mathematical competences attract more and more attention.

If text production in mathematics lessons becomes usual, the chance exists that learning of mathematics improves by cognitive and metacognitive activities – more than before. Organization clues to such mathematics lessons exist in different places (Sjuts 2004b). Lessons as contract, discursivity and problems are the main points that should be emphasized in this context (Cohors-Fresenborg & Kaune 2001, Sjuts 2002b, 2003a).

PISA 2003 substantiates that this isn't an inept expectation. The abilities of German students in solving problems show a cognitive potential that has been converted insufficiently to mathematical competences so far. The results in the domain of problem solving demonstrate that the lessons in Germany can and must be further developed (Prenzel et al. 2004, p. 367).

The educational standards rightly demand that mathematics lessons should aim to more difficult convergent and divergent problems – and earlier as has been done so far. Children should learn to solve such problems early enough. They should pursue text production and metacognition in general. They should get to higher cognitive achievements at an earlier age.

Spitzer (2004) illustrates the effect of such an understood learning with the help of the acquisition of language; at the same time he mentions mathematics as an evident example: "Please write down everything you learnt during your whole time at school on a piece of paper. I bet a small piece of paper will be enough. Have longstanding mathematics lessons been for nothing? Not at all!" (Spitzer 2004, p. 705) Because without remembering any details you have acquired competences, at the best organized mathematical thinking. "We can do a lot and know little." (Spitzer 2004, p. 704)

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