# On high-school students' use of graphic calculators in mathematics

John Berry, Plymouth (England)
Ted Graham, Plymouth (England)

**Abstract:** When students are working with hand held technology, such as graphic calculators, we usually only see the outcomes of their activities in the form of a contribution to a written solution of a mathematical problem. It is more difficult to capture their process of thinking or actions as they use the technology to solve the problem. In this paper we report on two case studies that follow the progress of students as they solve mathematical problems. We use software that works in the background of the graphic calculator capturing the students' keystrokes as they use the calculator. The aim of the research studies described in this paper was to provide insights into the working styles of these students. Through a detailed analysis of their graphic calculator keystrokes, interviews and associated written solutions we will discuss the effectiveness of their solution strategies and the efficiency of their use of the technology and identify some barriers to the use of graphic calculators in mathematical problem solving.

Keywords: graphic calculator, working styles

**Kurzreferat:** Wenn Schüler sich nicht netzbasierter Technologie bedienen, etwa graphikfähige Taschenrechner benutzen, so sehen wir gewöhnlich nur die Ergebnisse ihrer Aktivitäten als Beitrag zur ihrer schriftlichen Aufgabenlösung. Es ist schwierig, den Denkprozess als solchen einzufangen. In diesem Beitrag verfolgen wir in zwei Fallbeispielen diese Prozesse. Dabei bedienen wir uns einer Software im Hintergrund, durch die das Betätigen der Tasten aufgezeichnet wird. Ziel unserer Analyse ist es, Einsicht in die Arbeitsstile der Schüler zu gewinnen. Auf der Basis dieser Aufzeichnungen, unter Zuhilfenahme von Interviews und aufgrund der schriftlichen Aufgabenlösung diskutieren wir sowohl die Effektivität ihrer Lösungsstrategien als auch die der eingesetzten Technologie; dadurch können wir Barrieren identifizieren, die einem Einsatz des graphikfähigen Taschenrechners beim mathematischen Problemlösen entgegen stehen.

**ZDM-Classification**: C30, U70

## 1. Students' problem solving strategies working with graphic calculators.

Using technology effectively in the teaching and learning of mathematics requires, among other things, positive attitudes from and good personal competences of teachers and students. Numerous research studies into the impact of the use of graphical calculators in the teaching and learning of mathematics and solving problems in mathematics have been largely positive (see for example Dunham and Dick, 1994; Penglaze and Arnold, 1996; Hennessy, 1999; Burrill et. al., 2002 for surveys of research on handheld graphing technology in school mathematics). Dunham (2000) cites a few instances where calculator usage resulted in negative outcomes in respect to conceptual development. The findings over the past decade exploring the use of graphic calculators in mathematics education reveal the complex nature of the interaction of technology and teaching in the learning of mathematics.

How students actually work when solving mathematical problems with a graphic calculator available for use is a complicated question. Burrill et. al. (2002) report that there has been little research on students' spontaneous use of handheld technology to solve problems (i.e. individual choice of solution strategy with or without technology) and that there are few studies that report how students think mathematically with calculators.

One of the first research papers that investigated the problem solving strategies of students as they used a graphics calculator was by Ruthven (1990). The research study focussed on an analysis of the written answers students had given to six questions where they were asked to write down the equation to a given graph. Ruthven was able to identify three different strategies that the students adopted in reaching their solutions and he was able to link this to graphic calculator use. He noted that:
*Responses to the symbolisation items cannot be interpreted as accurate records of the reasoning processes of individual students. But taken together they do provide the outlines of three fundamentally different types of approach to symbolising a graph.*

The three approaches were summarised in the following way. The first step the students took was to use their existing knowledge to classify the graph as a member of a particular family of graphs e.g. quadratic. Ruthven talks of this as the process of recognition. This is followed up by the process of refinement as the students attempt to identify it precisely. The three approaches to refinement were described as:

- Analytic construction – students employ their existing knowledge to build up the function. This method was used by both groups of students but the graphic calculator group were able to check their solutions more speedily and effectively.
- Graphic trial – the student essentially uses the graphing facility of the calculator to continually modify the symbolic expression (i.e. trial and improvement.).
- Numeric trial – a symbolic conjecture is made, often guided by the numeric pattern of a small number of co-ordinates from the graph in addition to the overall picture of the graph. The conjecture is adjusted when they have seen the results of plotting the graph.

Ruthven's observation was carried out by studying the written work of the students. They were expressly asked to write down the reasoning for their solutions.

Another set of researchers who have been looking at students' problem solving strategies as they use a graphics calculator have been Harskamp et al (1998 and 2000). They particularly looked at student's use of the graphical features of the calculator. One of the research questions that they were interested in was: what effect does the use of the graphics calculator have on the solution approaches of students and on their knowledge of functions. Their study followed the progress of two experimental groups of students with different availability of graphic calculators; one group used the graphics calculator for the whole year, whereas the other group used the graphics calculator to study one out of the four topics. The experimental group's work was also compared with that of a control group using a scientific calculator. The solution methods identified by Harskamp et al. were:

- Heuristic – student uses their own solution method (may be trial and error)
- Graphic – student graphs the function and uses it to help solve the problem.
- Algorithmic (analytic) – student uses an analytic or algebraic approach.
- No solution – student offers no solution.

Again the student's strategies were inferred from the written solutions.

The work of Harskamp et. al., Ruthven and others that describe student problem solving strategies uses the student's written solutions and personal descriptions from their memory in post interview situations to describe how they arrived at those solutions. When providing such a written summary in this way there is a danger that the student could omit some steps in the use of the graphic calculator. We do not know from Ruthven's work, for example, how efficiently the calculator was used.

In the research studies reported here, we investigated the manner in which groups of students used a graphic calculator as a tool to aid the solution of mathematics and statistics tasks. By collecting data on the actual keystrokes used by the students we can observe exactly what the calculator did. One of the aims of this research is to gain an insight from the keystroke record into how effectively and efficiently a group of students used their graphic calculator in graph drawing and function finding situations.

### 1.1. Capturing Students' problem solving strategies

For the purposes of the research studies each student was given a Texas TI-83 Plus graphic calculator that contained specially written software (the Key-record software) for capturing the student's calculator working styles while they worked on various tasks. The software allows the data either to be replayed on the calculator or viewed as a list of keystrokes.

The software includes:

- a module which saves the keystrokes to the calculator's internal memory as an Application Variable;
- a module which allows the data to be replayed so that each individual keystroke can be viewed;
- the ability to take the data off the calculator and store it on the hard disk of a computer via the Graphlink.

The software runs in the background so that the students are not actually conscious of its presence. For more details of the key-record software see Smith (2003) and Berry and Smith (2002).

It is within this framework of teaching, learning and doing mathematics with graphic calculators that this paper is set. We will present two case studies demonstrating the use of the key record software to capture students' working methods with graphic calculators.

## 2. The Theory of Instrumentation

The theory of instrumentation has been developed and applied successfully by a number of researchers who have been looking at the impact of technology on the way that students learn. Instrumentation refers to the process of learning to use technological tools, in this case calculators. Drijvers and van Herwaarden (2000) use this idea in their research into the use of Computer Algebra systems (CAS). Guin and Trouch (1999) considered the specific case of graphic and symbolic calculators, using an analysis of the potential of the tool and observations of students. More recently these ideas have been developed further by Artigue (2002), who reflects on the complexity of instrumentation and its mathematical needs in a CAS environment. Ruthven (2002) has also discussed the role of instrumentation in the context of computer algebra systems. This body of knowledge has been developed because of the need to understand how students learn to use technology and in what ways they use the technology that is available to them. One feature of very much of this research has been the identification of unexpected ways of working or unpredicted approaches to problems. Drijvers (2004) describes unforeseen methods that students had developed.

The real strength of the instrumentation perspective is that it has been conceived to enhance the study of how students learn mathematics in a technological environment. It must be stressed that instrumentation is a developing theory, but it is one that has a massive potential for application in areas where technology is used in the teaching and learning of mathematics.

The aim of the studies reported here was to apply the instrumentation perspective to investigate the use of calculators by high-school (upper secondary) mathematics students, particularly in the context of individual use on tests or similar activities. One of the key ideas of instrumentation that is essential for these studies is the notion of a scheme, in which both technical and conceptual aspects interact. When a student begins to use an artefact for the first time, they are beginning an instrumental genesis, through which the artefact evolves to a tool and eventually to an instrument which is part of a scheme. An artefact is simply a material object that may have no purpose for the student. Once the student can use the tool to perform some tasks it has become a tool. It is useful to the student and can be used to carry out tasks in a more efficient way than could be done without it. A tool or an

artefact is a very concrete object that is a very easily observed. An instrument is a much more complex notion, it includes a psychological construct or scheme, rather than just a tool. A scheme allows the student not only to carry out a task, but also to plan and to understand. It is when the tool or artefact is integrated with the mental or conceptual processes of the student that it becomes an instrument or scheme. It is essential to recognise that there is a relationship between the student and the tool at the heart of every scheme. The tool has an impact on the student, but the students' knowledge and understanding impact the way in which the tool is used. Drijvers (2004) has identified the three key elements of a scheme as the tool or artefact, the task and the mental schemes of the student.
This can be represented diagrammatically in the context of this project as shown in Figure 1, in which it is the mental activity of the student that brings together the three elements into a scheme.

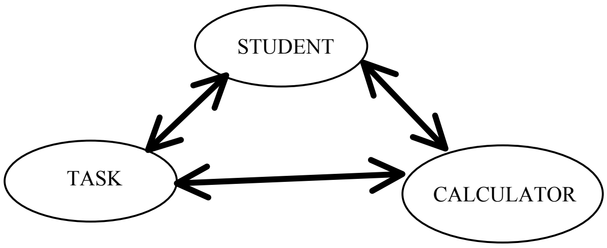


Figure 1 - The Elements of a Scheme

Different students will have different schemes. The real issue for educational researchers is that techniques can be observed but that schemes cannot be, for the simple reason that they are held in the minds of the students. In order to begin to understand the schemes that students are using it is necessary to formulate theories as to how the student and the technology are interacting. Drijvers (2004) and others, for example Artique (2002) have based much of their work on interviews with students about tasks that they have carried out. Drijvers has for example used his observations and interviews to "reconstruct" students' schemes for different tasks. The problem with this type of approach is the impact of the researcher on the activity of the student. Through the use of the key-recorder software, to be described below, there is an opportunity to gain an accurate record of all the calculations carried out by the students, without any interaction with a researcher during this process. These calculations can then be examined by researchers in order to try to develop models of the schemes used by the students while solving the problems. The insights gained from the key recorder data can be supplemented with data from student interviews. These interviews may be based on the key recorder data and can be used to investigate why students took their approach to solving the problem, and to reveal more of the mental processes that are taking place as the student works with the calculator.

**3. Case Study 1**
In this case study we report on the use of graphics calculators by a small group of students taking a mock statistics examination paper as part of their preparation for an A-level examination in the UK. The keyrecord software was installed on the calculators that the students would use in the examination and after the examination the students were interviewed about their approach to the examination and their use of calculators. For a full report on this study see Graham et al (2003).

**3.1. The Participants**
The participants were a group of seven students, who had been taught by a teacher who was very enthusiastic about graphics calculators and had encouraged the students to purchase and use a TI-83 calculator. The appropriate statistical features of the calculator had been demonstrated and used with the students as they covered the topics required for the examination.

**3.2. The Tasks**
The questions on the examination paper offered ample opportunities for the students to make use of their graphics calculators. The questions were considered by an expert user and opportunities to use the graphics calculator were identified. The ways in which the calculator could be used were then classified as quasi-scientific in which the calculator did nothing other than what could have been done on a standard scientific calculator; semi-proficient, where some use was made of the features of the graphics calculator, or proficient, where the student uses the graphics calculator in the most efficient way to obtain the required solution.

It is also important to note the regulations concerning the use of calculators in examinations at the time that this study was carried out, although these have now changed. For some of the early examination papers for A-level the students were only allowed to use a scientific calculator, while in others a graphics calculator was permitted.

**3.3. Results and Discussion**
When the key record data was examined it became clear that the students had not used any of the specialist statistical functions that were available on the graphics calculator. With a few exceptions, the only time that the graphics calculators had been used was for one question where the students were expected to sketch the graph of a probability density function. While most of the students were able to do this satisfactorily, two students produced incorrect graphs. The equation of the function was $f(x) = \frac{5}{4x^2}$. Figure 2 shows the screens that one student produced as she tried to produce the sketch.

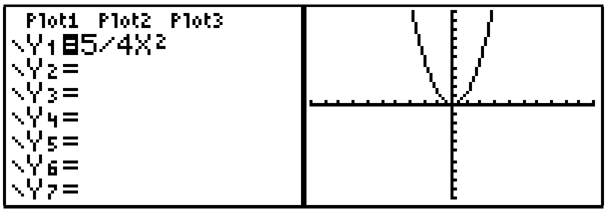


Figure 2: One Student's Graph

Figure 2 shows a graph that was produced due to an error in entering the function correctly. The fact that the student then copies this as her sketch indicates that she has not considered whether or not this is a sensible sketch for the given function. The other error was to include a correct sketch, but that included negative values of $x$, which were not appropriate in the context of the question. This type of working shows that some students have incorporated the graphics calculator into a scheme that allows them to sketch graphs of this type while others have not.

The fact that the students had not used the statistical functions on the calculators illustrates that they had not developed schemes for using the calculator's statistical functions.

Aside from the graph sketching the only other scheme that could be identified was for one of the students who effectively used the list facilities in the calculator to calculate the test statistic for a chi-squared test.

There was also very little use of the graphics calculators in a pseudo-scientific style. There were a number of questions on the examination paper that required this type of calculation, but virtually no evidence of these being done on the graphics calculators.

The key-recorder data indicated that the students were not creating schemes or ways of working that incorporated the graphics calculator, even though they had been encouraged to take this approach by their teacher. The interviews with the students were used to explore the reasons for this phenomenon. In the interviews it soon became clear that there were a number of reasons why the students were not using the graphics calculators and hence not building them into mental schemes. The first of these was the familiarity of the students with their scientific calculators, which had been built up during their 11-16 schooling and that had not been challenged due to the restrictions on the use of graphics calculators in their early module examinations. At interview, one of the students stated:
"*I think that one of the things that put me off using it is you can't take it into the first exams, so you get used to using the scientific one instead and stick with that.*"

The second reason, which is linked to the first, concerns the students' confidence. Some of the students stated that while they knew how to use the calculator to carry out statistical tests, they did not feel totally confident in doing this. When asked to explain the contents of a screen dump during their interview, one of the students said:
"*I remember seeing this, but I cannot remember how to do it.*"

The third reason was that the output from the calculators was in a different format to the official examination mark schemes that the students had seen. This had discouraged the students from using the graphics calculators. This view was expressed by one student at interview, who stated:
"*Well if you get the answer wrong the examiner can look back and see how you did it and you may get method marks. If you have done it on the calculator and just written down the answer and got it wrong, the examiner won't know where you went wrong and just mark it with no marks.*"

The fourth reason that became apparent in the interviews was that the textbook, written specifically for the module and examination, did not make any reference to graphics calculator methods. This sent out another negative signal to the students.

The student who had used the lists to carry out the chi-squared was asked about her approach. Interestingly it emerged that she had attended a biology field trip, but had taken only her graphics calculator. In order to complete the work required on the field trip she had made use of the lists and become very proficient in manipulating them.

The picture that emerges from his study is one of students failing to interact fully with their graphics calculator and not building up their confidence and working knowledge of the calculator. This has happened in spite of the teacher's enthusiasm for using the graphics calculators, and seems to be due to a number of negative influences associated with the examination that is to be taken at the end of the module.

This study indicates how there are many factors that can impede the development of student use of graphics calculators and which it is difficult for teachers to overcome. Interestingly the regulations for the use of calculators in A-level have recently been changed with the aim of encouraging more use of graphics calculators.

**4. Case Study 2**

In this Case Study we report on the use of the keyrecord software in observing the manner in which a group of students used a graphic calculator as a tool to aid the solution of twelve mathematics tasks. These tasks involved drawing graphs of given functions and finding expressions to describe given graphs. We will demonstrate the use of the software by analysing one of the twelve tasks. For a full analysis see Smith (2003).

### 4.1. The Participants

For the research project a group of 12 first year undergraduate students who were studying mathematics at the University of Plymouth were selected. All the students had passed the school leaving matriculation type programme for England, Wales and Northern Ireland (the General Certificate of Education, Advanced Level) in Mathematics which includes a reasonable amount of work on the topic of functions and calculus.

The students were a convenient sample from a cohort of 50 students on a calculus module. All of the students owned and were proficient users of a Texas Instruments TI-83 graphic calculator. The students at the time of the study were taking a calculus module, which as a prerequisite requires a good working knowledge of the properties of a variety of functions. The students would have been expected to have studied polynomial, rational, exponential and trigonometric functions as part of their school mathematics curriculum.

For the purposes of the research study each student was given a Texas TI-83 Plus graphic calculator that contained the Key-record software. The students were told that there were 12 tasks which related to the graphs of functions but stressed that although a graphics calculator was provided they weren't obliged to use it. No time limit was set on the problem solving session.

### 4.2. The Tasks

Two types of task were used in this research project. All the tasks were chosen as activities that the students would have met before. In the first type the students were given a function and asked to sketch its graph. There were six tasks of this type. In this research a complete graph of a function was interpreted as one that shows all of the important features of the function. Unlike Ruthven (1990), our students were not asked to write down their reasoning. The purpose of the research study was to investigate the written solutions in the context of the student calculator keystrokes. Figure 3 shows a typical task from this section of activities and some possible graphics calculator screens.

Sketch a complete graph of the function $y = 0.1x^3 + 2x^2 + x - 9$

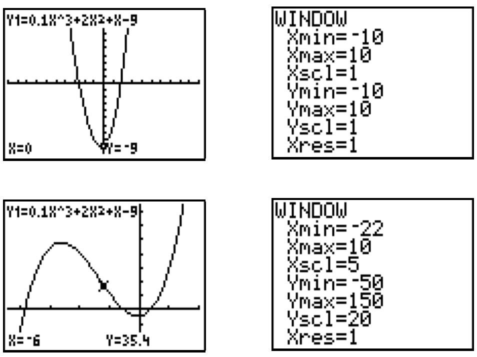


Figure 3. A Type 1 Task with the graphs shown in the calculator's standard window and user set window.

Type 2 Task: Find an expression for $y$ in terms of $x$, which describes the graph below.

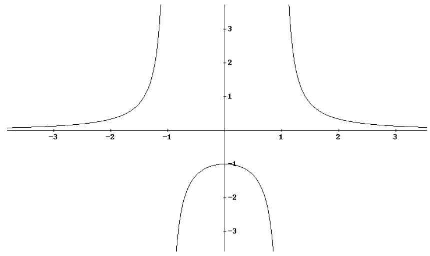

Figure 4. Graph for a Type 2 Task for which the answer is $y = \frac{1}{(x+1)(x-1)}$ .

### 4.3. Results and Discussion

In this article we show the results of three of the students working on the Task in Figure 3. Before the first task they were told that 'A complete graph of a function is one that shows all the important features of the function'. It should be noted that before the students started the activity they were advised that, although they had a graphics calculator, they didn't have to use it while doing the tasks. It was anticipated that the students would show and give numerical values to such features as:

- Where the function cuts the axes
- What happens to the function for large values of $x$
- Stationary points
- Asymptotes

No guidance was given to the students, other than a definition of a complete graph. It was left up to the individual student to decide on how much detail they gave. The information that was required could easily be found using the commands that are built into the calculator. Each of the solutions produced by the students were classified using the following categories:

- Correct – full detail given

The general shape of the sketch is correct. Numerical values have been given to all relevant features such as stationary points and where the function cuts the axes.

- Correct - some detail given

The general shape of the sketch is correct. Numerical values have been given to some but not all of the relevant features.

- Correct - minimal detail given

The general shape of the sketch is correct. The student may have put numerical values on the sketch but these have been taken directly from the calculator screen and so in most cases are approximate. No calculations have been attempted either by hand or by calculator.

- Incorrect

The student has been unable to sketch the general shape of the complete graph correctly. If there is a general feature running through the sketches this has been highlighted.

- No response

From the lack of keystrokes recorded on the calculator and no written evidence it was assumed that the student has not attempted to answer the task.

This particular Type 1 Task was chosen because as shown in Figure 3 the function looks like a quadratic function when viewed using the graphic calculator's standard window settings. The students should know the difference between the graph of a cubic function and that of a quadratic function and so it was surprising that 38% of the students actually sketched a quadratic function on their script. What seems likely is that the power of technology took over and that the students were happy to simply copy what they saw on the graphics calculator screen. The students who gave incorrect answers all sketched a graph that resembled a quadratic function.

In the following descriptions of student answers we provide a copy of the student's written script and a description of their use of the graphic calculator as derived from a replay and transcription of their calculator key strokes. We focus the discussion around the work of the students using the TI-83+ calculator.

#### 4.3.1. Correct Responses – Key-record Students

The students with correct responses to this task made appropriate use of the technology. They did this by either changing the window settings to produce a good graph or using the Zoom Out function. In addition several of the students used the Calc function on the calculator to identify some of the features.

Students E and G both used a standard calculus approach to find the local maximum and local minimum. It is likely that they are unaware that this can also be done easily by using the calculator. The solution produced by student G is shown in Figure 5.

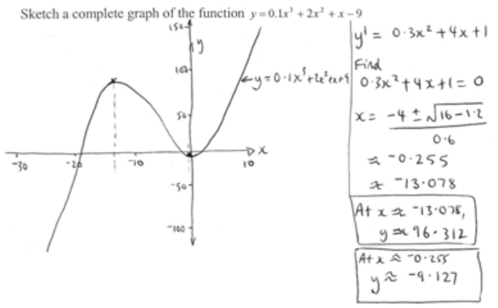


Figure 5 Response of Student G to the Type 1 Task

Student A initially graphs the function using the standard window and sees a quadratic. He Zooms Out twice and then changes the window to the one that produces the graph shown in Figure 6. This is identical to the sketch on his script, which is shown in Figure 7. The student finds the coordinates of the two stationary points, as can be seen in Figure 7. He writes down the derivative and forms an equation to find the *x*-coordinates and then uses the graphics calculator to solve his equation. It is interesting to note that he also uses the calculator to find the *y*-intercept, even though he could have got this value directly from the function. No attempt was made to find the coordinates of the points where the function cuts the *x* axis.

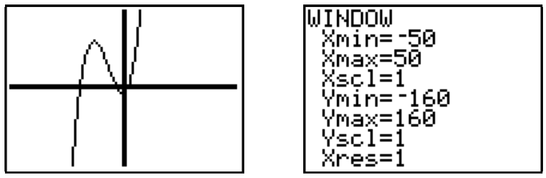


Figure 6. Screen dump showing what Student A saw before sketching his answer to the Task

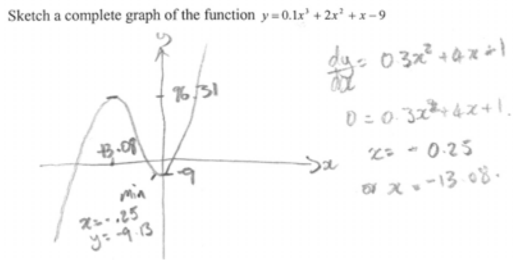


Figure 7. Response of Student A to the Type 1 Task

#### 4.3.2. Incorrect Responses – Key-record Students

The students who gave an incorrect answer were unable to classify the function as a cubic and relied solely on the graphics calculator to produce the sketch. Unfortunately for them if the window settings are near those of the calculator's standard window settings then the function will have the appearance of a quadratic function. This is fully illustrated by Student K whose script is shown in Figure 8 together with the last calculator screen he saw.

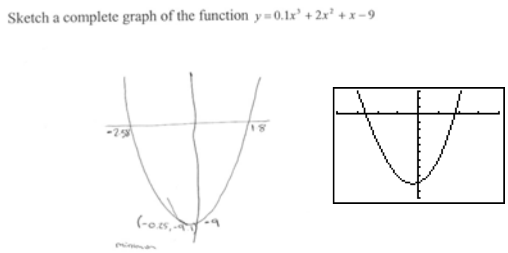


Figure 8. Response of Student K to the Type 1 Task

#### 4.3.3. Summary of the student work on Type 1 Tasks

The answers given by the students to the Type 1 Tasks in our study could reflect the way that curve sketching has changed in the curriculum of English schools. This probably reflects the view that mathematics teaching is often directed towards the examination. The Examination Boards, who are responsible for the assessment of school mathematics in the England, have influenced the use of graphic calculators through the examination questions. The authors have observed that very few curve sketching tasks are set on current examination papers. This lack of curve sketching in the examination may mean that the importance of this section of the syllabus is reduced by teachers. What is lost to the student is the ability to look at the equation of a function and effectively sketch it. This investigation highlights the need for students to be proficient in the use of the graphics calculator.

The different approaches used by the students for these tasks included the following:

- The student graphs the function using the calculator and copies the screen, without giving further consideration to the function. There was plenty of evidence of this from the twelve ‘keyrecord’ calculators.
- The student graphs the function using the calculator but uses the functions of the calculator such as changing the window settings and using the different zooms to get what they believe is a better picture. They then copy what is on the screen onto their script, even though this may not be correct. There was plenty of evidence of this.
- The student effectively knows what the function looks like and uses the calculator to confirm this. Very few students approached the tasks in this way.
- Several of the students attempted to use calculus to find values for the stationary points and included these on their sketches.

As we suspected at the outset of the research study, when asked to sketch a complete graph of a function, the majority of students are happy just to enter the function into the calculator and apart from a few adjustments to the window, basically just copy down the graph shown on the screen. This meant that very few numerical values are put on the sketches. We would argue that a good sketch of the graph of a function should identify the co-ordinates of the essential features that describe the graph such as the positions of the stationary points.

What we find is that the students are too reliant on the calculator without knowing many of the anomalies it may induce particularly in curve sketching. When a new function was graphed no initial thought was given to the window display. Almost all the students pressed graph immediately thus using the window they had set for the previous task which in many of the cases was totally unsuitable. In particular it was evident that the students in our research experiment are not skilled in the use of the zoom functions and setting a suitable viewing window. As Steele (1993) reported ten years ago:

*The importance of students asking themselves, “Is this result reasonable?” after drawing a graph with a calculator cannot be emphasized enough.*

## 5 Conclusions and Implications for Teaching and Research

The keyrecord software has proved to be an excellent tool for collecting data on the students’ natural use of a graphics calculator and has proved very successful in investigating how students work with a graphics calculator. In the first case study it was used to investigate graphic calculator use by students in an examination situation. What was learned from this was that although their teacher used the calculator regularly in class and showed them how to use the statistical functions of the calculator, the students were loath to use it under examination conditions. Taylor (1995) attributed the lack of use of graphics calculators in the summer 1994 matriculation examinations to the lack of training the students had received on the use of the calculator. This lack of training did not apply to our students. Here the students had been exposed to the use of the calculator for some time. Their teacher had regularly used the view-screen and the students had been encouraged to use the graphic calculators in classes. Initial training had also been provided for these students. In this case it seemed that pressures associated with the examining system had the greatest influence on the students. The keyrecord software now provides an innovative methodology for exploring how students actually use graphics calculator when solving mathematics problems.

By considering the strategies of the students we have identified a number of errors or problems that the students encountered with the calculator that are common to many of the students. Of some concern was evidence that the students were unaware of many of the features of the calculator even though they had used the technology regularly for several years. This was confirmed by the majority of the students not being able to enter the modulus function into the calculator or use the CALC function to obtain values of intercepts or turning points.

On the graph sketching tasks there was a tendency for the students to enter the function and graph it after possibly making a few changes to the window settings and then copy what appears on the calculator window onto their script. No real thought appeared to be given to what the basic shape of the function could have been. This could have been because of their lack of knowledge of basic functions but we suspect that this wasn’t the case. There were many examples of this, particularly where the students sketched what appeared to be a quadratic function for a cubic function and where a rational function was sketched as a continuous function. Furthermore the students nearly always graph a function using the window settings they had previously used, having given no thought to what they would have expected to see. When using a graphics calculator the students seem to have forgotten what they had learned when they first started out plotting graphs. The evidence from the tasks was that the students were happy to enter the function into the calculator and apart from the occasional change of window settings when the picture produced is poor, they just copy the calculator screen. When students first begin to use a graphics calculator for curve sketching we believe that it is beneficial to take them back to the days when they were first taught how to plot graphs.

This would involve them:

Entering the function into the calculator.
Using the TBLSET to set up the calculator table based on the likely domain they will use.
View the TABLE to find the range over this domain.
Set the calculator WINDOW settings.
Graph the function.

If this does not produce a satisfactory graph then we would expect them to reflect on what they have done and repeat the sequence of steps. Only then when they have mastered this method and have time to reflect on what they have done should they be introduced to the Zoom functions of the calculator.

One of the problems we observed was that students experienced difficulties with the scale on the calculator screen which they would possibly not experience with a graphing package on a computer. We believe this is because the calculator does not put any numerical values on the scale and so the students do not pay much attention to the scale settings. It is important and teachers need to continually remind students of this fact. A second feature that appears to be common among most of the students who took part in the study is a lack of feel for the shape of a graph of a function and the functional form of a given graph. Of course we would argue that students need to have the feel for graphs of functions in a traditional teaching approach, however our experience is that many students dive into the algebraic approach without producing a rough sketch first which identifies the key features. The implications here for the teaching of graphs with graphics calculators is that the need to be able to produce a rough sketch or 'hand waving curve' is an even greater because the technology will do the tedious algebraic work. We would argue that teaching programmes and assessment schemes need to focus on such mental skills.

As with computers, students are likely to adopt a button pressing experimental strategy to solve problems. So that teaching / learning materials produced for the graphic calculator should encourage students to regularly write down what they have discovered. This should encourage them to reflect on what they have been doing. Designing tasks that promote reflection and render button pushing ineffective could address this problem.

This article set out to describe two case studies that use a research tool for observing students' working with graphic calculators when solving mathematics problems. One of the key research questions that we have been exploring in the Centre for Teaching Mathematics at The University of Plymouth is 'How can we effectively observe students working in a naturalistic way with graphic calculators?' Some of the problems that can arise when students are observed using graphic calculators include, for example, when a video camera is used to record their use of the calculator, then the direct presence of the camera may have an effect on the student, and so the results that we obtain will have been influenced by the observation. If direct observations are used to see how the student is using the calculator our presence will raise the same sort of problems. If the student is asked to give a running commentary for a tape recorder of what they are doing then it will affect the data collected.

The keyrecord software was designed to overcome the problems of observing students while they are working. It allows us to observe students using a graphic calculator without them being continually reminded that they are being observed. The key-record software is an easy tool to use. All that needs to be done is for the software to be loaded onto the calculator and the calculator to be reset to some predetermined default settings. The calculator is then ready to be given to a student. As the calculator is fully portable, the student can take it anywhere they want. The researcher can be confident that any use of the calculator is being recorded. Once the calculator has been taken back from the student it is a simple matter to download the data in just a matter of seconds from the calculator to a PC using the TI Graph Link so that the calculator can be immediately used again and the data produced can be analysed at the researcher's leisure. The data can be viewed as many times as the researcher wants.

The keyrecord software has proved to be an excellent tool for collecting data on the students' natural use of a graphic calculator. Its success in investigating how students work with a graphic calculator has been demonstrated by the three pilot studies. Furthermore, students can be encouraged to reflect on their own learning of mathematics concepts and skills by replaying their key-strokes alongside their written solutions to mathematics tasks.
We have seen that the key-record software is an effective tool to help analyse the students' use of a graphic calculator as they engage in a piece of mathematics. There are many different research projects in which the key-record software could be used as a research tool. For example, further exploration of how students work with advanced calculators when doing mathematics in schoolwork, homework and in examinations in mathematics and in other subjects? Research on how students' working styles compare with the approach of their teacher and the training the students and teachers have received on the technology would also be valuable.

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**Authors**

Berry, John, Prof., The Centre for Teaching Mathematics, Faculty of Education, The University of Plymouth, Douglas Avenue, Exmouth, Devon, EX8 2AT, England.
Email: jberry@plymouth.ac.uk

Graham, Ted, Dr., The Centre for Teaching Mathematics, School of Mathematics and Statistics, The University of Plymouth, Plymouth, Devon PL4 8AA, England
Email: egraham@plymouth.ac.uk