Is the definition of mathematics as used in the PISA Assessment Framework applicable to the HarmoS Project?

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Abstract: The project known as the "Harmonisation of the Obligatory School", or in its shortened form as "HarmoS", has a high priority for Switzerland's educational policy in the coming years. Its purpose is to determine levels of competency, valid throughout Switzerland, for specific areas of study and including the subject of mathematics. The general theoretical basis of the overall HarmoS Project is constituted by the expertise written under the direction of Eckhard Klieme and entitled "Zur Entwicklung nationaler Bildungsstandards" (Klieme 2003) [i.e. "On the Development of National Education Standards"]. The proposal announcing the HarmoS partial project devoted to Mathematics includes references to the results and subsequent analysis of PISA 2003. It thus seems appropriate for us to begin our work on HarmoS with a critical consideration of the definition of mathematics and mathematical literacy as they are used in the PISA Study.
In a first part, we want to describe the core ideas of HarmoS. In a second part, we will address the meaning of general educational goals for the development of competency models and education standards to the extent that it is necessary to properly locate our problem. In a third part we will analyse the concept of mathematics which is at the basis of the PISA Study (OECD 2004) and more precisely defined in the publication "Assessment Framework".(OECD 2003) In the fourth and last part, we will try to provide a differentiated answer to the question posed in the title of this paper.

Kurzreferat: Das Projekt "Harmonisierung der obligatorischen Schule" oder kurz "HarmoS" hat für die Bildungspolitik der Schweiz in den nächsten Jahren hohe Priorität. Beabsichtigt ist eine gesamtschweizerische Festlegung von Kompetenzniveaus in bestimmten zentralen Fachbereichen, u. a. im Fach Mathematik. Die allgemeine theoretische Grundlage zum Gesamtprojekt HarmoS bildet die unter der Leitung von Eckhard Klieme entstandene Expertise "Zur Entwicklung nationaler Bildungsstandards"(Klieme 2003), für das HarmoS-Teilprojekt Mathematik wurde im Ausschreibungtext u.a. auf die Resultate und Folgeanalysen von PISA 2003 verwiesen. Es erscheint uns deshalb sinnvoll, zu Beginn unserer Arbeit an HarmoS den Begriff von Mathematik und "mathematical literacy", wie er der PISA-Studie zugrunde liegt, einer kritischen Reflexion zu unterziehen. Wir wollen dazu zunächst in einem ersten Teil die Kerngedanken von HarmoS darstellen. In einem zweiten Teil werden wir auf die Bedeutung allgemeiner Bildungsziele für die Entwicklung von Kompetenzmodellen und Bildungsstandards eingehen, soweit es zur Einordnung unseres Problems nötig ist. In einem dritten Teil werden wir den Mathematikbegriff, der der PISA Studie (OECD 2004) zugrunde liegt und in der Publication "Assessment Framework" (OECD 2003) genauer bestimmt wird, analysieren. Im vierten und letzten Teil werden wir versuchen, eine differenzierte Antwort auf die Titelfrage zu geben.

ZDM-Classification: A40, B10, B20, D30

1. HarmoS

"HarmoS" is an abbreviated form of "Harmonisation of the Obligatory School" and with it that which should finally be achieved with the HarmoS Project, or at least an initial and rough outline: it refers to a long-term harmonisation of the obligatory school in Switzerland. The necessity for harmonisation has been justified by the Swiss Conference of Cantonal Directors of Education (EDK) – a body corresponding to the Committee of the Ministers of Education and the Arts in Germany – in its policy paper on the objectives and conception of HarmoS as follows:

> "A growing mobility of the population, the permeability on the tertiary level, the organisation of education on the secondary level II (general education and vocational training) through basic curricula valid throughout Switzerland, the raised scholastic standards, requirements of parents as well as expectations from educational circles and from industry today necessitate a more precise statement and harmonisation of learning results.
> From this political background, an effective and binding intensification of the harmonisation efforts in the obligatory school provides an answer to the expectations of society for an improvement in the quality of the schools." (EDK 2004a, p.2)

This means, first of all, that the harmonisation and with it the anticipated precision should do justice to the standards, the requirements, and expectations which students, educational and training institutions, parents and industry place on the obligatory school education and, secondly, that it should contribute to an improvement of the quality of the schools.

This long-term and ambitious goal, which denotes the HarmoS Project, is opposed to a much more modest goal that actually characterises HarmoS: the determination of competency levels effective throughout Switzerland. This goal is to be achieved through a co-ordination between science and political policy.

> "The *HarmoS* Project intends to establish comprehensive competency levels in specific core areas for the obligatory schools in Switzerland. The necessary work in this respect is being accomplished on two levels:
> - The pedagogic-didactic level includes the development of competency models. This makes it possible to determine exactly which levels of competency can be expected at a specific time in the obligatory school (2nd, 6th, and 9th school years).
> - The juridical level relates to the conclusion of an inter-cantonal agreement on the harmonisation of the obligatory school. It constitutes an extension of the school concordat of 29 October 1970 and guarantees a binding character for the competency levels that have been stipulated." (EDK 2004a, p.1)

Therefore the course of the project, as planned, can be roughly divided into two phases:

- a scientific phase in which competency models are elaborated, empirically examined, and validated by practical experience in the school, and
- a political phase in which, on the basis of these models, a "specification of anticipated minimum competencies for all students for that particular school year" (EDK 2004b, p.12) is enacted in an inter-cantonal agreement".

This procedure in which the tasks are divided between scientific and political levels, or authorities, results in good part from the intent of providing adequate legitimacy for the competency levels that have been determined and thus for the greatest possible acceptance. According to this procedure, the determination of competency levels is neither a merely academic act nor a merely democratic act but rather an interaction of both. On the basis of scientifically substantiated competency models and empirical tests, the group responsible for a particular subject formulates a recommendation for competency levels which, in a second phase, is then subject to a democratic process (discussions in the cantonal parliaments, hearings, a motion if necessary, etc) and finally enacted – possibly in a revised form.

By and large, this summarises the HarmoS Project. If the role that is attributed to the scientific side of this project is more closely examined, two difficulties can be recognised that are systematic in character and linked to each other. The first relates to the criteria to be applied in choosing those mathematical themes, from the many partial components of the subject, that should be elaborated for the various grade levels. But even if, in a mathematical sense, it involves only finitely many partial components at one particular moment, the budget for HarmoS is too limited, in terms of finances and time, to investigate all of them. The second difficulty concerns the question as to which criteria, based on an elaborated competency model, should be chosen as a recommendation for the determination of minimum standards. In both cases, of course, it cannot simply be a matter of proposing just any criterion but rather of indicating criteria that can be satisfactorily legitimised. Although both problems were not given special attention in the EDK report on HarmoS, there are nevertheless certain references to them in the already-cited expertise "Zur Entwicklung nationaler Bildungsstandards" – an expertise on which the HarmoS Project should be focused as much as possible. We would like to address this in the second section that follows.

2. The Role of Educational Goals

In view of the allocation of tasks associated with HarmoS to a scientific and a political level and a corresponding division of the project into scientific and political phases in terms of time, the misunderstanding could arise that a scientific elaboration of competency levels could be achieved in a value-free way in the first phase, and that the normative part of the determination of minimum standards could be postponed to the second phase.

Let us first consider this in relationship to the first phase. Not only the economic argument mentioned above speaks against the opinion that the choice of partial dimensions could result independently of normative decisions, and especially of educational goals in general, but also the fact that only when we relate general educational goals to a subject or to an area of learning, the contours of this subject or of this area and its importance for the students usually become clear. The expertise of Klieme includes the following statement:

> "Educational goals are usually tied to a certain understanding for the importance that a subject or area of learning has with regard to personal development and wherein its social function exists. Is the learning of foreign languages aimed at communicative behaviour, or rather at a systematic introduction to a language and culture? Does mathematical training consist in the recognition of patterns for problem solving and the mastering of processes or rather in the capacity for modelling situations?" (Klieme 2003, p.20)

It is clarified by the following quotation from the PISA 2003 report that concerns an attempt to justify the poor results of students in individual partial dimensions:

> "In schools, mathematical content is often taught and assessed in ways that are removed from authentic contexts – *e.g.*, students are taught the techniques of arithmetic, then given an arithmetic computation to complete; they are shown how to solve particular types of equations, then given further similar equations to solve; they are taught about geometric properties and relationships, then given a theorem to prove. Having learned the relevant concepts, skills and techniques, students are typically given contrived mathematical problems that call for the application of that knowledge. The mathematics required is usually obvious. Students have either mastered the techniques needed, or they have not. The usefulness of mathematics in the real world may be given little attention." (OECD 2004, p.38)

In fact, it is not only occasionally that mathematical contents are taught and evaluated in this way. Criticism also pertains in a similar manner to certain curricula and teaching materials that are still in use in Switzerland and that allow this kind of instruction. Although a synopsis and analysis of the curricula presently valid in Switzerland is an important basis for the work being done on HarmoS, a harmonisation simply concentrating on a levelling of status quo requirements, would fall short. In fact, it is not a matter of merely harmonising the existing curricula nor one of a paradigmatical change from a system orientated on input and average performance to a system orientated on output and on minimum competency etc. Concealed or exposed, it is a matter of reflecting on the educational goals in general.

It follows from this that, as an initial requirement for the HarmoS Project, it must become transparent, that already the first – scientific – phase is subject to general educational goals as normative conditions, be it that such goals are explicitly formulated, be it that they are implicitly present within the concrete curricula. These goals as such are not the result of scientific research. At most, one or the other deviation of concrete goals from general goals reveals that they are in contradiction to other equally inferable goals, that they conflict with the basic understanding of the subject matter and its didactics or that it is unlikely that they will be attained by almost all of the students.[i] A second requirement ensues from the latter. One may not simply separate this normative part from the scientific phase because the intimated contradictions, discrepancies and unrealistic notions of objectives must be capable of being pointed out and criticised. In referring to the concrete work on HarmoS, we believe it is important for the normative parts of the scientific phase to be recognised as such and not suppressed or dismissed, and that there is a critical examination of alternative notions of mathematics and of alternative objectives of mathematics education and mathematics classes – specifically, a critical examination of the definition of mathematics and objectives of PISA on the one hand and that of the existing curricula in Switzerland on the other.

So much for the first difficulty, which concerns the concealed normativeness in the selection of partial dimensions. Now let us look at the second difficulty, which is linked to the specification of certain grade levels as a minimum competency. There is also the danger here of underestimating the meaning of general educational goals, or of normative influences. On the one hand, the expertise on the elaboration of national educational standards emphasises that the decision for a certain level of competency, in the sense of the theory of national standards of education, should not be decided at discretion:

> "If requirements are established as levels of a competency model, the decision is certainly not made as one simply thinks best. These are professional decisions that are focused on educational goals (such as a comprehension of the educational commitment related to a subject) and that also take account of the experiences and traditions of the relevant disciplines. If it is a matter of determining which requirements are reasonable and justifiable, there is above all a need for specialised didactics. Educational standards contribute to making these decisions transparent, and thus verifiable, according to scientific and professional criteria." (Klieme 2003, p.22)

On the other hand, it is clear that general educational goals are indispensable in this decision-making process.

> "Without reference to general educational goals, standards of competency would be purely arbitrary or merely reflect the opinions of experts. It is the orientation on these goals that legitimises the designation of the desired grade levels and the testing procedures that result from them. In particular, standards should originate from an understanding of the educational commitment related to the respective subjects that must be explicated." (Klieme 2003, p.23)

Once again, there are two requirements for the HarmoS "Mathematics" project resulting from this: firstly, that the recommendation for determining specific grade levels in the various partial dimensions must be focused on general educational goals and on the educational commitment for the specific subject, and hence receives its legitimation; and secondly, that the educational commitment relevant to the subject must be defined explicitly.

Thus general educational goals and the educational commitment relevant to the subject play an important role in reference to both, the choice of the partial dimension on the one hand and the determination of specific educational standards on the other. In the end, they are the instances to which the work on HarmoS must be directed if it is to achieve a broad basis of legitimation.

For Switzerland, as a country that sees itself strongly marked by federalism and direct democracy, this is more precarious than appears at first sight. From the federalist stand point, which views ideas regarding the harmonisation of the obligatory school with scepticism if not with rejection, seeking a "common denominator" for the existing curricula and extrapolating the educational commitment of mathematics from a reassessed synopsis of the existing mathematics curricula could be a solution to the first problem. From the democratic point of view, the second problem could easily be transformed into the question of defining the minimum standards that are capable of achieving a majority. There is also the danger here that the educational commitment for mathematics, which should be the actual legitimation criteria for the specifications according to the conception proposed in Klieme's expertise, will be extrapolated from the specifications only subsequently.

Neither is the professed goal of HarmoS. However there is a danger that HarmoS could be interpreted in this way and that the present work on the synopsis of the curricula in the first respect and a planned raising of the actual competency levels in the second respect could be misunderstood.

How should a legitimation of the selected partial dimensions and a legitimation of the definition of minimum standards be realised ? First of all by assuring that the understanding of the educational commitment of Mathematics, which should form the basis of legitimation for the selection of partial dimensions and for the specification of minimum standards, is made explicit. If it is made explicit, it will firstly provide a guideline for the concrete work of all of those participating in the HarmoS Project. Secondly, in respect to educational policy, it will offer the possibility of critically examining this understanding and whether it is in accord with general educational goals, if it conflicts with other socially important values or fails to take consideration of important social conditions. And thirdly, with regard to the scientific community, it will allow a critical and rational discussion as to whether this understanding can legitimise the choice of the partial dimensions as well as the proposal of certain competencies as minimum standards.

In the publications of the OECD devoted to PISA 2003 – and above all in the publication entitled "PISA 2003 Assessment Framework", (OECD 2003) the understanding of the educational commitment for mathematics, which is basic to the PISA project, was made explicit. We would like to more closely examine this understanding in the third part that follows.

3. The educational commitment for mathematics as specified by PISA

The understanding of the educational commitment, which is basic to the PISA 2003 study for mathematics, can be determined by the numerous and different variants of objectives that have been presented on the one hand and by the definition of mathematical literacy on the other.

> "The aim of the OECD/PISA study is to develop indicators that show how effectively countries have prepared their 15-years-olds to become active, reflective and intelligent citizens from the perspective of their uses of mathematics. To achieve this, OECD/PISA has developed assessments that focus on determining the extent to which students can use what they have learned." (OECD 2003, p.55)

The educational commitment *in general* is thus perceived as preparing students to become active, reflective and intelligent citizens, the educational commitment relevant to mathematics *in particular* is seen as enabling them to make the requisite use of mathematics. This aspect of "making use" is emphasised once again: whereupon PISA declines to examine the extent to which students can actually apply what they have learned. One can conclude from it that as far as the educational commitment is concerned, the purpose of teaching mathematics is largely seen in enabling students to acquire the necessary competencies. The basic capacity in general, or the basic competency that is concerned, becomes obvious from the definition of mathematical literacy:[ii]

"Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen." (OECD 2003, p.24)

Following this, certain expressions used in the definition such as "the world", "to use and engage with" and "that individual's life" are clarified in the Assessment Framework. We will return to the former somewhat later.

It is also interesting to note what is perceived as useful but does not belong to the definition of mathematical literacy according to this understanding of the term. It includes "attitudes" and "emotions" such as "self-confidence", "curiosity" and "feelings of interest and relevance" (OECD 2003, p.24).

What can now be said with regard to the understanding of the educational commitment relevant to teaching mathematics and, with a view to the HarmoS Project, what should be more closely examined, criticised, improved or elaborated? We would like to address just three of the thematic focal points:

Preparation for the role of citizen.

The central point in the PISA understanding of the educational commitment in general is that of adequately preparing students for their roles as citizens. That is certainly an important function but is it the only one? The term "citizen", as it is used here, is ambiguous. Is it a coincidence that in one part of the text, objectives are formulated with reference to "active, reflective and intelligent citizens" (OECD 2003, p.55) and in another part of the same text they are formulated with reference to "informed, reflective citizens and intelligent consumers"? (OECD 2003, p.24) One should not overextend the concept of citizen: even if we are (almost) all citizens, not everything that we do is a component of our role as citizen. Sociology distinguishes between a multitude of other roles that we want to – or must – fulfil competently, and that cannot be simply subordinated to the role of citizen, even though they may require the same or similar competencies.

Would it not be necessary for HarmoS to extend the educational commitment to the dimension of self-determination and self-realisation? Even if one believes that the same competencies are needed, it is important, first of all, to explicitly formulate (or to explicitly reject) this as an educational commitment, and secondly to explicitly show that the requisite competencies have already been covered by the dictates of the first educational commitment. A quotation from the expertise of Klieme refers to both dimensions:

"There is a practical consensus for the function of general education and for the expectations of its quality that should not be overlooked beyond the oft-renewed controversies. Viewed from a social point of view, this consensus refers both to the expectation that the educational system be included in the work of preparing for the role of citizen – in other words of enabling participation in public life – as well as to the expectation that capabilities will be acquired from the subjects within the educational system to allow the structuring of the individual's life as a learning process in spite of the uncertainty of profession and job, career and social standing." [iii]

The importance of mathematics

If one carefully reads the first part of the definition of "mathematical literacy", one begins to falter and wonder if the standards that it puts forward have not been set too high and if that which it formulates is indeed meant in this way. "To identify and understand the role that mathematics plays in the world" is a very demanding task if it is not simply limited to the recognition and understanding that mathematics plays a *very large* role in the world. The broad definition of "world" given in the commentary encompasses the "natural, social and cultural setting in which the individual lives" and is related to the dictum of H. Freundenthal, "Our mathematical concepts, structures, ideas have been invented as tools to organise the phenomena of the physical, social and mental world." (OECD 2003, p.25) With reference to HarmoS, which is concerned with a harmonisation on the level of minimum standards, we believe that to recognise *that* mathematics plays a role in each of these areas and at least, in part, to personally comprehend it is asking too much.

It is possible, however, that the first part of the definition is only intended to create a counterpole to an understanding of mathematics, which primarily places the solving of traditional mathematical problems in the centre, and fails to take consideration of the role that mathematics plays in the world, or represents its role only distortedly. In this case, the question arises as to whether one should not take up, or even augment, some portion of that which has been formulated in this part of the definition. Here, of course, the significance of mathematics is being addressed – and for a basic education in mathematics, it would be important to know and understand something about it in this respect. The significance of mathematics amounts to something more than a purely auxiliary function for other sciences, for technology and for the "natural, social and cultural setting in which the individual lives". Mathematics also has an importance for, and a value, in itself. Thus the phrase "the role that mathematics plays in the world" is also in a certain way ambiguous. The world in which mathematics plays a large role is only seemingly identical to the world of mathematics. Would it not be necessary for HarmoS to go even beyond the conception of PISA in this point and to take into consideration the intrinsic value of mathematics and of the occupation with mathematics as well?

Problem solving

A crucial capacity implied by the notion of mathematical literacy is the ability to pose, formulate, solve, and interpret problems using mathematics within a variety of situations or contexts. (OECD 2003, p.25) In addition to "making use of", "problem solving" is in fact *the* central paradigm for that which should be understood as "mathematical literacy", and thus it also has a leading function in the construction of mathematical problems. But also this concept as it is used here is ambiguous.

Based on the commentary to the definition of "mathematical literacy", one can gain the impression that the notion of problem solving is very broadly interpreted, as though the contexts would extend from the purely mathematical to those in which the mathematical structure can hardly be recognised.

In consideration of these statements, one could think that the inner-mathematical problems and their solutions – such as posing, assuming, and formulating mathematical correlations, defining appropriate terms, and posing, assuming, formulating processes, and outlining possible proofs – would be included.

However the test problems that have been published and the concept behind them reveal another tendency. Although the solving of aesthetic problems, of problems that serve as recreation or of inner-mathematical problems is included in the definition of mathematical literacy,[iv] these problems have to be transformed and remodelled. A problem solving situation in the sense of PISA 2003 emerges only after having been masked in the professed framework of possible categories reflecting situations or contexts, such as "educational/occupational", "public" and "scientific". (OECD 2003, p.32) Viewed in this way, the notion of problem solving is rather narrowly defined. An example from the Assessment Framework can illustrate this. It pertains to "Mathematics Example 2 Fairground Gameboard". (OECD 2003, p.28f.) Instead of immediately asking about the probability of a coin 6cm in diameter, tossed at a chessboard, staying in one of the 10x10 cm squares without touching the adjacent one, the problem is transposed into a game of chance at a fairground. Is the problem not of sufficient interest to present it without its being disguised ? The commentary that follows states that: "Clearly this exercise is situated in reality". There is no doubt that such a situation can occur in reality, but the answer to the question "What is the probability of winning at this game?" is, in a *realistic situation*, dependent not only on mathematical probability (in the strict sense) but also on the honesty of the players at the fairground who are not only opponents but also referees. By disguising the mathematical problem in this fictitious-realistic context, or situation, the achievement of the general educational goals mentioned above are hindered rather than furthered. With regard to HarmoS, one must ask whether an embedding in the context of a fictitious-realistic problem solving is always necessary or advantageous.

4. Applicability of the definition of mathematics as used by PISA for the HarmoS Project.

The harmonisation of the obligatory school in Switzerland – strongly marked by federalist thinking – can, with help of the HarmoS Project, only be successful if there is a wide and sustainable degree of acceptance for the proposed educational standards and the competency models on which they are based. In addition to the requisite transparency mentioned above, there is a need for a comprehensible justification of the educational standards proposed for the various subjects, on the basis of paramount educational goals in general as well as on educational goals related to a specific subject. Although a synopsis and analysis of the existing curricula for mathematics in Switzerland can make an important contribution, it is not sufficient in itself. The underlying concept of mathematics and of teaching mathematics, and thus the understanding of the educational commitment of the subject, are not always sufficiently clear in these curricula and not always sufficiently related to the concrete contents and objectives. In PISA 2003, through the objectives of the examination and the definition of mathematical literacy, an educational commitment has been determined and, in the construction and description of test problems, a relationship has been established to concrete contents and goals. Thus, is the definition of mathematics and mathematics instruction underlying the PISA study a valid alternative or supplement?

In our opinion, in each of the three thematic focal points there is one aspect in the conception of PISA 2003 that should be taken into consideration and elaborated for the HarmoS partial project devoted to mathematics.[v] In the case of "Preparation for the role of citizen", it is the attempt to legitimise the educational commitment of the subject through the contribution made by the subject to preparing for life in general. Under the heading of "Importance of mathematics", it is the aspect that competency in mathematics should also include the capability to evaluate the importance of mathematics in the world, and to express sound mathematical judgements. In the case of "Problem solving" as a thematic focal point, it is the revaluation of mathematising as opposed to merely using it for calculations and algorithms.

At the same time, in each of the three thematic focal points, there are ambiguous terms ("citizen", "world", "problem solving") that, according to how they are understood, can occasion irritation and misunderstanding and, according to the interpretation, lead to a conception that is inapplicable for HarmoS in our opinion. It is something else to prepare students for life and for the role as "active, reflective and intelligent citizens". First of all, as mentioned above, there are a number of other important roles that differ from these, and secondly, it is questionable if an educational commitment can only be legitimised if it serves as the preparation for a particular role in the "real world". In a certain way, there is already a tension here between the objective of PISA and the definition of mathematical literacy: the "capacity to identify and understand the role that mathematics plays in the world" or the "capacity to make well-founded judgements" can only be translated with a certain loss of meaning when referring to roles.

An overly restrictive interpretation of the Freudenthal dictum cited above, in which the meaning of mathematics is seen *exclusively* as providing "tools to organise the phenomena of the physical, social and mental world", is also too narrow and of no use for HarmoS if one understands the designated three worlds in such a way that the first represents the object of the natural sciences, the second the object of the social sciences and the third the object of psychology. The efforts that have been made, and will be made, to prove mathematical theorems that reveal no benefit whatsoever with regard to the three designated worlds could not be explained.

Finally, the standpoint that perceives the competencies to be acquired or furthered through instruction in mathematics exclusively in the capacity to solve problems is inadequate for HarmoS. Either this way of viewing leads to an artificial mutation such as with the "Fairground Gameboard" (see above); or the concept of problem solving is so broadly understood that even the solving of a mathematical exercise, the proof of a mathematical theorem, or the comprehension of a mathematical text can be interpreted as problem solving; or it leads to a strong constriction of what is regarded as essential competencies so that the educational commitment related to mathematics is limited exclusively to furthering the competency of problem solving.

Thus we believe that the PISA conception of mathematics and mathematics instruction, as the only and comprehensive basis of legitimation, is not applicable to the HarmoS project devoted to mathematics, but that the above mentioned aspects should nevertheless be recognised within the project.

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i The situation here is similar to the one that was analysed and criticised by Max Weber (1917, p.40ff.) with regard to value judgements in the social sciences.

ii The choice of the expression "mathematical literacy" is justified by wanting to emphasise that it concerns a functional use of mathematical knowledge in a multitude of different situations and in various ways that reflect, and are based on, comprehension. In addition to the knowledge of mathematical terminology, facts and procedures, mathematical literacy also presumes facility in the execution of certain operations and in the implementation of certain methods, and calls for a creative combination of all of these elements in order to cope with the requirements of external situations.

iii (Klieme 2003, p.62f.) – The spectrum of possible educational goals beyond that of the role of citizen is illustrated by the following quotation. "The educational objectives reveal the chances for developing the individual personality, for the inclination towards cultural and academic traditions, for coping with the practical demands of life and for an active participation in the life of the community that we want to give to our children and youth. The educational objectives of the school should also be focused on sustainability and transfer. They must permit life-long learning, and they must be related to the demands of everyday life, profession and society." (Klieme 2003, p.20)

iv The expression "to use and engage with" is explained in such a way that both areas of "application of mathematics" and "solving of mathematical problems" are covered. But it also implies that "a broader personal involvement through communicating, relating to, assessing and even appreciating and enjoying mathematics. Thus the definition of mathematical literacy encompasses the functional use of mathematics in a narrow sense as well as preparedness for further study, and the aesthetic and recreational elements of mathematics." (OECD 2003, p.25)

v At this point, we are dispensing with an analysis of whether, and to what extent, these aspects had already appeared or were prepared in earlier didactic or specialised didactic formulations.