

Beyond the Dichotomies Semiotics in Mathematics Education Research

Falk Seeger, Bielefeld (Germany)

Abstract: The present paper starts from the assumption that a semiotic approach might provide a fresh start in reconciling prevailing dichotomies in educational reflection and research. Especially as regards the popular dichotomies of individual against social learning and constructive against receptive learning. Three exemplary sections will illustrate the salient features of a semiotic approach: sign process seen as mediating means; sign processes as creating networks of objects, signs and interpretants; and the metaphor of map and territory as a relation of sign and activity. Throughout the paper it is tried to capitalize from the tension between the semiotic approaches of Peirce and Vygotskij.

Abstract: Der vorliegende Beitrag gründet sich auf die Annahme, dass ein semiotischer Ansatz möglicherweise dem Versuch zu neuem Erfolg verhelfen kann, herrschende Dichotomien in der pädagogischen und didaktischen Forschung zu überwinden. Dies betrifft besonders die beliebte Gegenüberstellung von sozial und individuell und von konstruktiv und rezeptiv beim Lernen. In drei exemplarischen Abschnitten wird die Fruchtbarkeit einer semiotischen Perspektive demonstriert: Zeichenprozesse als Mittel und Vermittlung; Zeichenprozesse als Vernetzung von Objekten, Zeichen und Interpretanten; und die Beziehung von Landkarte und Territorium als Metapher für die Beziehung von Zeichen und Tätigkeit. Die Spannung zwischen den semiotischen Perspektiven von Peirce und Vygotskij bildet dabei einen permanenten Bezugspunkt.

The following remarks pertain to the role of semiotics in research on mathematics teaching and learning. It follows from the interdisciplinary nature of mathematics education research that diverse questions can be asked, depending on the respective discipline, about what could be the benefit of adopting semiotics as a paradigm. It seems especially interesting to ask what could be gained to view the teaching and learning of mathematics as sign processes. Is semiotics especially relevant for the view of mathematics as a sign process or does it relate more to the philosophical grounding of mathematics? Are the language related aspects of mathematics a field of application for semiotics – and is it more applicable to the linguistic or more applicable to the discursive features? Maybe, semiotics is primarily pertinent to the diverse forms of mathematical representations and the use of these representations, especially mathematical signs and sign systems pervasive in the mathematics classroom? In what follows, I would like to sketch a framework that could be helpful in answering the questions mentioned above and questions that have not yet been asked.

The questions put above come to mind if one thinks from a disciplinary perspective about what could be a contribution of semiotics to the respective discipline. Now, we have to take into account that semiotics is in principle a trans-disciplinary approach and we do not have a clear-cut systematic of semiotic approaches that would allow to point to the contributions of a discipline

to semiotics. If this picture is correct, it seems more appropriate to choose a descriptive procedure focussing on paradigm cases and problems of mathematics education research. Accordingly, I will discuss in the following four sections four exemplary problems of what semiotics might have to offer to mathematics education research.

I would like to begin with a short review of the recent theoretical debate.¹ The purpose of this review will be to elaborate the chances of a semiotic approach. In the ensuing three sections I will expand on the following themes: signs as means; mediation and networking; and the map as a metaphor of educational representation.

„Acquisition“ and „Participation“ as Metaphors in Educational Research

Discussions on theoretical questions of teaching and learning, especially as regards the teaching and learning of mathematics, often are centered around the dichotomies of „social“ vs. „individual“ or „construction“ vs. „reception“. Classically, a constructivist approach emphasizing the individual and the constructive aspects is opposed to a cultural-historical conception underlining the social and the reception-related features. The struggle for a theoretical orientation is focussed on the juxtaposition of learning as acquisition versus learning as participation (see, e.g., Salomon & Perkins 1998; Sfard 1998; and the rigorous discussion between Anderson et al. 1996 and Greeno 1997).

In her analysis, Sfard (1998) comes to the conclusion that there are two fundamental metaphors, acquisition and participation, as illustrated in Table 1. There seems to be an intriguing parallel in the opposition of „mediating“ and „weaving“, a thought I have elaborated earlier (Seeger 2003). It will be interesting to see whether the levels of these two pairs of metaphors come to align.

Sfard does not only state some sort of complementarity of the metaphors she cites, but also points to the fact that a more general level of theory-building is in fact absent, a more general level of theorizing that would encourage reflection on a more comprehensive view on mathematics teaching and learning. The two metaphors, in her eyes, represent two divergent perspectives which are for principle reasons immune to every criticism put forward by the other perspective. In a sense, the theoretical territory of teaching-learning research thus must remain, by necessity, a patchwork of diverse perspectives and metaphors.²

¹ It has to be noted that „recent“ in this context means that part of the discussion is going back more than ten years. I am referring here to the discussion between „constructivists“ and the „cultural-historical“ or the „information-processing“ approach. Today, it is hard to find a comparatively intense discussion within mathematics education.

² How difficult it is to relate concrete approaches to specific metaphors becomes obvious if one considers that groundbreaking approaches of the „participation“ metaphor see themselves as being rooted firmly in an „acquisition“ metaphor. This becomes particularly obvious in the work of Jean Lave (see, e.g., Lave 1988; Lave and Wenger 1991).

Table 1: The research metaphors of acquisition and participation according to Sfard (1998)

	acquisition	participation
Goal of learning	Individual enrichment	Community building
Learning	Acquisition of something	Becoming a participant
Student	Recipient (Consumer), (re-)constructor	Peripheral participant, apprentice
Teacher	Provider, mediator, facilitator	Expert participant, preserver of practice / discourse
Knowledge, concept	Property, possession, commodity (individual, public)	Aspect of practice / discourse / activity
Knowing	Having, possessing	Belonging, participating, communicating

When we try to find out more about the underlying dimension in the discussions of the metaphors, we can say that the dimensions „construction“ vs. „reception“ and „individual“ vs. „social“ seem to be decisive. Put into a simple 2x2 table the combinations of these dimensions are displayed in Figure 1.

Constructivism could be put most simply into the upper left field A – as constructivism emphasizes an understanding of learning as constructive and individual, whereas the cultural-historical approach sits in field D emphasizing the features of social and acquisition in their understanding of learning. What then is to be found in the fields B and C?

Within these fields we find approaches to learning which, in a sense, incorporate the criticisms launched against its propositions into a revision of the approach. In the case of constructivism this means that a revision of the overemphasis of the individual dimension has led to the idea of „social constructivism“. And in the case of the cultural-historical approach the revision of the overemphasis on the reception dimension has led to turn again back to the „communicative“ dimensions of learning. This communicative interpretation of the cultural-historical school, for example, has been developed in the work of James V. Wertsch starting from Vygotskij and the literary-linguistic situation of the early Soviet Union (see, e.g., Wertsch 1991, 1995).³

I have already mentioned the „social constructivism“. Heinrich Bauersfeld and his co-workers with the advent

of their studies on interaction in the mathematics classroom are representing this approach (see, e.g., Bauersfeld 1983, 1988, 1995; see also, e.g., Cobb and Yackel 1991, Neth and Voigt 1991, Voigt 1994). Their approach heavily influenced by the sociological school of symbolic interactionism has developed an approach which tried to avoid to get entangled in the pitfalls of solipsism often following from an overemphasis of „pure“ constructivism. But also the „social“ continuation of Piaget's project of genetic epistemology represented by Annie Perret-Clermont must be mentioned here (see, e.g., Doise et al. 1975; Perret-Clermont 2000; Perret-Clermont et al. 1991). The aspect of individual acquisition might be seen in the center of attention of such approaches as Kruteckij's approach (Kruteckij 1966) or the usual instruction-oriented educational psychology (e.g., in the sense of the paradigm of information-processing of Anderson et al. 1996).

	individual	social
construction	A	B
reception	C	D

Figure 1. The dimensions of construction – acquisition and individual – social

The purpose of the above discussion has not been to fit the possible combinations of field in Figure 1 to certain schools and approaches in mathematics education or educational psychology in general. Rather it was meant to point to the necessity that any comprehensive approach to teaching-learning-processes has to take into account four essential complexes of conditions given in the fields A – D. This simple representation of a 2x2 table produces the immediate insight that in any attempt to grasp a teaching-learning process it cannot be possible that there is only one field under study while the others remain empty. In other words, in a non-reductionist discourse on teaching-learning processes all four conditions have to be taken into account. I assume that a semiotic approach is especially suited to meet this criterion because the focus in sign processes provides a solid basis for reconstructing conceptually the necessary complementarity of construction and reception – e.g. as the relation of the exterior, material quality of signs and their mental qualities – and at the same time grasping the complementarity of individual and social processes contributing to semiosis – e.g. in the sense of the well-known distinction of „internal“ and „external“ representations. A semiotic approach will also be especially fruitful as regards the fundamental problem of old vs. new in a learning process.

In a sense, the semiotic perspective on the dichotomies is meant to lead to the insight that they are obsolete. The view of something as a „sign“ or „sign-process“ (semiosis) in terms of a binary splitting seems to be doomed to failure from the start, because to

³ It should be noted that the concept of acquisition as found in the cultural-historical approach of A.N. Leont'ev is equally taken into account the constructive and the receptive elements of learning. In an acquisition process in learning the learner has to struggle for active construction of meaning, e.g., in the creation of the cognitively new or in the integration of what is to be learned into the existing motivational structure or in the development of new motivational structures (see Leont'ev 1971). An acquisition approach is meant here as a label for learning situations where primarily existing facts and processes are in focus and not the construction of „new“, not yet existing, knowledge relations. This notion of the acquisition metaphor is, obviously, well-known to Leont'ev's dialectic of „old“ vs. „new.“

understand something in terms of a sign means to see it as constructive and receptive, as individual and social in a fundamental sense. The present paper, thus, advocates the idea of breaking with the idea of the popular dichotomies. It is not very illuminating to ask constantly what the difference between social and individual learning might be, or whether learning might be more based on construction or on reception. Learning, from the start, is both: individual/social and reception/construction. Of course, this does not mean to deny that it might be very interesting and worthwhile to compare, e.g., classical solo performance with social processes of learning. What I want to underline here is that it might produce no further or only small insights into the learning process if these dichotomies are continued to be used and not given up. A semiotic perspective might lead the way into overcoming these binary splittings and dichotomies.

Signs as Means and the Triad of Signs

In the following two sections I will primarily try to compare the Vygotskian and the Peircean approach. One cannot say that this will be a serious comparison, because in a strict sense the two approaches appear to be incomparable – even though many similar features and parallels can be found. Although they depart from different philosophical starting points, e.g., both Vygotskij and Peirce underline the importance of „self-control“ as a major concept in development – Peirce within his approach of the pragmatic maxim (see, e.g., Colapietro 1989) and Vygotskij sees it as the final goal of ontogenetic development. Both thinkers are heavily influenced by Darwin's theory of evolution, although it appears that the influence of Darwin is not equally expressed in their writings. The evolutionary features are expressed much stronger in Vygotskij's writings, where the idea of development, the connection between phylogenesis and ontogenesis is pervasive. What both approaches have in common beyond these similarities is a struggle to come to terms with the idea of an „exterior“ mind. I shall begin with a discussion on what might be central concepts for Peirce and for Vygotskij. In Vygotskij's writings we find this perspective particularly in his conception of the ontogenetic development of higher psychological functions like cognition, emotion, volition condensed in his „genetic law of cultural development“:

We can formulate the general genetic law of cultural development as follows: every function in the cultural development of the child appears on the stage twice, in two planes, first, the social, then the psychological, first between people as an intermental category, then within the child as a intramental category. ... We are justified in considering the thesis presented as a law, but it is understood that the transition from outside inward transforms the process itself, changes its structure and function. Genetically, social relations, real relations of people stand behind all the higher functions and their relations. (Vygotskij 1997, 106).

While in Vygotskij's approach we find the progression from the outside to the inside, from the interpsychologi-

cal to the intrapsychological, Peirce's foundation of the sign concepts appears from the start as a complementarity of „inside“ and „outside“. This becomes apparent above all in Peirce *dialogical* foundation of the sign and the self (Colapietro 1989, Uslucan 2004).

If we are looking for points of difference between the two approaches we could choose many aspects that would help to set up a contrasting view. To begin with, I would like to choose intrasemiotic aspects, that is about the role of sign and sign-processes or semioses. Confronting what appears to be the essence one could say that Vygotskij concentrates on the character of signs as means⁴ while Peirce focuses on the ubiquity of signs. The character of signs as means has been expressed by Vygotskij using the triangle depicted in Figure 2. Stimulus and response are shown here to be mediated by some X, an „auxiliary“ stimulus – the medium. The triangular structure of behavior is in Vygotskij's view typical for all forms of higher psychological functions.

It is plausible to assume that among other things Vygotskij's understanding of the sign is essentially connected to his work on children with learning and developmental handicaps (cf. Veresov 1999, Keiler 2002). Because Vygotskij understood being handicapped in learning and development as a collapse of the structure of behavior, the goal of helping and therapeutic intervention was the reconstruction of that behavior. Vygotskij's approach was characterised by the idea that the relation of stimulus and response⁵ has to be re-mediated, that is, new means have to be found for mediating between the social and physical environment and the activity of the subject. These means are signs. The difference between handicapped and non-handicapped persons is not in the use of signs. It is not that handicapped persons do not use signs, while the „normals“ do. The difference between handicapped and non-handicapped is only in terms what kind of signs they use and what kind of sign-processes they create. If new signs mediating stimulus and response can be integrated successfully into the structure of behaviour, the handicap becomes more or less obsolete – and simultaneously this means that „higher functions,“ the specifically human functions, are back in operation again. In a sense, it is also the attempt to give back to the handicapped their self-image as humans. „For higher functions, the central feature is self-generated stimulation, that is, the creation and use of artificial stimuli“ (Vygotskij 1978, p.39).

⁴ Stating here that Vygotskij puts an emphasis on the signs as means has to be restricted here as a relative statement which is not applicable to his whole work. As Peter Keiler (2002) has shown in detail, the social character of psychological processes comes more and more to be dominant in Vygotskij's thinking. It is important here to avoid unproductive confrontations between the idea of signs as means and the primacy of social processes. Both processes afford each other.

⁵ Although Vygotskij uses the two classical categories of behaviorism here, he is nothing less than a behaviorist. For historical reasons and for reasons lying beyond the scope of this article, Vygotskij is using here the categories of Pavlovian reflexology while simultaneously criticizing the approach (for more detail cf. van der Veer & Valsiner 1991; Veresov 1999).

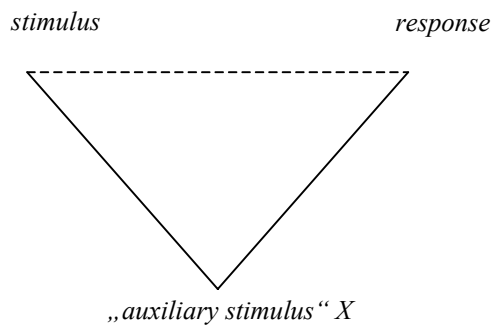


Figure 2. Vygotskij's version of the mediated response

Vygotskij's approach to the role of signs could not be more mean-related, more instrumentalist. Culture, in a sense, is also the aggregate of means, means to develop and to foster higher psychological functions.⁶ In a sense, even the social side of Vygotskij's main metaphor for appropriation, the „zone of proximal development“ can be seen as a means for appropriation: Adults or „more capable peers“ form the zone of proximal development in which learning and development meet (see Vygotskij 1987). In this light, adults and peers appear as instrumental for appropriation, in the first place, because they display the next step in the development of an ability and of knowledge. The „power of the sign“ for Vygotskij does not so much spring from the sign itself but from using it as a means, from arbitrarily creating new signs to be used as means in novel situations.⁷

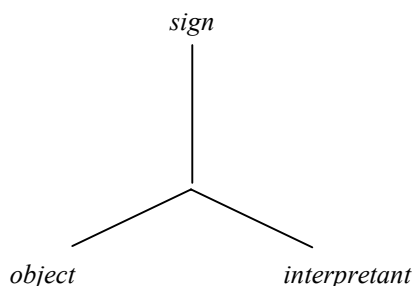


Figure 3. The semiotic triad according to Peirce

The semiotic triad is for Peirce constructed in a totally different way. This can be read from Figure 3 with a quick glance. In contrast to Vygotskij's triad, the sign is not the mediating link between two otherwise independent entities. Paradoxically, the sign *is* the total triad and *part* of the triad. The sign itself is to be understood as a triad as it is expressed in the following quote:

⁶ Elaborated as the idea of the „cultural tool kit“ (Cole 1996) or in Kozulin's book (1998) on psychological tools.

⁷ It is interesting to note that from this vantage point, Vygotskij's educational-psychological approach had a very strong „constructivist,“ productive orientation. This is surprising, because the dominant tone of the constructivist critique on Vygotskij had been that his conception of appropriation was too much based on the notion of a „reception“ of knowledge instead of its construction.

A sign, or *representamen*, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the *interpretant* of the first sign. The sign stands for something, its *object*. It stands for that object, not in all respects, but in reference to a sort of idea, which I have sometimes called the *ground* of the representamen (Peirce 1897, CP2.228)

Peirce, in contradistinction to Vygotskij, did not view the sign as being located „in the middle“ between object and subject – a view today still very popular. He did not believe that signs have to be understood as means or tools. This, in turn, did not keep him from investigating how sign-processes might become tools of the intellect in the course of scientific research, e.g., in the form of „diagrammatic thinking“ (cf. Hoffmann 2004). Peirce devoted a large part of his efforts to the development of schemes of classification of different signs and sign-systems. For him, sign-processes are the absolute „ground“ of human understanding, as expressed in the sentence: „It is that the word or sign which man uses is the man himself. ... my language is the sum total of my self.“⁸

It is said that the semiotic of Peirce is primarily a theory of reading signs (Trabant 1996). If this is true, the process of reading and understanding signs becomes especially interesting and, with that, abductive reasoning (see Hoffmann 1999).

Two things appear interesting in this context. First, it is the fact that abductive reasoning within a „syllogistic action“ (Hubig 2002) can be understood as a means. Equally interesting is it that abduction does not find its paradigm cases of application in particular situations like inventions or scientific discoveries, but in everyday situations. The use of everyday language, speaking and understanding requires an exceedingly powerful abductive reasoning.

Peirce is dealing with the „new“ in development and in thinking primarily as it appears in connection with a form of logical conjecturing. This logical form does not codify the successful past of human thinking and obser-

⁸ The complete quote is: ... it is sufficient to say that there is no element whatever of man's consciousness which has not something corresponding to it in the word; and the reason is obvious. It is that the word or sign which man uses is the man himself. For, as the fact that every thought is a sign, taken in conjunction with the fact that life is a train of thought, proves that man is a sign; so, that every thought is an external sign, proves that man is an external sign. That is to say, the man and the external sign are identical, in the same sense in which the words homo and man are identical. Thus my language is the sum total of myself; for the man is the thought (Peirce CP 5.314). Again it is surprising to note the obvious parallels in Vygotskij's thought as expressed in the following quote: „In consciousness, the word is what – in Feuerbach's words – is absolutely impossible for one person but possible for two. The word is the most direct manifestation of the historical nature of human consciousness“ (Vygotskij 1987, p. 285). This quote from Vygotskij quoting Feuerbach echoes also the emphasis on the fundamental role of dialogicality that Peirce is expressing as we have seen above.

vation but is directed toward the anticipation and development of the new. But abduction is not only directed toward the discovery of totally new entities. It is the essence of language that abductions are necessary for understanding, just because language and speaking are and remain ambiguous. Understanding what is spoken is possible just because a specific interpretation of an ambiguous expression is applied via abduction from the context of what has been spoken and what is given as a situation. If, e.g., in the course of a conversation the sentence is uttered: „He came home“, but many male persons have been mentioned during the ongoing conversation, abductive inferences from the context of speech and situation become necessary leading to the identification of the person. This abductive processes are running in the background of the conversation and have to be executed practically all the time. There is an interesting aspect of the relation between abduction and context which cannot be persecuted here.

Peirce had been distinguishing between many different forms of interpretants, was we shall see below. The difference between an immediate, „selective“ and a „contextual“, „environmental“ interpretant in a way reflects an old problem in the conceptualization of learning, that is, the notion that learning has to be understood also as learning about learning. This problem has been extensively analyzed by Bateson (1972) in his theory of different levels of learning.⁹

I shall not go into the details of discussing whether Peirce actually was discriminating between an „immediate“ and a „contextual“ interpretant. I will focus here on how he was thinking about the role of „collateral experience“ in semiotic processes.

Mediating and Weaving: The Role of the Interpretant

In what follows, I would like to present a diagram in Figure 4 and 5 that tries to capture the idea of a contextual interpretant as a variation of the original Peircean triad. This variation attempts to picture an aspect of learning situations that is of such fundamental importance that it can be called a necessary and critical feature of human learning. I am talking here about metacognition, metaknowledge, decentering – something that makes learning possible in the first place and thus can be called a fundament for education in schools. With his vision of „deutero learning“, Gregory Bateson (1972) drew attention to a characteristic feature of learning processes, namely, that progress in learning always aims at „learning to learn.“ Or, in other words, progress in learning is made possible because the learner can act from a higher or meta-level on to a previous level of learning. If one tries to imagine how climbing to a „higher“ level might be achieved, it seems plausible to assume that this works according to a „metaphoric“ principle as „something is

seen as something else.“ This, in a certain sense, can be understood as a link to Peirce’s idea of abduction.

What conceptual development has to achieve here is, basically, to show that the sign perspective or the semiotic view or, in Merlin Donald’s terms (1991), the view of culture as representational makes it possible to describe the centered, basic, elementary process of meaning making as well as the secondary process operating on the elementary process and thus express „the meaning of meaning.“

In what follows I would like to make some remarks on the role of the interpretant in Peirce’s thinking particularly in relation to the interpretant as context-dependent and the role of „collateral experience“ – as Peirce called it.

It is well-known that Peirce differentiates between an immediate, a dynamic, and a final interpretant. We cannot try to discuss the many different types of interpretants Peirce had thought about before – especially in his notes put down in his diary (see MS 339, July 7 – October 30, 1905). Here, I would like to concentrate not so much on the evolution of this final classification of interpretants, but more on an attribute of the interpretant that makes it possible that the semiotic web is in principle extended into infinity. Peirce has described this web-like structure in one of his definitions of a sign:¹⁰

Anything which determines something else (its interpretant) to refer to an object to which itself refers (its object) in the same way, the interpretant becoming in turn a sign, and so on *ad infinitum* (Peirce 1901, P. 527).

Learning could be understood, following Bateson’s approach to grasp the essence of forms of higher learning, as looking back from a more advanced level back to what has been learned. The development of learning, then, takes place if a learner can look onto their learning from a higher (maybe better: from a different) level. If one tries to imagine how such a level could look like it is obvious to search from something operating according to the metaphorical principle – an abductive process as a metaphor that includes viewing abduction as managing to translate something, as managing to „see something as something else,“ then it seems possible to view context as a decisive moment enabling metaphorical reflection. Here, too, Bateson (1972) has prepared the ground for understanding „higher“ forms of learning or the development of the new in terms of the „development of contexts.“

In Peirce’s thought, we find context in relation to the interpretant primarily connected to what he called „collateral experience,“ as he writes in a letter to William James:

Now let us pass to the Interpretant. I am far from having fully explained what the Object of a Sign is; but I have reached the point where further explanation must suppose some understanding of what the Interpretant is. The Sign

⁹ Of course, Bateson (1972) was not the first to mention the aspects of meta-learning or „deutero-learning“ as he called it. However, he was one of the first to try to understand that it wasn’t the „symbolic“ or the „power of the human mind“ that was responsible for reflective thinking – in the sense that animals would not be able to learn reflectively. As an impressive account on the reflective powers of chimpanzees see Fouts (1997).

¹⁰ See 75 more definitions Peirce had written down collected by Robert Marty on the following webpage: <http://members.door.net/arisbe/menu/library/rsources/76defs/76defs.htm>

creates something in the Mind of the Interpreter, which something, in that it has been so created by the sign, has been, in a mediate and relative way, also created by the Object of the Sign, although the Object is essentially other than the Sign. And this creature of the sign is called the Interpretant. It is created by the Sign; but not by the Sign *quâ* member of whichever of the Universes it belongs to; but it has been created by the Sign in its capacity of bearing the determination by the Object. It is created in a Mind (how far this mind must be real we shall see). All that part of the understanding of the Sign which the Interpreting Mind has needed collateral observation for is outside the Interpretant. I do not mean by „collateral observation“ acquaintance with the system of signs. What is so gathered is not COLLATERAL. It is on the contrary the prerequisite for getting any idea signified by the sign. But by collateral observation, I mean previous acquaintance with what the sign denotes. Thus if the Sign be the sentence „Hamlet was mad,“ to understand what this means one must know that men are sometimes in that strange state; one must have seen madmen or read about them; and it will be all the better if one specifically knows (and need not be driven to presume) what Shakespeare’s notion of insanity was. All that is collateral observation and is no part of the Interpretant. But to put together the different subjects as the sign represents them as related – that is the main of the Interpretant-forming. Take as an example of a Sign a genre painting. There is usually a lot in such a picture which can only be understood by virtue of acquaintance with customs. The style of the dresses for example, is no part of the significance, i.e. the deliverance, of the painting. It only tells what the subject of it is. Subject and Object are the same thing except for trifling distinctions. [---] But that which the writer aimed to point out to you, presuming you to have all the requisite collateral information, that is to say just the quality of the sympathetic element of the situation, generally a very familiar one – a something you probably never did so clearly realize before – that is the Interpretant of the Sign, – its „significance“ (Peirce 1998c, 493-494)

This idea of collateral experience, in a sense, explains why and how the interpretant is functioning. Collateral experience grows over time. It could be seen as some form of habit-formation – also for the final interpretant habitualization is the fundament of its functioning. Here, we meet an old paradox of learning: in order to learn something new it is necessary to have already learned something, one cannot start learning from scratch.

Peirce begins in the above quote with a description of the basic structure of the sign, and semiosis respectively. This is the description of relational structure, a structure of relations, where the relationships and not the „substance“ is the essential element of knowing. It is not only a dialogical perspective, but also seen from the perspective of an observer, from a very general standpoint.¹¹

¹¹ It would be tempting to discuss here in more detail the connection with the „pragmatic maxim“: „Consider what effects,

In other words, the semiotic triad expresses that all we can know of is not only a relation between an object and a sign, but is also tied to the condition that any relation between an object and a sign can only have meaning if it is formulated in relation to a third element: something cannot be a sign for an object if it is at the same time a sign for an interpretant.

Peirce’s conception has an impact on the metaphorical and interpretative ground of knowing. It entails that all we know about the world is at first an interpretation and only insofar as it is an interpretation it can say something about the epistemological objects. In contradistinction to hermeneutics, however, where we find an emphasis on fallibility and subjectivity Peirce’s idea does not imply or lead to a position of relativity. In the conception of the sign as a triad the complementarity of objectivity and interpretation is addressed by the interpretant. In his diverse forms as immediate, dynamic, and final, it represents a link allowing other triads to dock on. This idea is tentatively represented in Figure 4.

The diagram in Figure 4 is an illustration of a beautiful idea Michael Cole has presented in his work on cultural psychology (Cole 1996). In this book where he takes up Wundt’s idea of a cultural psychology, he elaborates that context can also be more than surrounding. In context is the most salient explanation for what happens inside thinking and learning processes, than we have to arrive at a new understanding of context, an understanding of context as more than something which „surrounds.“ It could be understood as something which is „woven into“ activity and is in this way explaining why context does not remain at the surface but reaches deep inside the learner.

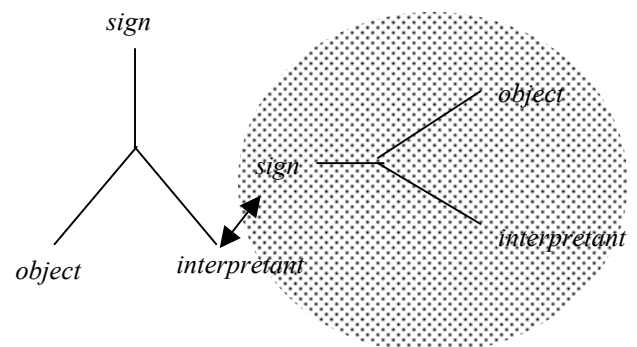


Figure 4. The „context“ of a triad

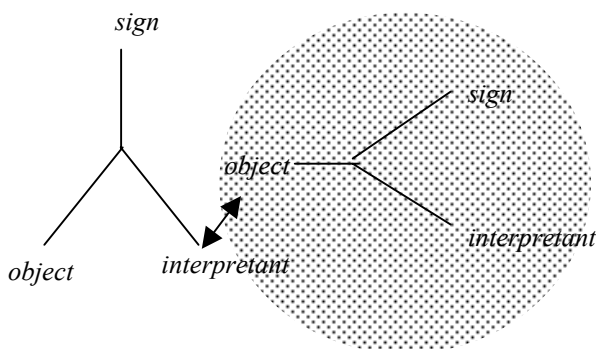
Figure 5 is a variation of Figure 4. In Figure 5 the interpretant of the secondary triad is not functioning as a sign, but as an object. Examples for the two figures could be found easily. I will offer only two small exemplary inter-

that might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object." ('How to Make Our Ideas Clear', Peirce 1998a, p.132). It must be enough to point out that later versions of the pragmatic maxim, e.g. the version in the Manuscript 682 from 1913 (Peirce 1998b, p.465), indicate a deep relationship to Vygotskij.

pretations. Figure 4 can be read as an example where the interpretant could be spoken word, being in relation to a written word (sign) and an object. The spoken word (interpretant) then is taken as a sign and is understood by a listener (interpretant) in a certain way. In Figure 5 a situation could be depicted where again the interpretant is a spoken word, but for reasons being in the acoustic environment like noise the word cannot be interpreted as standing for a certain object. The interpretant then becomes the object insofar better conditions have to be found where signs could be yielded that are hearable.

If we were to try to incorporate „context“ into that diagram, we would be wanting to express the specific quality of the interpretant to become the object of another triad. This seems to be exactly the point characterizing a learning meta-perspective because learning is made an object of learning. The two diagrams seem to capture this specific situation.

Figure 5. The germ cell of the semiotic web



The diagrams illustrate that a meta-perspective can primarily be taken because the interpretant is changing its position: Now, it has become the object of another contextual triad with yet another interpretant. However, if we look closely at the following excerpt from Peirce, we can see that this quality of the „meshing“ of triads seems to be an effect of the fact that, ultimately, the „meaning of a representation can be nothing but a representation.“

A sign stands for something to the idea which it produces, or modifies. Or, it is a vehicle conveying into the mind something from without. That for which it stands is called its object; that which it conveys, its meaning; and the idea to which it gives rise, its interpretant. The object of representation can be nothing but a representation of which the first representation is the interpretant. But an endless series of representations, each representing the one behind it, may be conceived to have an absolute object at its limit. The meaning of a representation can be nothing but a representation. In fact, it is nothing but the representation itself conceived as stripped of irrelevant clothing. But this clothing never can be completely stripped off; it is only changed for something more diaphanous. So there is an infinite regression here. Finally, the interpretant is nothing but another representation to which the torch of truth is handed along;

and as representation, it has its interpretant again. Lo, another infinite series. (A Fragment, CP 1.339, Not dated)

Something important becomes apparent when the original Peircian triad is extended as in Figure 4 and 5. It shows only the first step of an endless spreading of these meshed triads: The triads are woven together in infinite processes of semiosis.¹² An end of the actual and potential weaving cannot be determined.

This web-like structure, this weaving, surprisingly echoes much of the idea of a *rhizome* put forward by Gilles Deleuze.¹³ For the present discussion, only the following features of rhizomatic structures should be briefly mentioned: Each point of a rhizome can be connected to any other point; a rhizome starts to grow from the middle, but it has no center; if a rhizome is cut or interrupted, it continues to grow at any given place. The rhizomatic form of representation *par excellence* is the map: representations are no longer layered and hierarchically organized, but spread over on the flat surface of the map.

In contrast to centered (even polycentric) systems with hierarchical modes of communication and pre-established paths, the rhizome is an acentered, non-hierarchical, non-signifying system without a General and without an organizing memory or central automation, defined solely by a circulation of states. (Deleuze & Guattari 1987, 21)

The Semiotics of the Map

But there are also some difficulties with the attempt to interpret a map as some kind of rhizom. These difficulties are pertaining above all to the actual use of the map which is determined by the relation to the territory, to navigation and orientation in the territory.¹⁴ The essential feature of a map is usually said to be that it represents the territory iconically – according to a principle of similarity. But iconicity is only one characteristic feature of maps or more general of plans representing a certain territory as a spatial arrangement of features of interest to the user. This other feature of maps has already been called into attention by Gregory Bateson's monitum: „The map is not the territory!“¹⁵

What is striking is not only that maps have to be similar to the territory but that their essential feature is

¹² For another attempt to elaborate the educational implications of this expanding web-like structure, see Presmeg (2004).

¹³ But also other approaches could be mentioned here, such as the concept of dissipative structures put forward in the context of a theory of self-organization by Prigogine (see Nicolis & Prigogine 1977; Prigogine & Stengers 1981) or similar ideas on the manufacturing of social order without a central steering ordering power formulated by Bourdieu (1979).

¹⁴ In their erratic boundary-breaking volume „A Thousand Plateaus“ Deleuze and Guattari (1992) also discuss the Peircian triad of Icon, Index and Symbol and relate it to the concept of territory (see, e.g., page 92 ff. of the German translation). We will not discuss this here for reasons of space.

¹⁵ Bateson is quoting here Alfred Korzybski (1941) and calls this sentence the idea that made Korzybski generally famous.

that they are different from the territory. Also the nature of maps as signs is in a sense not strictly to define. That maps make orientation and navigation possible is based, on the one hand, on their iconic representation of the territory, but at the same time the map gives an indexical representation of the landscape. While indexical representation is lending to a map a certain history, relevance, and usefulness for specific purposes, as it is typically the case for maps (see Wood 1992), the map as an iconic representation is embedded into the continuous flow of activity, the search for an alignment of map and territory.

Now, it is interesting to see that obviously different processes are involved in learning to use these two semiotic functions – although it is understood that both functions have to be executed together and have to be simultaneously present if the map is supposed to work as a means of way-finding and navigation.

Starting from a passage in Ed Hutchins' work on navigation (Hutchins 1992) the difference in these learning processes can be described as follows. Some of these learning processes have been realized as interpsychological processes in the sense of Vygotskij's genetic law of cultural development discussed above and some of these processes obviously have not been realized in this form because they appear to be simply too complex to be represented with the usual communicative means. Hutchins gives the following example (cf. Hutchins 1992, p. 285):

The task of reconciling a map to a surrounding territory has as subparts the parsing of two rich visual scenes (the chart and the world) and then the establishment of a set of correspondences between them on the basis of a complicated set of conventions for the depiction of geographic and cultural features on maps. As performed by an individual, it requires very high bandwidth communication among the representations of the two visual scenes. Very occasionally, this task appears as a socially distributed task when a pelorus operator has no idea of how to find a particular landmark. In that case, the restricted bandwidth of communication between the pelorus operator (who can see the world) and the bearing recorder (who can see the chart) makes the task virtually impossible. The spatial relations implied by the locations of the symbols on the chart are simply too rich to be communicated verbally in such a way that the pelorus operator can discover the correspondences between those verbally expressed relations and the relations among the objects he can see in the world.

This example gives rise to the question whether we have to assume the existence of learning processes that do not progress from the outside to the inside, from the social to the individual. If this question is justified, one can also state that not everything has to be interiorized but that certain means have to stay at the outside in order to support the psychological functions of orientation, of navigation, and of learning. Exactly because certain means remain exterior they can fulfil their purpose. This is especially true for maps as means of orientation and navigation: they serve their purpose by staying an exterior means and so render the continuous alignment of territory

and map possible. If the map would be internalized, the territory would be familiar and the map obsolete.

We have to ask now, of course, if the map could be applied as a metaphor to orientation and navigation problems in the teaching and learning of mathematics. Would it be a helpful metaphor? Or would it hide more than it makes visible?

I find the map-metaphor helpful in a certain way, if one agrees that it is not a metaphor about the relationship between the world and mathematics (map). A multitude of maps seems to be possible representing a particular territory or an object as a combination of iconicity and indexicality. Often those maps can be understood as supporting or as complementary to classroom discourse: their iconic features help to render things visible that could not be made visible through verbal communication. As an example we could look at how students represent with some kind of maps their arithmetic conceptions having no other sign-related way to express those conceptions. A beautiful example are the so-called „Eigenproduktionen,“ mathematical conceptions and strategies primary students create on their own. These conceptions express the need for orientation in number space in conjunction with the aspect of rule-orientation.

In Figure 6 one of those *eigenproductions* is shown. These idiosyncratic mathematical creations have been extensively documented in the work of Selter and Spiegel (2001). It is not easy to follow the course of evolution of these creations. This seemed to be also partly felt by the student who included indexical elements into the self-created representations which makes them look like maps for way-finding. On the upper part of the map four three-digit numbers can be found: this is the addition task. These numbers have been the points of students collected in some play or sport event. Now, they have to be added making the addition as easy as possible.

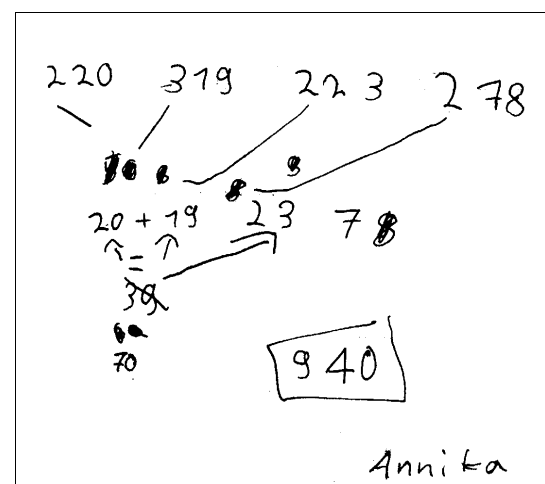


Figure 6: The territory of addition in the „Eigenproduktion“ of a primary student (from Selter & Spiegel 2001, p. 37)

Selter and Spiegel are giving the following overview of the course the arithmetic operations of the student had probably taken (see Table 3). As one can see there is only one small slip of the pen, the rest of the calculation is correct.

Table 3: The likely course of the calculation shown in Figure 6

	written down	calculated
1	50	$200+300=500$
2	6 (50 crossed out)	$500+200=600$
3	8 (6 crossed out)	$600+200=800$
4	20+19=39 (arrows to the 20 and 19 as well as an equation sign); in addition 23 and 78	$20+19=39$
5	62 (39 crossed out)	$39+23=62$
6	70 (the 8 of 78 crossed out; 62 the same)	$62+8=70$
7		$70+70=140$
8	9 (8 crossed out)	$800+100=900$
9	940 (9 crossed out)	$900+40=940$

The communicational and educational benefit of this type of maps requires some sort of a culture of reading these map-like diagrams. Often they can only be deciphered using detective skills and with experiences gathered over the time with the idiosyncrasies of the map makers. The problems teachers have to understand these kind of idiosyncratic diagrams are mirrored in the problem primary students often have to understand what seems obvious in diagrams meant to support and enhance learning. A now classical case of such a misunderstanding has been cited by Christel Manske presenting the learning history of disabled youth (cf. Fingerhut and Manske 1984).

If maps and diagrams are meant to comply with requirements of orientation and navigation, one cannot look at the relationship of iconicity and indexicality alone. If the full meaning of these devices is to come to bear on teaching and learning mathematics, one has to take into account that they are symbols incorporating and representing a certain relation between sign-processes and a user.

Coda

The work of Peirce and Vygotskij appear close to each other insofar as they both reject what Ernst Cassirer (1969) termed „substance“ thinking: in the passage from the concept of „substance“ to the concept of „function“ he saw the decisive feature in the evolution of modern science. It has already been mentioned that both thinkers have a close relationship to Darwin's ideas.

We have to ask now whether the distinction between intrasemiotic and intersemiotic made in the beginning has been helpful. Especially, has it been helpful as regards the difference between Peirce and Vygotskij. Does it make sense at all to differentiate between a sign as such and the semiotic web?

The discussion has shown that in a way the intra- and intersemiotic perspective are supplementing each other adding up to some form of complementarity. At the beginning of this paper I assumed that it would be a good start to understand Vygotskij approach as intersemiotic

and Peirce's approach as intrasemiotic. It seems to be clear now that this is an oversimplification. Now it seems more like Peirce's conception begins, not necessarily chronologically, with the intrasemiotic processes and then lead to some form of a semiotic web as a rhizom. Schönrich (1990) has shown that „reflection of reflection,“ which is the germ cell of the semiotic web, is already present in Peirce's triadic concept of the sign. For Vygotskij the development runs counter to Peirce's logic of development from the intersemiotic to the intrasemiotic sign processes. That for both conceptions the relation to the respective discipline, philosophy, logic and mathematics for Peirce and psychology for Vygotskij, leads to considerable differences becomes obvious (cf. Ort 2001).

Many questions remain unanswered, and some of them appear to be pursued as worthwhile problems. Among these question I would rank the idea that the function of a sign as a means for „syllogistic action“ (Hubig 2002) as well as the problem the role of the observer in semiotics (see, e.g., Jahraus und Ort 2001). To me the sign processes in maps and diagrams appears especially attractive and deserves attention.

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Author

Falk Seeger, Dr. phil., Senior Lecturer at Institut für Didaktik der Mathematik (IDM), Universität Bielefeld, D-33501 Bielefeld, Germany.
Email: Falk.Seeger@uni-bielefeld.de