

Mathematics achievement and interest in mathematics from a differential perspective

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Abstract: In this article, we present results of an empirical study with 500 German students of grades 7 and 8. The study focussed on students' mathematics achievement and their interest in mathematics as well as on the relation between these two constructs. In particular, the results show that the development of an individual student's achievement between grade 7 and grade 8 depends on the achievement level of the specific classroom and therefore on the specific mathematics instruction. Interest in mathematics could be regarded a predictor for mathematics achievement. Moreover, our findings suggest that the students show hardly any fear of mathematics independent of their achievement level.

Kurzreferat: In diesem Beitrag wird über eine empirische Studie mit über 500 Schülerinnen und Schülern der Jahrgangsstufen 7 und 8 berichtet. Im Fokus stehen dabei die fachlichen Leistungen und das fachspezifische Interesse der Jugendlichen sowie der Zusammenhang zwischen diesen beiden Konstrukten. Die Ergebnisse zeigen unter anderem, dass die individuelle Leistungsentwicklung von Jahrgangsstufe 7 zu 8 abhängig von der Klassenebene und damit vom Unterricht ist. Interesse und Leistung korrelieren erwartungsgemäß. Zudem ist auffällig, dass die Schülerinnen und Schüler unabhängig von ihrer individuellen Leistung kaum Angst vor dem Fach Mathematik haben.

ZDM-Classification: C23, D53, E 53

The learning processes of students are influenced by a variety of factors. Individual prerequisites do not only interact with variables of the teachers and their instruction, they are also integrated into the context of the specific classroom. As we may take into account a student's individual achievement, we may consider the achievement spectrum of a whole class. Accordingly, non cognitive variables such as interest and motivation are individual characteristics, though their occurrence differs within different classrooms. In this paper, we will discuss the relation between mathematics achievement and interest as well as motivation related to mathematics on the individual level and on the classroom level.

1. Theoretical background

Recently, school instruction and its effects on students' learning are studied from an integrative perspective. This perspective includes aspects of the teacher personality, the instruction, the individual prerequisites as well as classroom and domain related mediation processes and learning activities of the students. Helmke (2003) describes the complex relations in which instruction and learning are integrated (Figure 1). Instruction is regarded as an offer and students may choose to benefit from this offer.

The model serves as a framework how cognitive and non cognitive aspects of teaching and learning are related to each other. Different studies utilized this perspective to investigate the learning of mathematics and focused on the relationship between subject specific interest and learning motivation on the one hand and (mathematical) achievement on the other hand.

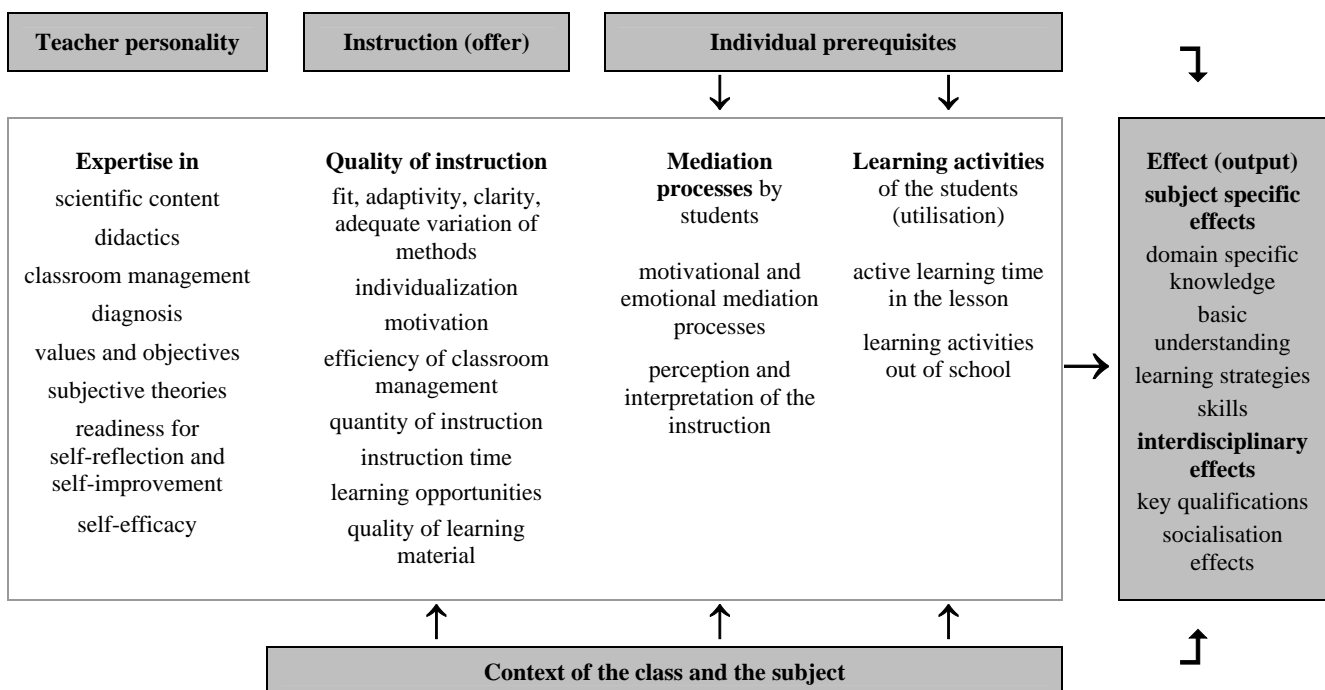


Figure 1: A model for teaching and learning (Helmke, 2003, S. 42)

1.1 Interest, motivation, and mathematical achievement

There are two substantial theoretical approaches for the investigation of subject specific interest and learning motivation: the self-determination theory of motivation (Deci & Ryan, 1993) and the subject-object theory of interest (Krapp, 1992; Prenzel, 1988). The latter is based on a concept of interest and motivation described as a relationship between an individual (e.g. learner) and an object (e.g. learning topic). In this theoretical context, it has to be differentiated between a current situational relation to an object and the interest in an object. This interest is long-term and independent of present situations. Another approach is described by the self-determination theory. It is based on the assumption that individuals, who achieve the highest level of internalisation regarding the adoption of values and objectives, tend to integrate the objectives related to a learning topic consistently and permanently into their value and belief system (Wild, Hofer, & Pekrun, 2001). This assumption can be traced back to the difference between intrinsic motivation (oriented at actions determined by interest) and extrinsic motivation (oriented towards actions carried out in order to attain certain consequences). Both theories, the subject-object theory and the self-determination theory, are characterised by the fact that they encompass the influence of the social environment on the individual intrinsic motivation. Thus, these theories are of particular importance for analysing school instruction, in which a variety of social processes are interacting.

In the context of the subject-object theory it was investigated which conditions of school instruction are fostering individual motivation. It may be assumed that the students' perception substantially depends on the particular teacher who is responsible for organizing teaching and learning in the classroom. It turns out to be crucial that students experience the classroom as supportive with respect to autonomy and competence and that they have the feeling of being socially integrated (Lewalter, Krapp, Schreyer, & Wild, 1998). Moreover, step-by-step individual experiences regarding interest and motivation can lead to the development of an orientation of interest towards a school subject (Prenzel, Krapp, & Schiefele, 1986).

The importance of intrinsic motivation for the learning process was investigated in a series of research projects. Generally, it is assumed that intrinsic learning motivation induces a deeper elaboration of the learning contents whereas boredom and fear are related to rather superficial learning strategies (Hoffmann, 1997). In a meta-analysis, it was pointed out that there are substantial correlations between subject-specific interest and subject-specific school performance (Schiefele, Krapp, & Schreyer, 1993).

There are many studies that give evidence of a correlation between achievement and aspects of interest. The meta-analysis by Schiefele et al. (1993) previously mentioned shows an average correlation of $r = 0.30$ between interest and performance data. However, some studies used a selected non-representative sample.

Considering a restricted range of variance with respect to performance (e.g. only high achieving students) may imply slightly lower correlations. Accordingly, the performance variance explained by interest and motivation might be higher than the average correlation of $r = 0.30$ would indicate.

In this kind of correlation it is not clear whether performance was influenced by interest or interest was influenced by performance. Krapp (1998) assumed that there exists a feedback effect between interest and performance in such a way that interest promotes the preoccupation with the learning topic thus inducing a better performance and in turn increasing interest. In a long-term study (grade 7 to 12, $N = 602$), there is some empirical evidence that does not support this theory. Köller, Baumert, & Schnabel (2001) showed that subject-specific interest in mathematics had no significant influence on the development of performance if the previous knowledge was controlled. Nevertheless, the subject-specific interest was important for the students' choice of mathematics as a basic or as an advanced course at the upper secondary level.

1.2 Interest and motivation regarding mathematics

In recent years several investigations did not focus the correlation between interest and performance but rather on the descriptive analysis of interest and motivation with respect to mathematics. The PISA 2000 study showed that German students' interest in mathematics was below the international average. For reading literacy it turned out that the subject-specific interest was a good predictor. There was a difference of 84 points on the reading literacy scale between students with low interest and high interest in reading ($r = 0.33$). For mathematics the correlation was clearly weaker, and the difference between students with low and high interest in mathematics was only 17 points on the mathematics literacy scale (Artelt, Demmrich, & Baumert, 2001). A possible reason for these results might be that reading literacy is not only acquired in the classroom but also by interest-related reading activities outside of the classroom. In contrast to that, mathematical interest may be strongly related to mathematics instruction and rarely lead to mathematical activities beyond the classroom, which could influence the students' mathematical literacy.

Several other studies support the result that German students' interest in mathematics is not well developed. In the survey QuASUM¹ (Lehmann & Peek, 2000) a moderate subject-specific interest in mathematics was shown for grade 9 students ($M = 2.56$, $SD = 0.55$ on a scale from 1 to 4). Moreover, the interest decreased from grade 5 to grade 9. The 5th-grade students still showed a higher interest in mathematics ($M = 2.85$, $SD = 0.54$) (Lehmann et al., 2000).

Differences between grade-5 and grade-9 students could also be found in the correlation between mathe-

¹ QuASUM means „Qualitätsuntersuchung an Schulen zum Unterricht in Mathematik“ (evaluation of the quality of mathematics instruction in schools). This representative study was conducted in the German state of Brandenburg.

mathematics achievement and interest in mathematics: The data for the grade-5 students yielded a correlation of $r = 0.42$, whereas the correlation for the grade-9 students was $r = 0.33$ resp. $r = 0.36$ (depending on the achievement test). However, in grade 9 there was a large range between school tracks²; the strongest relation was found for the Gymnasium ($r = 0.43$ respectively $r = 0.50$). The findings of the survey MARKUS (Helmke & Jäger, 2002) are consistent with these results. The survey MARKUS comprised all grade-8 students in Rheinland-Pfalz (Rhineland-Palatinate, one of the 16 states in Germany) and collected data for mathematics achievement and several other constructs like *learning motivation*. This concept included aspects like intrinsic motivation and the affective value of the school subject mathematics. The results indicate that the learning motivation of the students from Gymnasium and Realschule was below average whereas the values for the students of Hauptschule were above average. The average of all students was approximately the same as the mean of the scale (Schrader & Helmke, 2002). The correlation between achievement and learning motivation differed from $r = 0.18$ (Hauptschule) to $r = 0.26$ (Gymnasium). In contrast to other studies MARKUS analysed the classroom level. Depending on the educational background there was a moderate relation between $r = 0.15$ (Hauptschule) and $r = 0.17$ (Gymnasium). The high achieving classes in Hauptschule and Realschule showed a high motivation compared to low achieving classes in these school tracks; this effect was less prominent for the Gymnasium.

1.3 The influence of the classroom on mathematical achievement

Regarding the individual student achievement, different studies indicate that there is a strong influence of the classroom. It is frequently discussed that a reason for these differences might be traced back to the composition of a specific class. Hosenfeld, Helmke, Ridder, and Schrader (2002) identified a negative influence of student variables like "German native speaker", "having repeated a school term", "age" and "TV consumption" on the mathematics achievement of the class. These effects were substantially stronger for classes in the Hauptschule, however, they became weaker if the different school tracks were considered separately. Within the tracks Hauptschule, Realschule and Gymnasium, there was hardly any connection between the proportion of German native speakers in a classroom and the mathematics achievement of this class. Additionally, the number of students in a class had no influence on mathematics achievement.

Differences in mathematics achievement between classes in the same school track can be quite substantial as our own research shows. Results of a study on students' competencies in reasoning and proof in geometry

indicate that this is the case particularly for non-routine problems, i.e., tasks on the upper competency level in terms of PISA (Reiss, Klieme, & Heinze, 2001; Klieme, Neubrand, & Lüdtke, 2001).

A study with students from 27 classrooms at the lower secondary level confirmed these results in greater detail (Reiss, Hellmich, & Thomas, 2002). The mathematical performance of the students was evaluated by means of an achievement test based on a three-step model for competencies in reasoning and proof. With respect to achievement, substantial differences between the 27 classes of the sample could be identified. The class scores (aggregation of the individual results) ranged between $M_{\min} = 5.7$ and $M_{\max} = 17.7$ points; the mean over all 27 classes is $M = 12.42$ ($SD = 3.07$) where the possible maximum score was 26 points. Thus, the difference between low-achieving and high-achieving classrooms was about four standard deviations (Reiss, Hellmich, & Thomas, 2002).

In mathematics achievement, the classroom level has an explicit influence on the individual achievement (though the relation between cause and effect could not be clarified in this study). In three classrooms showing the lowest performance, all students except one scored less than half of the possible points. There were no high-achieving students in these classrooms. In high-achieving classrooms we found a wide range of individual achievement results. The basic tasks of the first competence level were solved satisfactorily or very well by the high-achieving classes though the weaker classes answered satisfactorily as well. The important differences with respect to performance were caused by items requiring students' argumentative abilities. With respect to these items, students of low-achieving classrooms showed hardly any correct solutions (Reiss, Hellmich, & Thomas, 2002).

1.4 Research questions

In summary, one can state that there are reliable and replicated results that there is a correlation between individual interest respectively motivation and school achievement. There are, however, open questions concerning the complex structure of the effects and the interaction of interest, motivation and achievement. There is no sufficient explanation which processes can foster interest in mathematics. There are no generally accepted criteria for the *best* instruction, and it is unlikely that there will be clear criteria according to the *most* motivating instruction. More consideration should be given to aspects like the influence of a specific classroom on individual motivation and interest. Thus, in this paper we will describe the classroom-specific interest and motivation with respect to mathematics and will then address the question how interest and motivation are related to the achievement of classes.

2. Objectives and design of study

2.1 Objectives

The theoretical approaches and the results discussed in the preceding section suggest that mathematics

² The lower secondary level in Germany is divided into three tracks depending on the students' performance: the lower track is called Hauptschule (grades 5 to 9), the medium track Realschule (grades 5 to 10) and the upper track is called Gymnasium (grades 5 to 12 resp. 13).

instruction has not only an important influence on the students' individual achievement, but also on their individual subject-specific interest and motivation. This can also be assumed for the classroom level: There are differences in achievement, and there might be differences between classrooms with respect to interest and motivation. This hypothesis is plausible since both of the mentioned theories, namely the subject-object-theory and the self-determination theory, rely on the assumption that social environment influences the intrinsic motivation.

The study described in the following investigates the question to what extent a specific classroom (here referred to as level of classroom) is significantly influencing the achievement of a student (particularly in the domain of reasoning and proof) as well as the subject-specific interest respectively subject-specific motivation. Firstly, we will consider the individual mathematical interest respectively the motivation of eighth-grade students and the relation between interest and motivation on the one hand and mathematics achievement on the other hand. The main research question refers to the comparison between the individual and the classroom level. We focus on the development of classroom-specific mathematical interest and motivation and the relation between interest, motivation, and achievement.

2.2 Sample and design of the study

The sample consisted of 524 students from 27 classrooms of different schools (Gymnasium). At the end of grade 7 and in the middle of grade 8 an achievement test was administered. This test measured basic and argumentative competencies in geometry. In the weeks before the second test the classes participated in a regular teaching unit on reasoning and proof (congruence geometry); some of the lessons were videotaped. After the second test in grade 8, a student questionnaire on the development of interest and motivation in mathematics respectively mathematics instruction was administered.

The achievement tests for reasoning and proof consisted of items covering basic mathematical facts and argumentation and proof problems. While the basic items required elementary knowledge of mathematical concepts and the application of rules, the argumentative items required one-step and multi-step argumentations. The items in both tests followed the usual curriculum guidelines and textbooks. Pre-test and post-test differed insofar as there were only a few identical or similar items.

The items of the first test as well as those of the second achievement test could be arranged on one latent dimension. In order to test unidimensionality for both tests the dichotomous Rasch model was applied; the parameters were computed by the software WINMIRA (Davies, 1999). The goodness-of-fit evaluation by graphical tests and the application of resimulated datasets (bootstrapping) provided acceptable fit values.

The data for interest and motivation related to mathematics were assessed by a student questionnaire developed in the context of the PALMA study (cf. vom Hofe, Pekrun, Kleine, & Götz, 2002). The questionnaire

consisted of 48 items and evaluated topics like interest, motivation, pleasure, fear, boredom, and self-regulation on a five-point Likert scale ("strongly disagree", "disagree", "uncertain", "agree" and "strongly agree").

3. Results

3.1 Mathematics achievement - results

Both achievement tests were developed on the basis of a three-level model regarding competencies in argumentation, reasoning, and proof (Reiss, Hellmich, & Thomas, 2002). Each item was assigned to a specific level of competency. In this model the competency level I included tasks for elementary knowledge of mathematical concepts and the application of rules, whereas the competency levels II and III were operationalized by argumentation tasks that required one-step respectively multi-step argumentations. As it was shown in Table 1, the theory based competency model was confirmed empirically by both achievement tests. In addition, dividing the sample in an upper-, a middle-, and a lower-achievement group of similar size (regarding the test scores), gives evidence that each of these achievement groups may be assigned to a particular level of competency (cf. Table 1).

Table 1: Percentages of correct items for the competency levels and the different achievement groups of students

Achievement groups	Competency levels in Test 1		
	C I	C II	C III
Lower-achievement group	46	25	6
Middle-achievement group	64	62	19
Upper-achievement group	79	88	50
Average	64	60	26
N = 524	Test 1: M = 0.51 (SD = 0.19)		
Achievement groups	Competency levels in Test 2		
	C I	C II	C III
Lower-achievement group	30	23	4
Middle-achievement group	57	36	8
Upper-achievement group	74	55	26
Average	54	38	13
N = 524	Test 2: M = 0.37 (SD = 0.16)		

Table 1 indicates that the second achievement test (mid of grade 8) turned out to be more difficult than the first test (M = 37 respectively M = 51). This can be explained by the fact that the second test emphasised argumentation tasks. Moreover, most of the items were related to the specific content of the teaching unit on reasoning and proof (e.g., congruence theorems). Additionally, the latter might be the reason for the moderate correlation between the two tests ($r = 0.435$, $p < 0.001$).

Supplementary interviews with grade 8 students supported the findings in Table 1. Five low-achieving and five high-achieving students (regarding their scores in both tests) were asked to work on geometrical tasks and

to think aloud when solving the problems. Even high-achieving students had particular problems with generating a proof idea and combining facts respectively statements to a chain of arguments (competency level III). The low-achieving students showed additional deficits with respect to the knowledge of mathematical facts and the methodological knowledge regarding mathematical proving (competency level I and II). They were hardly able to master argumentation tasks requiring multi-step solutions.

The pre-test in grade 7 as well as the post-test following the teaching unit for reasoning and proof in grade 8 showed significant classroom effects (cf. Section 1.3 for the pre-test). The classroom means based on the individual post-test scores were located in a range between the minimum $M_{\min} = 3.9$ points and the maximum $M_{\max} = 18.9$ points (the maximal possible test score is 32 points). The mean of the class values was $M = 11.8$ points ($SD = 3.07$).

The Rasch-scaled class values of the pre-test and the post-test had a correlation of $r = 0.604$ ($p < 0.001$), i.e., the results on the classroom level were more stable than the results on the individual level ($r = 0.435^{**}$). At this point the question arises to what extent the classroom level influenced the individual performance. For both, pre-test and post-test, the values of the individual performance showed a great variance. The variance on the class level was restricted by the higher reliability of the mean values. Thus, individual students might improve their performance significantly above the average of the classroom they belong to.

In order to describe the development of the individual achievement between the two times of measurement, we computed the expected values for the post-test (respectively the residuals) on the basis of the individual pre-test scores and linear regression. These values have to be interpreted carefully, because there is only a moderate correlation of $r = 0.435$ between the results of both achievement test. Nevertheless, a linear regression can refer to tendencies in the individual achievement development and may be helpful for identifying extreme cases.

The residuals indicate that 55 students (= 10.5%) achieved post-test scores that are more than one standard deviation greater than the predicted scores. On the other hand, 66 participants (= 12.6%) achieved post-test result about more than one standard deviation lower than their pre-test results predicted. Many of those extreme cases cumulate in certain classes: Nearly half (49.1%) of the strongly improving students can be found in three of the 27 classrooms. Two of these classrooms are high achieving and one shows a medium achievement level. Similarly, about half (53%) of the very low-achieving students belong to five classrooms; among those there are the two weakest performing classes. Thus, it seems that the regression-to-the-middle effect did not take place.

Based on the findings we assume that classrooms with a low-achievement in reasoning and proof at the end of grade 7 could hardly improve (in comparison to the higher-achieving classes) during the regular teaching unit on this topic. A multi-level-analysis shows that

the pre-test on the classroom level is a significant predictor of the individual performance ($\chi^2 = 312.2^{**}$, $df = 25$). On the other hand, the influence of the individual pre-test results turns out to be less important ($\chi^2 = 45.3^*$, $df = 26$).

3.2 Interest and motivation results

For the analysis of the data regarding interest and motivation we conducted a principal component analysis. On the basis of the screetest six factors were determined and extracted. The explained variance adds up to 57.5%. For each factor we selected those items with a loading greater than 0.5. The reliability analysis permitted the aggregation of the different item groups to six scales denoted by *interest in mathematics*, *fear of failure*, *boredom* (in mathematics lessons), *achievement motivation*, *self-determined learning*, and *fear of mathematics (classroom)* (cf. Table 2).

Table 2: Values and parameter for interest und motivation related to mathematics.

	N	Mean*	SD	Reliability (Cronbachs α)
Interest in mathematics	467	-0.27	0.44	.96 (17 items)
Fear of failure	481	-0.26	0.51	.92 (9 items)
Boredom	498	-0.45	0.48	.82 (4 items)
Achievement motivation	498	0.40	0.44	.79 (3 items)
Self-determined learning	491	0.15	0.42	.71 (3 items)
Fear of mathematics classroom	504	-0.76	0.33	.76 (4 items)
* Values transformed to a scale from -1 to 1.				

Overall, the mean values in Table 2 indicate that students rather have no fear of mathematics classroom ($M = -0.76$; $SD = 0.33$), show an acceptable achievement motivation ($M = 0.40$; $SD = 0.44$) and are not bored in the mathematics lessons ($M = -0.45$; $SD = 0.48$). Moreover, they have little interest in mathematics ($M = -0.27$; $SD = 0.44$) and no great fear of achievement failure ($M = -0.26$; $SD = 0.51$). However, it must be mentioned that these scales have no absolute point of origin and already small variations in the formulation of the item texts may let students agree or disagree to an item. Furthermore, the problem of the social desirability may have an influence.

As expected the different scales correlate with each other. For example, there is a significant correlation between the scales *fear of failure* and *fear of mathematics classroom* ($r = 0.601$, $p < 0.001$) as well as a negative correlation between *interest in mathematics* and *boredom* ($r = -0.439$, $p < 0.001$).

If we aggregate the individual values of the six main components on the classroom level, then we may

observe a broad range similar to the achievement tests (cf. Table 3).

Table 3: Class means regarding interest and motivation (scale – 1 bis 1).

Class means	Min.	Max.	Mean*	SD
Interest in mathematics	-0.62	0.03	-0.27	0.17
Fear of failure	-0.55	0.27	-0.26	0.18
Boredom	-0.71	0.08	-0.46	0.17
Achievement motivation	0.07	0.71	0.40	0.13
Self-determined learning	-0.25	0.37	0.14	0.14
Fear of mathematics classroom	-0.96	-0.53	-0.75	0.11

* Means for the 27 class means

It is noticeable that some classes showed extreme values for nearly all scales. However, in some cases there was a broad distribution of the individual values within the respective class. The students of one class rarely agreed in their opinion regarding one scale. Nevertheless, in many classes there were tendencies for agreeing respectively disagreeing to some scales and it can be assumed that those tendencies were influenced by mathematics instruction.

As the multi-level analysis revealed neither on the classroom level nor on the individual level *interest in mathematics* and *fear of failure* could be identified as predictors for the achievement ($\chi^2 = 19.2$, $df = 26$, n.s.; $\chi^2 = 18.4$, $df = 26$, n.s.).

3.3 The relation between mathematics achievement and the components of interest

We calculated correlations between achievement scores and scores for the six factors extracted from the questionnaire on interest and motivation regarding mathematics on the individual level. Only in three cases we found a significant correlation with the results of the achievement tests (cf. Table 4).

Table 4: Correlations between achievement and interest/motivation.

Correlations	Interest in mathematics	Fear of failure	Fear of mathematics classroom
Achievement (End of grade 7)	.195**	-.209**	(-.130*)
Achievement (Mid of grade 8)	.212**	-.218**	(-.174**)

* $p < 0.05$, ** $p < 0.01$

A deeper analysis indicated that only the correlations between *interest* and achievement and between *fear of failure* and achievement were stable. While the relation between achievement and *fear of mathematics classroom* disappeared after partialling out the other two components, the correlations of *interest in mathematics* and *fear of failure* to the achievement still existed after

partialling out other components. Since interest and motivation data were collected in parallel to the second test in grade 8, the correlations with the first achievement test were noticeable. They still existed after partialling out the second achievement test.

The relations became a little more apparent, if we considered the values of the components of interest/motivation for the different achievement groups. Here, the achievement groups were determined by the second achievement test on the basis of the non-dichotomous test scores (which provides a more differentiated separation of the population). The high-achieving group showed a somewhat higher value for the component *self-determined learning*, whereas the lower-achieving group seemed to have more *fear of mathematics classroom* (in both cases the differences are significant, but not substantial). As far as the components *interest in mathematics* and *fear of failure* were concerned the differences between all achievement groups were significant (cf. Figure 2).

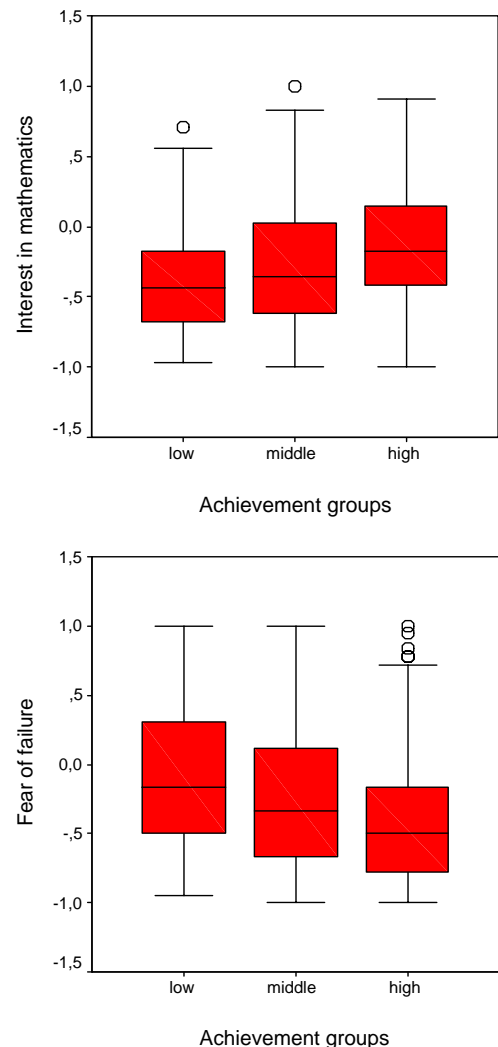


Figure 2: Parameter values of the principal components *interest in mathematics* and *fear of failure* for the different achievement groups.

As already described in the previous sections we found strong classroom effects for the achievement tests as well as for the interest and motivation scales. These

classroom effects indicate that the conditions of mathematics instruction (and, in particular, the teacher) influence the students. However, if we consider the classroom means for the six scales of interest and motivation, then we could not identify strong correlations between the classroom means of the two achievement tests. In contrast to the individual level where correlations between the factors *interest in mathematics* and *fear of failure* as well as the achievement results were observed, there seems to be no similar correlation for the data on the classroom level. Neither correlation analysis nor dispersion diagrams revealed a significant relationship between these variables. Accordingly, the differences for interest and motivation on the class level had hardly any influence on the individual achievement.

4. Discussion

The results of our study confirm that the competencies of students in the area of reasoning, proof and argumentation are unsatisfactory. It is a remarkable fact that a teaching unit on proving and reasoning in the geometry classroom of grade 8 is hardly apt to foster mathematics achievement. As was pointed out by the competency model (cf. Table 1) most of the students in grade 7 as well as in grade 8 can apply simple rules and concepts and are able to solve problems which require only a one-step argumentation. However, they fail if they have to combine two or more arguments to a chain of arguments. As supplementary interviews indicate, the latter is due to the difficulty of generating a proof idea or a proof strategy for an existing hypothesis. Thus, our findings correspond with the results of TIMSS and PISA.

Similar to the MARKUS study we observed explicit classroom effects for mathematics achievement. Furthermore, there was a high stability of the classroom means between both times of measurement ($r = .604^{**}$). If we take into account that students with a strong achievement decrease respectively achievement increase cumulate in particular classes, then this fact indicates that the classroom level has an influence on the individual achievement. Certainly, there will be varying prerequisites of instruction in the different classrooms. Moreover, reasoning and proof in geometry is a challenging part of the mathematics curriculum that requires a sound basic knowledge. So it might be expected that this topic will lead to a larger achievement increase for the high-achieving students. However, the classroom effects may be caused by teachers in low-achieving classrooms. They might teach this topic less ambitiously and avoid arguing on a complex level. Consequently, an above-average achievement increase of the students in these classrooms would be hindered due to the specific instruction.

The investigation of interest and motivation indicates that students, on the one hand, face mathematics instruction comparatively fearless and, on the other hand, pay a certain amount of attention (cf. Table 2 the mean values for the components *fear of failure*, *fear of mathematics classroom*, *boredom* and *achievement motivation*). However, this does not imply an above-

average interest in mathematics. It may be assumed that for many students learning mathematics at the lower secondary level is mainly extrinsically motivated. Only a few students develop an above-average interest in mathematics. Once more, we observed a remarkably broad distribution of the classroom means. There are obvious differences for some of the scales (cf. Table 3). For example, with respect to the component *fear of mathematics* there are just small variations (e.g., from “no fear” to “no fear at all”), whereas other components lead to absolute classroom differences of the range “rather agree” to “rather disagree”. This is the case for *fear of failure* and *interest in mathematics*. According to the subject-object theory of interest and the self-determination theory presented in Section 2 (both emphasize the importance of the social environment) the classroom conditions determined by the teacher can be assumed as substantial for the classroom differences. It is unclear how classroom conditions affect the students as individuals. The data reveal that the classes are not homogeneous for example in their opinion to the *interest in mathematics*. Therefore, we assume that the instruction has different motivational effects on different groups of students. At this point it would make sense to conduct further studies to clarify specific significant variables.

Though we observed a broad distribution of the data for mathematics achievement, interest and motivation, our findings replicate that the relation between achievement and interest is rather small. Basically, we found a significant correlation between the components *interest in mathematics* resp. *fear of failure* and the achievement on the individual level (cf. Table 4). It is remarkable that both components correlate with the results of the first achievement test (despite partialling out the data of the second test), although the first test already took place half a year before. This may indicate that these two components describe long-term attitudes to mathematics or to the particular topic reasoning and proof.

In contrast to the results of the MARKUS study (Helmke & Jäger, 2002) our investigation did not reveal a correlative relation between achievement and interest and motivation on the classroom level. Here, it must be considered that the achievement tests essentially covered argumentation and reasoning competences and, thus, included only one piece of the content of the mathematics curriculum. Classes that mainly experience an instruction focusing on calculations and algorithms can show high values for *interest in mathematics*. However, students from such classrooms will probably have poor achievement results in a test going beyond the application of simple rules.

In summary, we can conclude from the results of our investigation that the classroom level and, thus, probably the instruction has an influence on the development of achievement in reasoning and proof. To be successful in solving reasoning and proof tasks the learners have to meet a combination of different prerequisites. They should rely on a sound knowledge base for solving proof tasks, but they should also be able to generate proving strategies and to pursue them by adequate mathematical methods. In particular, the last two

aspects are obtained primarily within the mathematics classroom. Since mathematical proving is certainly a difficult area in mathematics, we can expect a motivational influence on the individual level. However, it should probably be differentiated between a general interest in mathematics and an interest in mathematical activities which go beyond the level of calculations and simple algorithms.

Although there are clear differences between the classrooms as far as their achievement and their interest and motivation are concerned, no significant correlation between these variables could be identified in this study. Even if a class shows poor interest in mathematics, this does not necessarily imply negative effects on the achievement of this class. In contrast, the achievement of a specific class obviously has an important influence. This becomes apparent particularly for low-achieving classes. A possible research question for a further study is: to what extent does the proficiency level within a class affect the motivation of the students?

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