Features of mathematical activities in interdisciplinary, project-based learning

Stefan Halverscheid[1], Universität Bremen (Germany)

Abstract: Project-based learning in mathematics education leads to mathematical activities that are uncommon in regular lessons at school. Among these activities, the following are identified and examined more closely:

- the elaboration and formulation of relevant mathematical problems, including necessary definitions,
- the search for the mathematically feasible, and
- the recognition of opportunities to apply mathematical methods.

Also, implications for the design of project-based learning environments are developed.

Kurzreferat: Projektlernen in Mathematik führt zu mathematischen Aktivitäten, die in traditionellen Unterrichtsformen nur selten angetroffen werden. Unter diesen werden

- die eigenständige Formulierung problemrelevanter Aufgaben samt erforderlicher Definitionen,
- die Suche nach dem mathematisch Machbaren und
- das Erkennen von Gelegenheiten, mathematische Methoden anzuwenden,

herausgearbeitet und genauer untersucht. Darauf aufbauend werden Folgerungen für das Design von Umgebungen für das Projektlernen entwickelt.

ZDM-Classifikation: C39, D44, D53

1. Project-based learning at the high-school level in mathematics

1.1 The concept of project-based learning

In recent years, school curricula in Germany have stressed the importance of project work. Although the organisations differ among the federal states, learning environments of this sort seem to be established in the curricula at the high-school level. In order to avoid giving a distorted impression, we stress that these activities should be seen as exceptional events rather than as characteristic of everyday school life. Among those, project weeks (‘Projektwochen’) have been popular for many years. During a project week, ordinary school life is replaced by work in teams, which are comprehensive in age and which are united by a common interest and a motto. A more individualistic approach is to demand that in the final school year every student write an essay that may be based on some project. This is seen as preparatory for studying at the university. Some states in Germany reserve lessons for project work in the last or next-to-last school year; in Bremen, for instance, 80 hours in grade 12 are reserved for this purpose.

In view of the open concepts, it is not surprising that there is no fixed notion of project-based learning. We are, therefore, attempting here to define a somewhat radical approach that aims at minimizing the teacher’s guidance of the students. Roughly speaking, the teacher’s role is to pick out a suitable topic and, later, to serve as a sort of encyclopaedia, answering students’ questions. Ideally, authority is not used to influence the direction of the project.

There are many examples of learning environments with mathematical contents in which the project model is used. Applied to mathematics education, the project model has not achieved the same level of attention as it has in other areas, such as history and science. In particular, it is not clearly defined what is meant by the project character. Quite often, project work develops naturally from regular coursework. After an introduction, the students are acquainted with the basic techniques and tools for a certain area of mathematics and are then given the opportunity to use these abilities within a certain context. The problems or tasks given under these circumstances are not as open as they may seem at first glance. The students have a common mathematical and social background to which the design of the project refers.

Open problems in a curricular context do not necessarily satisfy the following definition of project-based learning: ‘Generally speaking, in project-based learning the students themselves define the challenges and questions they want to work on’ (Völker 2001).

This definition may be seen as unrealistic for the learning of mathematics. It should be taken into consideration, however, that projects in humanities, from which this definition is taken, often reach a point where they too cannot be carried out in accordance with scientific standards. The projects discussed in this paper come from extracurricular activities within the Saturday University (‘SamstagsUni’) project, which brings together students of different schools and ages to work on a certain topic over a weekend.

1.2 Concepts and aims related to project-based learning

The literature for this paper reflects the diversity of contributions to project-based learning. Several publications have a nonmathematical background. That is because research into project-based learning has been conducted in general contexts, the idea being that criteria worked out are valid generally. It seems clear, however, that mathematics was not the authors’ main interest.

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Some work on project-based learning at elementary school in mathematics investigates motivational aspects, e.g., Meyer (1997).

The history of project-based learning, in its various forms, is long (Knoll & Calvin 1988). John Dewey, who called for a more practical approach to education, is considered a pioneer of project-based learning; with Kilpatrick (Dewey & Kilpatrick 1982) he worked out conditions for project work. The concept of reflective thinking indicates which learning processes Dewey aims at (Dewey 1933). Dewey considers a thought process complete if the problem is made precise, a method of resolution is conceptually planned, and a solution simulated and experimentally checked.

Kilpatrick (1925) speaks of the 'project method … to treat children in order to stir the best in them and then to trust them to themselves as much as possible'. For Kilpatrick (1918), a project was a 'wholehearted purposeful activity proceeding in a social environment'. He pinpoints the crucial element as a 'purposeful act' that guides the process of experiences and determines the degree and the direction of the internal motivation. He distinguishes four types of projects: the producer's project, the consumer's project, the problem project, and the learning project.

The following aims are connected with project-based learning (Emert & Lenzen 1997): reference to society and life, self-determined, holistic, interdisciplinary learning, solution-oriented thinking, and knowledge transfer by communicative competence.

1.3 Other aspects of project-based learning

In project-based learning, students are motivated to switch roles and to assume responsibility (Duncker 1998). In the field of mathematics, this aspect takes on even greater importance. Laypersons become experts, and teachers often can learn from their students in a dynamic project. This stands in close relationship to paradigm shifts that concern the social forms of mathematical learning.

Multidisciplinarity is a characteristic of such learning environments; sometimes mathematics is integrated into the concept. Too often, the different subjects are not linked, and it is unclear whether the resulting spirit can be used fruitfully for mathematics. Relationships between several subjects are complicated, insofar as they contain aspects of coexistence and cooperation (Huber 1999[2]). When we speak here of interdisciplinary project-based learning in mathematics, we mean that mathematical activities come to the fore from a starting point within nonmathematical material.

1.4 Outlined theses for the arrangement of project-based learning in mathematical projects

With the definition of project-based learning given above, we now turn to ways to structure such projects. Probably, none of the following guidelines is new. They are collected here to give an impression of how the conceptual designs analysed in this article are set up.

- An important characteristic of project-based learning is independence. Students take the steps necessary to solve the problem on their own. The journey is the reward. Therefore, the potential of themes concerning the variety of learning possibilities has to be assessed. Throughout the project, instructors should follow the principle of giving minimal help; assistance should be given only when students ask for it.
- Another key characteristic is multidisciplinarity. Project-based learning is designed in a multidisciplinary way. A project becomes mathematical by making use of mathematical approaches. However, students should be given the opportunity to determine which approach to choose—sometimes at the cost of losing the original mathematical richness of the task.
- The goal of project-based learning is for students to experience research in a microcosm. Mathematical project-based learning, therefore, aims at creating a research atmosphere. This can hardly be achieved under time pressure, which would tend to re-establish a typical classroom setting.
- Most learning experiences come about when students explore different ways of approaching a certain topic and evaluate those strategies. Time should be reserved for presenting these approaches. It is crucial that both teachers and students appreciate and respect one another's contributions.
- Participants can choose when to work in a team and when to work individually. Hence, they are eager to find a balance between meeting challenges as a group or individually.
- It is the students' choice as to whether media and other auxiliary means, such as personal computers, are used. Their access should be guaranteed.[2]
- The mathematical approaches used during the project have to be supervised, and students should be accompanied by an instructor who can answer their questions. Therefore, the fields of exploration should be chosen at a level that allows some preparation for this task.
- Since research activities can be successful only with an appropriate background, the areas of mathematics should be elementary. The classical elements of school mathematics seem appropriate for the final years of high school.

[2] Experiences with the use of the Internet, however, are mixed, because some students spend more time searching for solutions than thinking independently.

2. Remarks on methods

This essay aims at identifying mathematical ways of working that are typical of project-based learning. In particular, learning processes are analysed not from the point of view of content; the idea, rather, is to work out features of mathematical activities that can be found in different contents.

Students have been observed in several project-based learning environments at the Saturday University (Halverscheid, Hiltawsky & Sibbertsen 2004)[3]. A video camera was used to record some of the activities of the students. To be sure, the enormous technical effort that would have been necessary to record all activities made such a goal unreachable. The groups in which the students work change very often, as do phases of individual work and discussions. We therefore asked the students to write down their impressions in research journals, which we will frequently quote in the sequel.

In view of the methodological difficulties inherent in documenting the learning processes comprehensibly, three projects are introduced to represent three features we singled out over the years as typical of project-based learning:

- In the billiard environment, the lapse of a billiard ball or the passing of a beam of light is studied on differently shaped billiard tables or mirror boxes. We will see that the activities in this environment are characterized by *the students' elaboration and formulation of relevant mathematical problems, including necessary definitions*. The project was carried out in three settings: in two extracurricular activities for students in grade 9 in four sessions of 90 minutes each and two sessions of three hours, and in a regular course (grade 11) in six sessions of 90 minutes each.
- In the lion-sheep environment, students look for a model in a predator-prey situation. This is a modelling task that gets to the heart of project-based learning, because the learners carry out the *search for the mathematically feasible* themselves. This has been tried in two extracurricular activities, each a session lasting three hours. In another session of the same length, the results were presented and discussed by the group.
- The opinion-poll project was carried out along with regular coursework in 11th grade over five months. As a particularly mathematical activity, the focus was on *the recognition of opportunities to apply mathematical methods*.

Finally, we discuss these features of mathematical activities in relation to classic problem-solving. This comparison is appropriate because analyses of mathematical problem-solving also tend to work out principles spanning different areas of mathematics.

3. The elaboration and formulation of relevant mathematical problems, including necessary definitions

Problem-solving with its demands is widely accepted as an important mediator in the development of mathematical abilities. In the following project-based learning environment, the students have posed problems several times that seemed relevant to them for the success of the project. In addition, they formulated definitions that they considered necessary for their description. In the following section, we will examine more closely the students' studies of billiards on differently shaped tables.

3.1 The billiard learning environment

In the billiard learning environment, the trajectory of a billiard ball should be described given a particular shape of the billiard table. The following introductory text has been used to get the activities started:

'A rectangular[4] billiard table has no holes. We assume that the balls roll on it without losing speed and never hit any of the corners directly. Describe the trajectories of the billiard balls.'

A variant of this has been formulated in the context of optics:

'In a rectangular box, mirrors are placed at its border showing to the interior. A ray of light parallel to the billiard table is reflected at these mirrors, never hitting the corners directly. Describe the trajectory of the ray.'

This format has been carried out with the following groups:

- In the project Saturday University in three sessions of three, two and three hours with students from grade 9 to 12 who joined in because they were interested in the topic. The choice of the geometry of the billiard table was left to the students. Students in this group are used to work with computer algebra systems and dynamic geometry software.
- In grade 9 of a grammar school ('Realschule') after the introduction of linear equations in the plane. We restricted ourselves to rectangular billiards in two sessions of 90 minutes and two sessions of 45 minutes.
- In a course of grade 11 on rectangular, circular and elliptic billiards, where we used the optical model.

[3] The concept of Saturday University shifted from formats typical of university life to project-based learning, the aim being to give students a taste of research in mathematical areas.

[4] It will become clear in the sequel that other geometries have been used as well: particular and general triangles, circles, and ellipses. In the course of grade 11, the students were asked to choose the geometry themselves.

3.2 Three strategies and their realisation by the independent elaboration of problems

3.2.1 Strategies at the work with linear equations

Almost all students of degree 9 oder 10 who had the choice picked the rectangular billiard to approach the problem by some heuristic strategies. They examined

- trajectories in their own sketches,
- differences of trajectories with varying initial conditions, and
- sometimes special cases of trajectories, e.g. those parallel to some side of the rectangle.

The majority of younger students picked out a specific example for which the problem was considered. 'We have solved it', Dirk and Jochen, in the group at the grammar school, shouted when they had finished their calculation to prove that their trajectory in the square touching the mid-points of opposite sides is indeed periodic. For them, the problem they picked out was *the* problem, although it is quite special. After some time, they turned to variations of this problem:

- they started with 'difficult' points, as Dirk said, and discussed for some time the equations after the first time the boundary is hit,
- then Jochen suggested to consider a trajectory of a ball from the mid-point of the first to mid-point of the adjacent side because he could read off the slope of the line,
- in this manner they worked the way through to the fourth point; only a sign mistake at the third point prevented them from getting the expected answer.

When they celebrated their solution, members of the other groups came to see what 'the' solution looked like. After five minutes of discussion, the other teams returned to their projects. Dirk and Jochen considered other special cases. After the third calculation, their project had turned into a routine problem. Dirk justified their restriction to the case of a square by giving a conjecture: 'If the numbers are not strange, the ball returns to the place it started from.' They felt it would be necessary to find a word to discribe this. But there was not enough time to proceed.

3.2.2 Approaches with the use of computers

All students in the three groups were used to work with some dynamic geometry software. In the class of grade 9, dynamic geometry software had just been used to visualise linear equations in the plane. The choice of computer software and the decision to use computers at all was left to the students but made easy by the choice of classroom with access to several computer desks.

The groups opting for the use of computers posed themselves the following problems:

- all groups defined the law of reflection at a line explicitly or implicitly,
- the different geometries led to attempts to realize ellipses with dynamic geometry software,
- the qualitative change of trajectories was debated even if it was not clear to the students whether non-periodic trajectories exist,
- in the case of ellipses, the construction of tangents proved to be difficult; one group decided to continue individually to search in the internet for a solution of the problem,
- a group of students in grade 10 had just learnt to write computer programs with LOGO in a computer science course; after some time, modules for a LOGO program were worked out as tasks for the team members; a long night of work led to quite a comfortable and elegant computer program, which illustrates the trajectories for the case of the ellipse.

The program of the last group was used for the final session in which all groups joined in. It leads to new phenomena, which are not easy to describe. All trajectories are tangential to "a smaller ellipse", as a student said. In fact, a confocal ellipse has the role of constant of motion: all trajectories are tangential to it. This can be proved with elementary plane geometry.

3.2.3 Working on geometric tasks following exploratively gained insights

The small teams, which the students formed at certain stages of the project, worked in one or two school rooms. They did not act completely independently from one other. A certain spirit of competition does play a role, too. News on progress in certain teams spreads quickly and can change other teams' plans. The individuality of the concepts and the pride students take in them, however, make it less likely that projects will completely change direction in response to new developments.

After gaining insights with dynamic geometry software, the teams tackled geometrical tasks, which the students worked out themselves. This is one of the fundamental ideas behind the use of dynamic geometry software; however, the results of the exploration are not suggested by the design of the given problem. They are based, rather, on the individual investigations, or on those of other students. The following geometrical questions were discussed:

- the verification of reflection properties in rectangular billiards,
- the examination of the relations between angles and the question of when orbits are closed,
- the question of the parameterization of the trajectories, when it was examined, at which time the billiard balls reach the boundary, and
- the verification of the constants of motion properties. In the case of the circle, these seem to be easier to prove than to discover. Two groups found this result

> and proved it. In the case of an ellipse, the result can be proved with elementary considerations using congruence geometry and the properties of the foci of the ellipse; one group discovered the result, but did not manage to give a proof. Already, the formulation of this property seems to be mathematically challenging.

In this environment it was striking that the students looked for links to areas of mathematics which were recently topics in the regular lessons.

4. The search for the mathematically feasible

4.1 A word on the silent search for the mathematically feasible

Although books and articles on mathematics tend not to mention it, the search for the mathematically feasible is a frequent feature of mathematical research. If a topic, but not its mathematical realisation, is fixed, what can be achieved depends largely on mathematical abilities as well as the progress of mathematical science. It is a commonly known by researchers in the area of mathematical modelling that there is an opposition between the wish to find models as realistic as possible and the mathematics available. For this as a research experience, it is not very important whether the students have reached the appropriate level in mathematics or whether the mathematical science is simply not yet developed enough for the problem.

4.2 The lion-sheep environment

The following learning environment seems to create a situation in which students search for the mathematically feasible.

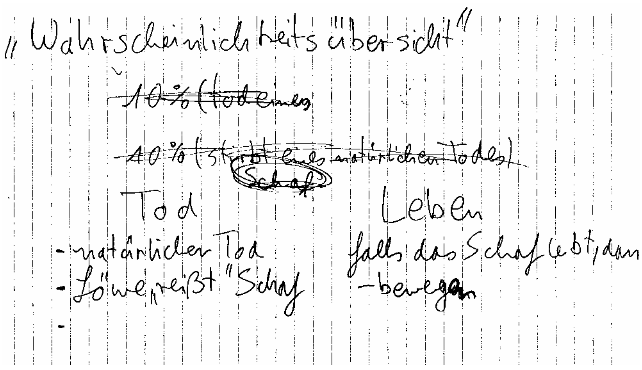
‘A sheep is in the wilderness. There is a lion, too, waiting for breakfast. What is the probability that the sheep survives for a certain, given time? Work out a model with the help of which you give an answer! Feel free to make assumptions which seem appropriate to you!’

The students were given three hours to work on this environment at two different occasions: at the Saturday University in a group of students from grade 9 to grade 12 and in an afternoon session with interested students of grade 10. The students had access to computers and were accustomed to using them. In the following, we look at the work of two groups at the Saturday University.

4.3 Two strategies

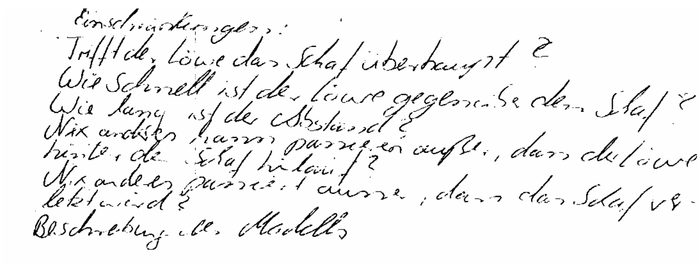
4.3.1 Reduction of the complexity of the problem

Miroslav, grade 9, and Peter, grade 10, work together in a group of two. For about 45 minutes, their discussion does not seem to focus on mathematics. They are rather interested in philosophical remarks concerning life and death, the life expectancy of a sheep and the question as to whether other dangers are looming in the wilderness.



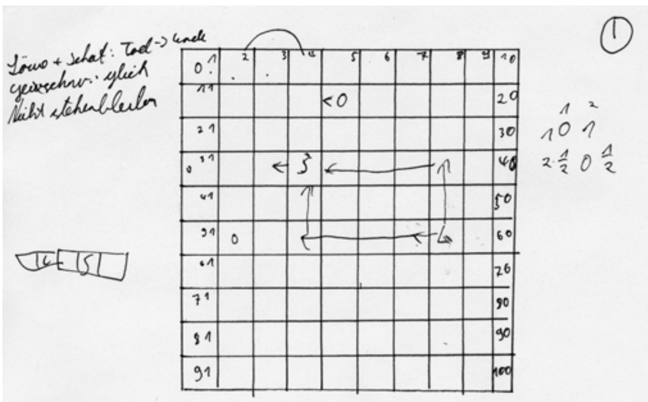
They only write down very few lines within this period, which they call ‘probability outline’. They write something about ’10 % (death)’ and ’10 % (by natural death)’ and confront the situations of ‘death’ and ‘life’.

Suddenly, they work out questions which are written down by the other student:

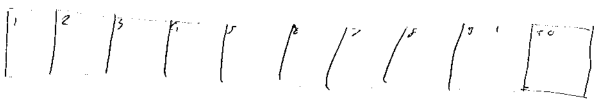


‘Does the lion meet the sheep at all? How fast is the lion in comparison to the sheep? How long is the distance? Nothing else can happen except for the situation that the lion traces the sheep? Nothing else can happen except for the situation that the sheep is hurt? Description of the model.’

The discussion becomes more mathematical and a two-dimensional model is worked out:



They consider probabilties to which the lion and the sheep move from box to box. They assume that the lion tears the sheep if and only if sheep and lion are in the same box. Despite many attempts, they do not make progress on the problem how to keep track of the movements and how to make use of the probabilities. After two hours of work, they decide to restrict themselves on the x-direction of the square and to neglect the second dimension.

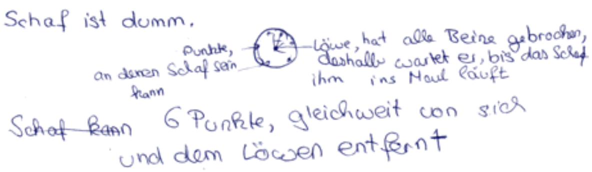


Miroslav and Peter discuss now that the probabilities describing the possible movements of the animals should be smaller at the border because there were less options to get to the boarder in the two-dimensional setting, too.

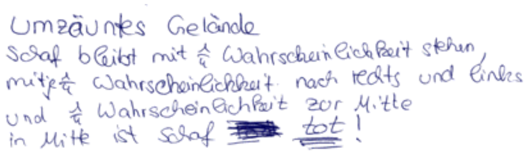
They decide to simulate this with a computer program they want to write in BASIC; but they do not complete this within the given time.

4.3.2 The search for a very simple, but interesting model

A group of five students of grade 10 to 12, two ladies and three gentlemen, also start with a discussion of about half an hour. They discuss how to approach the problem. Someone had the idea to 'do something which works out fine for sure'. The group decides to construct first a simple model. The first ideas prove to be much too trivial to be worthwhile considering in their opinion. Then the following is written down.



'The sheep is stupid. The lion has all legs broken. This is why he waits until the sheep runs directly into his mouth. 6 points have the same distance to the lion.'



They define the area to be surrounded by a fence to prevent the sheep from running away. Then they define the sheep's options within the following probability model: 'The sheep stays with a probability of ¼ at its place and does not move, with a probability of ¼ each, it turns right or left. And with a probability of ¼ it goes to the center.' The students stress the tragical end of the sheep's life: 'in the center, the sheep is dead.'

But they go beyond these amusing descriptions. Within the simple mathematical model, the group works out a couple of questions whose answers they want to strive for:

- after some probability experiments, which they carry out with two distinguishable coins (because of the probability of ¼), they conjecture that the sheep has „practically no chance" (of surviving) (the originally given problem to start with a certain given time whithin which the sheep's destiny is determined does not play any role anymore for the group),
- they ask how the answers change if the situation is generalized,
- the students change the question by varying the number of lions and sheeps and by, thus, creating more complex predator-prey models.

One might appreciate the mathematical results the group obtains less than those in the billiard environment. The experience, however, to start with very elementary models and to have no alternative than to use simulations for the solutions of slight generalizations is quite realistic for mathematical modelling.

5. The recognition of opportunities to apply mathematical methods: remarks on the limits of mathematical contents in producers' projects

In this paragraph, we discuss a problem that appeared in some groups working on the lion-sheep environment: It is a challenge in problem-based learning in mathematics to identify starting points that allow for the application of mathematical methods. This paragraph is included not only for the sake of completeness, but also to avoid wrongly giving the impression that possible mathematical activities are always realised by the students.

In a city of some 50,000 residents, 11th-grade students were asked to plan, carry out, and evaluate an opinion poll. The topic, the opening hours of stores, was chosen because it was being discussed frequently in the local media and because the city council had shown some interest in such a survey. The planning and the implementation of the opinion poll required a high degree of organisational work. In standardised interviews, the students stressed the low proportion of activities they considered mathematical. The self-evaluation of their activities will be investigated more closely elsewhere.

The students decided to carry out a representative opinion poll among residents, shop owners, and the staff of the shops. The following mathematical problems linked to this purpose were identified by the students:

- finding a standard to define representativeness,
- developing efficient and systematic strategies for carrying out the poll, and
- selecting and adopting appropriate statistical means for the evaluation of the data.

Although help from university researchers and the city statistics bureau was available, the interest in the mathematical background was more and more opposed to the pragmatic aim of a producer's project. The development of the project cannot be worked out here in detail. Roughly speaking, time pressure and the expenditure of students' labour became so big that even rather elementary mathematical questions could hardly be thought through. In view of reports of mathematicians working in industry, it is worth asking whether this difficulty is because of a lack of experience or whether it is simply a reflection of the reality of mathematics in the business world.

6. Discussion – Why project-based learning complements classical problem-solving

6.1 Main features of the work with problem-solving questions

There is more to mathematical skills than the ability to solve routine and nonroutine problems. The concept of mathematical literacy has to do with the relationship of mathematics to the students' needs in the future: 'Mathematical literacy is an individual's ability in dealing with the world to identify, to understand, to engage in, and to make well-founded judgements about the role that mathematics plays, as needed for that individual's current and future life as a constructive, concerned, and reflective citizen.' (OECD 1999). That description, however, does not answer and does not claim to answer the question of how the skill can be acquired. In this paragraph, we return to the starting point: As far as mathematics is concerned, does problem-based learning offer anything different from classical problem-solving questions and modelling exercises?

The merits of nonroutine problem questions for the learning of mathematics have been studied from various points of view (Pehkonen 1991). The following points seem characteristic for them:

- A nonroutine problem asks more from the students than the time-tested work along a scheme.
- The text of the problem question states clearly perequisites and the aim of what is to show by which means.
- The learners can be sure that a solution is possible under the perequisites stated in the text.
- The text of the question typically defines the scope of the area of mathematics; the main motivation to work on it is to find a solution.

From an instructor's point of view, nonroutine problem questions have the following advantages:

- These problem questions exist at various mathematical areas and levels of difficulty.
- Problem questions with a limited amount of mathematics allow for effective preparation.
- In case of difficulties and frustration, students can be given hints and solutions. It is also possible to vary problem questions for a group according to members' mathematical abilities.
- Guidelines for correction and marking of the problems can be defined clearly.

There is a long tradition of nonroutine problem questions in which a canon of heuristic strategies has been established (Engel 1979), e.g. the pigeonhole principle, the principle of extremes, the invariance principle, the symmetry principle, the strategy of working backwards, and the strategy of systematic trail and error.

These principles and strategies are part of the mathematical research and exist in different settings, even though samples of problem questions often stress a logical context. Comparing Polya's stages of problem-solving with the examples illustrated here, the students go through parts of problem-solving processes, which are, however, linked in a complex way with other processes like modelling and simulation.

6.2 The suitability of project-based learning environments

The experience of mathematical research can hardly be restricted to the solution of several problem questions, which are more or less elegantly strung together. It is part of the liveliness of mathematics and originality to approach a problem by the elaboration of several small problems, by attempts to solve those, by working from and by replacing assumptions, and by discovering mathematical phenomena and formulating definitions for them. Many circumstances (e.g., sociological circumstances) cause the significant drop in the number of participants for problem-solving competitions from grade 8 in Germany. It could be asked, however, whether the design of such problem questions is attractive enough for adolescents. Moreover, the author's studies indicate that those who are very successful at solving nonroutine problem questions tend to be reluctant to work in more openly designed learning environments (Halverscheid 2004).

The NCTM standards (NCTM 1991) state several principles that could be seen as supporting the point of view that it is necessary to complement classical nonroutine problem questions: 'Mathematics has broad content encompassing many fields' is one of the stated visions of mathematics. Among the visions of the learning of mathematics, one finds: 'Students should learn to value mathematics' and 'Students should learn to reason mathematically' as visions that stress the universal aspects of mathematics.

In this sense, it would be necessary to develop more project-based learning environments that offer everyone in class a broader view of what mathematics is and can be. The examples described here are not meant to breed mathematical researchers, but to give both weak and strong students a taste of the richness of mathematics. Project-based learning—especially in an interdisciplinary context—is roomy enough for various levels of talent. It would surely be one of the many challenges of the instructor to guarantee that weaker students are given the same attention in the groups.

6.3 Perspectives

At the moment, reports and questions to the author indicate that the recently developed activities in the curricula are only very carefully realized practically. This may be due to the enormous change in the teachers' roles and in the careful preperation of such projects. It takes effort to examine learning processes in project-based learning more systematically and to develop and evaluate suitable environments for them.

It has been examined in several articles (Anzengruber et al. 1994; Frey 1982) that interdisciplinary project-based learning soon reaches certain limits. We have seen in the fifth paragraph that project-based learning in its radical form could lead to less mathematical contents than planned. The discussions of classical problem-solving questions are not meant to contest their educational value, but to justify the starting point of our investigation here. In other words: Problem-solving cannot be replaced entirely by more open learning environments like project-based learning. On the other hand, students who do not have the opportunity to experience project-based learning in mathematics are missing something.

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Author

Halverscheid, Stefan, Fachbereich Mathematik-Informatik, AG Didaktik, Universität Bremen, Bibliothekstr. 1, D-28359 Bremen, Germany.
Email: sth@math.uni-bremen.de