The interactive development of mathematical inscriptions - a semiotic perspective on pupils externalisation in an internet chat about mathematical problems

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Abstract: In internet-chat situations about mathematical word-problems, the students are confronted with the fundamental issue of presenting their solving-attempts in a written or graphic form. This circumstance raises the opportunity to study the use of inscriptions as defined by Latour and Woolgar (1986). In my essay I present results of a pilot study in which primary school students allocated in two separate rooms solve mathematical problems by means of internet chatting. In order to analyse the inscriptions emerging during the chat sessions Charles S. Peirce's semiotic approach is applied, based on research methods of Interpretative Classroom Research.

Kurzreferat: Die schriftlich-graphische Darstellung ihrer Lösungsbemühungen ist ein grundsätzliches Problem, das sich Schülern und Schülerinnen in der laufenden „Pilotstudie zur chat-unterstützten Erstellung mathematischer Inskriptionen unter Grundschülern" stellt. Dies eröffnet Möglichkeiten Entwurf und Interpretation sowie die Nutzung und Weiterentwicklung gemeinsamer Inskriptionen zu untersuchen und deren Beitrag zur Genese mathematischen Wissens zu ermitteln. Ein Ausschnitt aus einer Chat-Session aus der Pilotstudie wird auf der Basis eines interpretativen Ansatzes aus semiotischer Perspektive mit Charles S. Peirce triadischer Zeichenrelation analysiert.

ZDM-Classification: C30, C50

1 Introduction

The project „Mathematical Internet-Chat"[1] is about the genesis of ‚mathematical inscriptions': In an experimental situation, an internet-chat-setting, the communication between pupils, solving given word-problems together, depends on the use of written/graphical representations. This setting potentially offers new insights into fundamental problems of teaching and learning mathematics. Mathematics seems to depend on written forms of communication more than any other subject (Pimm 1987). Writing constitutes an integral part of mathematical communication. It has been argued that students' understanding would benefit if they were asked to fix their solutions in written form and reflect upon them (e.g. Pimm 1987; Morgan 1998; Fetzer 2003). Fixing ideas in a written form changes their status and makes them more explicit and conveyable. One could say that 'private' thoughts are externalised into 'public' ones. According to Bruner (1996), one can call this the "externalization tenet" (pp. 22-25).

The special status of internet-chat-communication, a status between verbal and written communication, is the point from which Götz Krummheuer and myself started the pilot-study. Internet-chat-communication is a written form of communication regarding the medial dimension but similar to spoken (vocal) communication referring to the conceptual aspect (Koch & Österreicher 1985; Achenbach 2002). Hence it is based on both, written language through its medium and spoken language in its structure. This property of internet-chatting allows initiating and observing a conversation that is conceptually close to verbally based "face-to-face" interaction among pupils and at the same time medially based on written representations.

The interactively evoked chat products are called ‚inscriptions' referring to Latour & Woolgar (Latour & Woolgar 1986; Latour 1987; 1990). Vocal interaction between the chat partners on the two sides of the setting is not possible, so there is the demand to externalise questions, hints, and solving-attempts in the chat-box or the whiteboard. This process is based on the "chat-interactive" development of shared inscriptions. Other publications have also focussed on the interactive development of inscriptions (s. Roth & McGinn 1998; Lehrer et al. 2000; Sherin 2000; Meira 1995, 2002; Gravemeijer 2000, 2002), however in original face-to-face situations. In our project, the difference is the focus on the exclusively inscription-based communication between the two poles of the chat-setting, which prevents verbal communication by means of the experimental design.

More concretely, we investigate:

- how pupils use and develop inscriptions when solving mathematical tasks,
- to what extent the process of interactive problem-solving is structured by using these inscriptions,
- in what way this structuring fosters the learning process, and
- how far the use of the inscriptions contributes to the genesis of mathematical knowledge.

In the following chapters, first the term "inscription" is introduced. Afterwards, on the basis of Hoffmann's work, the triadic relation of signs by Charles S. Peirce is presented as a tool of semiotic analysis. Then an example of the chat sessions is depicted by a transcription. Finally, this example is analysed on the basis of the developed instrument in order to reconstruct semiotic aspects of chat-situations.

2 Inscriptions as a joint production

By Roth & McGinn (1998) inscriptions are described as scriptures, pictures, signs, graphs, lists, and diagrams embodied in some medium, such as paper or computer monitors. The authors describe these as private thoughts, which are carried into 'public arenas'. The interest here is in what way these ´private thoughts´ are communicable and how they contribute the genesis of shared

[1] This study is supported by Müller-Reitz-Stiftung (T009 12245/02) entitled „Pilotstudie zur Chat-unterstützten Erstellung mathematischer Inskriptionen unter Grundschülern".

mathematical knowledge. Latour and Woolgar study the development and evolution of knowledge in laboratories. The different kinds of models, pictures, icons, and notations used in the laboratories are classified by Latour and Woolgar as 'inscriptions'. They describe several characteristics of inscriptions (Latour & Woolgar 1986; Latour 1987; 1990):

- Inscriptions are mobile because they are recorded in materials and can be sent by mail, courier, facsimile, or computer networks.
- They are immutable during the process of moving to different places. Inscriptions remain intact and do not change their properties.
- The fact that they can be integrated in publications just after a little cleaning up is described as one of the most important advantages of inscriptions.
- The scale of inscriptions can be modified without changing internal relations.
- It is possible to superimpose several inscriptions of different origins.
- They can be reproduced and spread at low cost in an economical, cognitive and temporal sense.
- Inscriptions can be merged with geometry because of their two-dimensional character. Latour (1990) mentions this advantage as the greatest one.

Inscriptions are seen by Latour and Woolgar as a very ductile means of representation that is continuously changing and improving. By this way they represent aspects of the conceptual development during the research process. Also in our project the focus is on the development of inscriptions in the interactional course of internet-chatting between pupils. Theoretically this process is seen as a part of a chat-based interaction process which produces among others taken-as-shared-meanings (Cobb & Bauersfeld, 1995). With regard to Gravemeijer, one can describe such a development as a "cascade of ever more simplified inscriptions" (2002, p.18). He describes a tendency in such cascades moving in the direction of a greater merging of figures, numbers, and letters towards even more meaningful and simpler inscriptions.

Roth & McGinn allude to the fact that using inscriptions is closely connected to the social practice in which they are produced: "Inscriptions are pieces of craftwork, constructed in the interest of making things visible for material, rhetorical, institutional, and political purpose. The things made visible in this manner can be registered, talked about and manipulated. Because the relationship between inscriptions and their referents is the matter of social practice … students need to appropriate the use of inscriptions by participating in related social practices." (1998, p. 54)

Our approach focuses especially on the genesis of particular inscriptions: In an internet-chat-based dialog the pupils externalise their ideas by means of alphanumerical and/or graphical notations. They receive reactions of the chat partners whereby, step by step, the inscriptions evolve into a shared inscription. Precisely in the case of using the internet-chat-setting, the chat-dialog-box and the whiteboard-frame enable this process by an interactive exchange based on inscriptions. Internet-chatting supports the process of creating a text as the distinction between the writer and the reader evaporates and becomes replaced by the process of a collaborative production of a text.

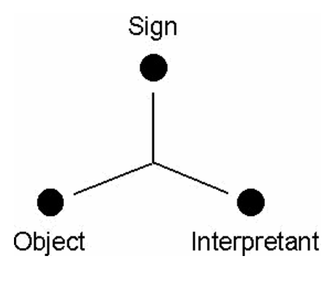
Meira demonstrates empirically the dialectical process between the designer's activity and the representation: „… a representation on paper has the important function of shaping its designer's activity at the same time that the designer shapes the representation itself." (2002, p. 101) This dialectical process takes place during the joint problem-solving when learners negotiate the meanings of representations. Referring to Meira's examples these negotiations are orally based. In contrast, in the mathematical-internet-chat the negotiation of meaning emerges from written and graphic representations themselves. The chat partners have to rely exclusively on the inscription-based communication.

3 The triadic sign-relation

For the analysis of the commonly accomplished inscriptions in the chat-based solving processes we refer to Peirce's sign model. The peircean sign model is a very differentiated classification and it is applied by some researchers of didactics of mathematics (f. ex. Volkert 1990; Hoffmann 1996, 2003a; Dörfler 2003, 2004) as well as by pedagogical researchers (f. ex. Zellmer 1979). In comparison to other semiotic approaches (f. ex. Saussure and Lacan: s. Gravemeijer 2002), it seems to be a more powerful instrument for my analyses. Among other arguments, the peircean approach is less oriented towards the language as the ones of Saussure or Lacan (s. detailed discussion in: Hoffmann 2003a, p. 7-12), but it integrates the individual as interpreting instance and opens by this the option of integrating this approach with an interactional theory of learning and teaching mathematics. Also Eco points out the advantages of peircean semiotics in "A Theory of Semiotics" (s. discussion in Eco 1976, p. 14-16), where he describes the peircean approach as more complete and semiotically more fruitful.

The peircean sign-relation consists of "a triple connection of sign, thing signified and cognition produced in the mind" (Peirce, 1.372). The three correlates in this triadic relation are specified in an elaborated definition (pic. 1):

Peircean triadic sign-relation



pic.1

"A sign, or representamen, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the interpretant of the first sign. The sign stands for something, its object. It stands for that object not in all respects, but in reference

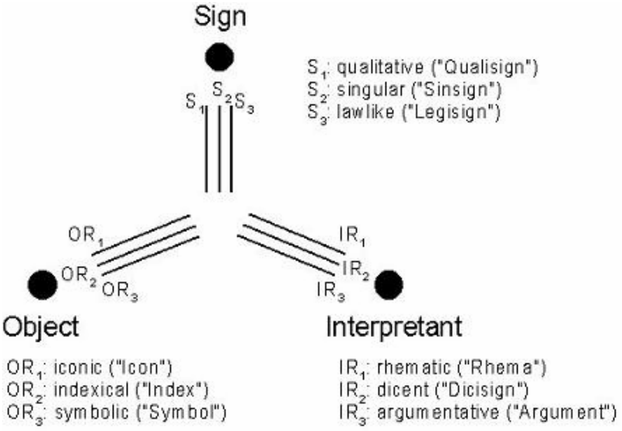
to a sort of idea, which I have sometimes called the ground of the representamen." (Peirce, 2.228)

Peirce characterizes the three correlates, the representamen or sign, the interpretant, and the object, in three ways as follows (see pic. 2):

The representamen (sign) can be subdivided into qualisign, a quality, which is a sign; sinsign, or token, which is an actual existing thing or event, a kind of singular sign; and legisign, or type, which is a law that is a sign.

The relation of the interpretant is divided into rhematic, dicentic, and argumentative. A rhematic sign is a single word, which is not true or false, almost any word or number. For Peirce "the readiest characteristic test showing whether a sign is a Dicisign (a dicentic sign) or not is that a Dicisign is either true or false, but does not directly furnish reasons for being so" (Peirce, 2.310). An argumentative sign is a "Sign of law" (Peirce, 2.252). The dicentic sign affirmes the existence of an object, but the argument proves its truth.

The object can be embedded in the sign-relation in an iconic, an indexical, or a symbolic category. Iconic signs represent similarities, as - for example - fotos, cartoons, footprints etc. "The index … forces the attention to the particular object intended without describing it." (Peirce, 1.369) For example, smoke is understood as a sign of fire. The symbol is a sign representing meaning. If the relation between the sign and its object is symbolic, it must be mediated by an interpretant.



pic. 2 (Hoffmann)

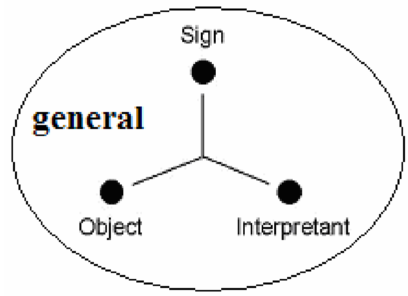
Hence there are three correlates and three ways in which the correlates are characterized, but due to some restrictions there are only ten valid combinations for distinct classes of signs (for more information see Nöth 1990, p. 45).

In the below presented chat session, sinsigns and legisigns occur: according to Peirce, every conventional sign, a word in a language or a number, is a legisign. But in a single utterance it is a sinsign. The sinsign is called a replica of its legisign. The relation of the different interpretants in my example is in some cases rhematic and argumentative in others. All the objects are embedded in a symbolic kind in the sign triads. The triads are depicted in section 5.3.

Applying and developing the peircean approach, Hoffmann (e.g. Hoffmann 1996) focuses on the 'idea' or 'ground' in the peircean sign model, whereas he calls „das Allgemeine", the „general" (translation by Schreiber), what Peirce calls „ground", or „idea". Hoffmann mentions as examples for the ´general´: concepts, theories, habits, competences etc., which are given mentally or physically.

In comparison to the analysis with peircean triads, the concept of the `general` seems to be advantageous for the analysis of the examples from the mathematical-internet-chat project. Therefore I combined Hoffmann's approach (1996) with the classical peircean triadic sign-relation:

The `general` as ground of the peircean triadic signrelation



pic. 3

the peircean triad is underlain with Hoffmann's 'general', because the interpretant is determined by the concepts, theories, habits, competences etc of the observer (pic. 3).

Semiotics as a tool of analysis appears to be appropriate to reconstruct aspects of the inscriptions evolving during the chat sessions. The communication between the two parties working jointly on word-problems is based on inscriptions. The initial analysis of interaction is supplemented by a semiotic approach. The analysis of interaction is the basis for the application of the semiotic analysis. Especially with regard to the reconstruction of the 'general' the analysis of interaction is of great help. It provides a stable ground to stand on concerning interactional aspects and the negotiation of meaning (s. a. Schreiber, 2004b).

4 Organizational aspects of the pilot study

In order to offer an appropriate setting for the pupils to communicate via chat, we use two Tablet PCs with touch-screens and wireless connection. Using the software NetMeeting (Microsoft) the pupils have the possibility to write in the chat-dialog-box and to draw in the whiteboard-frame. All these activities on both computers are recorded as a screen video by the software Camtasia – Studio (Techsmith). Furthermore, the verbal communication of the pupils working together at the same computer is saved with a digital voice recorder which is embedded in the computer. The following three chat-constellations have been realised:

1 pupil	⇔	1 pupil;
2 pupils	⇔	2 pupils;
1 pupil	⇔	2 pupils.

The pupils are at the age of 9 – 10 and go to several public primary schools in Frankfurt am Main (Germany).

In 5 series from October 2002 to July 2004, round about 60 sessions each with a duration of approximate 40 minutes were taped. In this period, the setting, hard- and software, and also the word-problems were improved continuously. Up to now, more than a dozen scenes have been transcribed and analysed. Early examples are described in Schreiber 2003a and 2003b.

5 An example

In this paper, an example of my analysis with regard to Peirce's triads is given. The excerpt is about two pupils on each side of the setting (chat-setting: 2 pupils ⇔ 2 pupils) solving jointly the following word problem via internet-chat: „A snail is on the bottom of a 3,20 m deep well. Each day it climbs up 80 cm. Each night it slights down 20 cm. How many days does it take to crawl up?" (This scene is also described in Schreiber 2004a, using different categories and a less developed analysis).

The chat session of these 2 x 2 students (with their self-chosen nicknames „flippers" and „sleepers") were recorded as described in chapter 4. We can look at the excerpt from three different perspectives: first, the creating of the inscription by Sleepers; second, the chat-communication based on this inscription; and third, the interpretation of the inscription by Flippers. The focus here is on the third perspective.

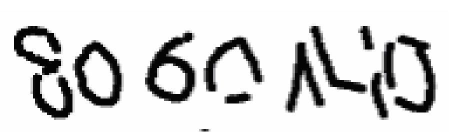
The excerpt is depicted below by a transcript, showing both sides of the setting and the jointly used whiteboard (transcription rules s. appendix). In a first step, the scene is described, focusing on the perspective of the Flippers (5.1). In a second step, it is analysed with the above developed approach (5.2).

5.1 First step of the analysis

In the following, I will describe the example from the perspective of the Flippers. They are the recipients of the inscription produced by their chat-partners, the Sleepers.

For the Flippers, the digits 8 and 0 evoke on the ground ('general I') of the knowledge of decimal system and the implicit knowledge of the coherence of digits and numbers the interpretant "eighty." When the inscription is continued (pic. 4) with the digits 6 and 0, two alternative

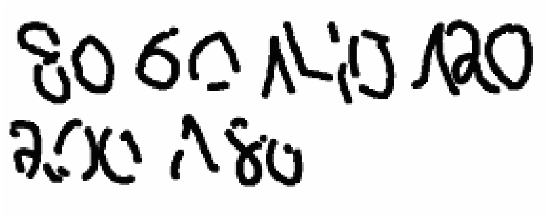
Whiteboard 0:15 min (sleepers ⇔ flippers)



pic. 4

interpretants are evoked, referring to the 'general I', too: „eighty, sixty", as two two-digit numbers, and „eight thousand and sixty", as one four-digit number. In conjunction with the next digits 1, 4, and 0 the inscription evokes, also referring to 'general I', the interpretant „eighty, sixty, one hundred and forty-three", as three distinct numbers. The successively appearing digits 1 and 2 first evoke the interpretant „twelve", with the next appearing digit 0 indeed „one hundred and twenty". The 'general' in this case is just as before the knowledge of the decimal system and the implicit knowledge of the relation between digits and numbers ('general I'). But the Flippers do not refer to the seven digits given before. Correspondingly, when the digits 2, 0, 0 and later 1, 8, 0 appear (see pic. 5), they do not refer to the digits given before, too.

Whiteboard 0:30 min (sleepers ⇔ flippers)

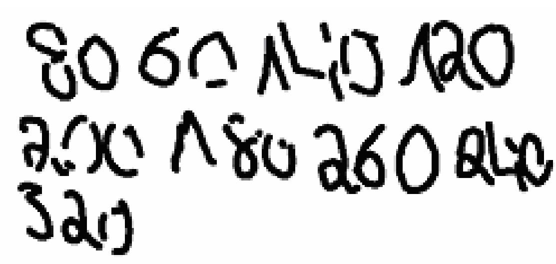


pic. 5

There is an important change on the conceptual level regarding to the 'general': The Flippers refer to the whole up to now created inscription (pic. 5), respectively to the interpretants evoked by the inscription before. The new 'general' is an idea of these numbers as a numerical order and the idea that it will represent the process of the solution of the task in a chronological manner ('general II'). There is the interpretant evoked that these several numbers signify steps on the way up.

The conclusion is abductive, and this abductive conclusion is proved now (pic. 6) by the Flippers by means of the following digits (260 240 320), and it is deductively verified. The evoked interpretant of these three numbers is the confirmation of the abductive conclusion. When observing the whole whiteboard and referring to the 'general II', particularly the number 320 evokes the repetition of a part of the question in the given

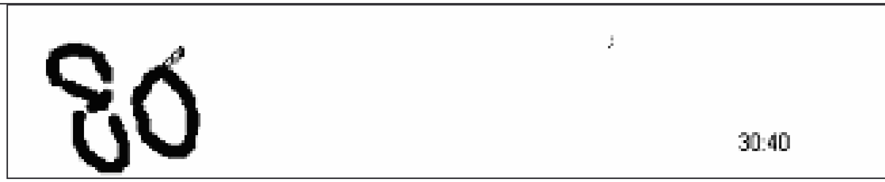
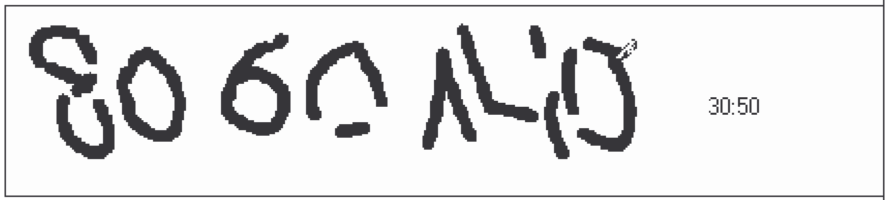
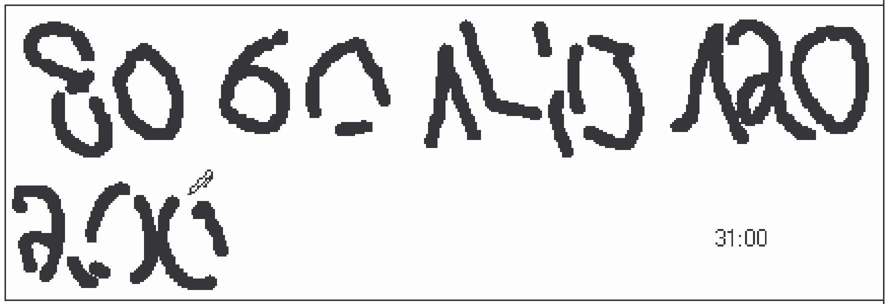
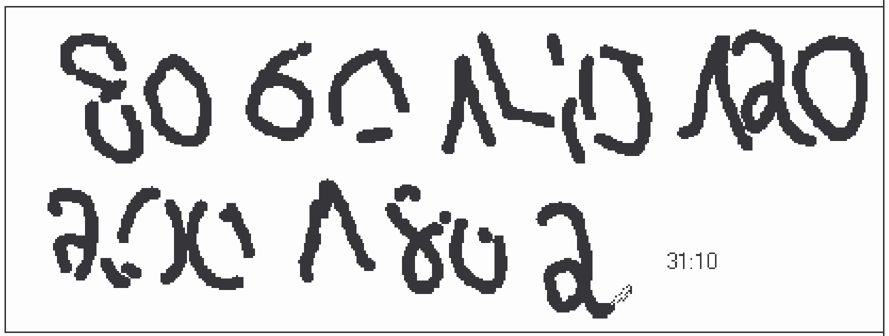
Whiteboard 0:55 min (sleepers ⇔ flippers)

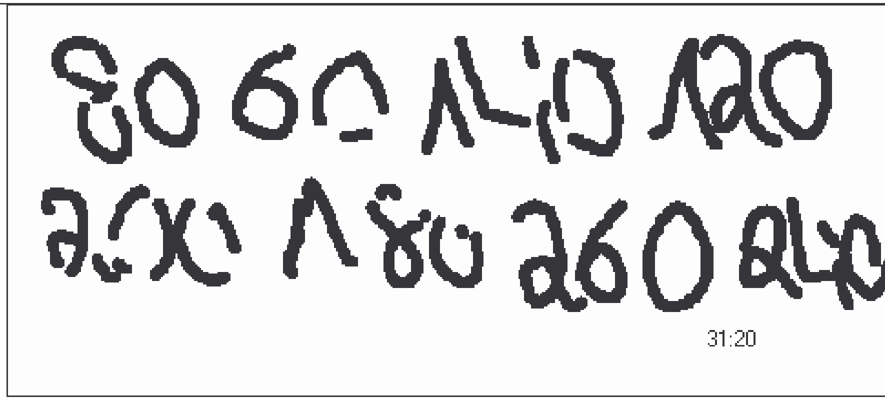
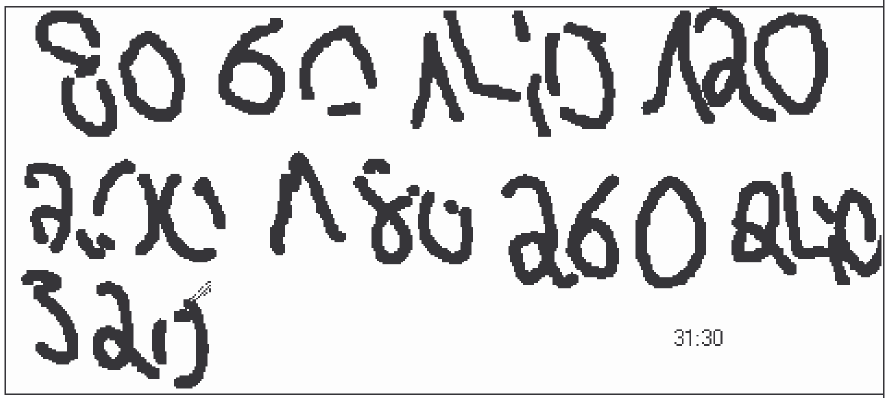


pic. 6

task: "how many?". This number is identified as the destination on the way up, whereas the other numbers are seen as stages and the gaps are seen as turning points. Flippers continue to refer to the 'general II' (idea of these numbers as a numerical order and the idea that it will represent the task in a chronological manner), when the whole inscription in the context of the task is used to count the days: "one, two, three, four, fifth". They count pairs of numbers as one day and the last day as just one number.

Transcript of the scene

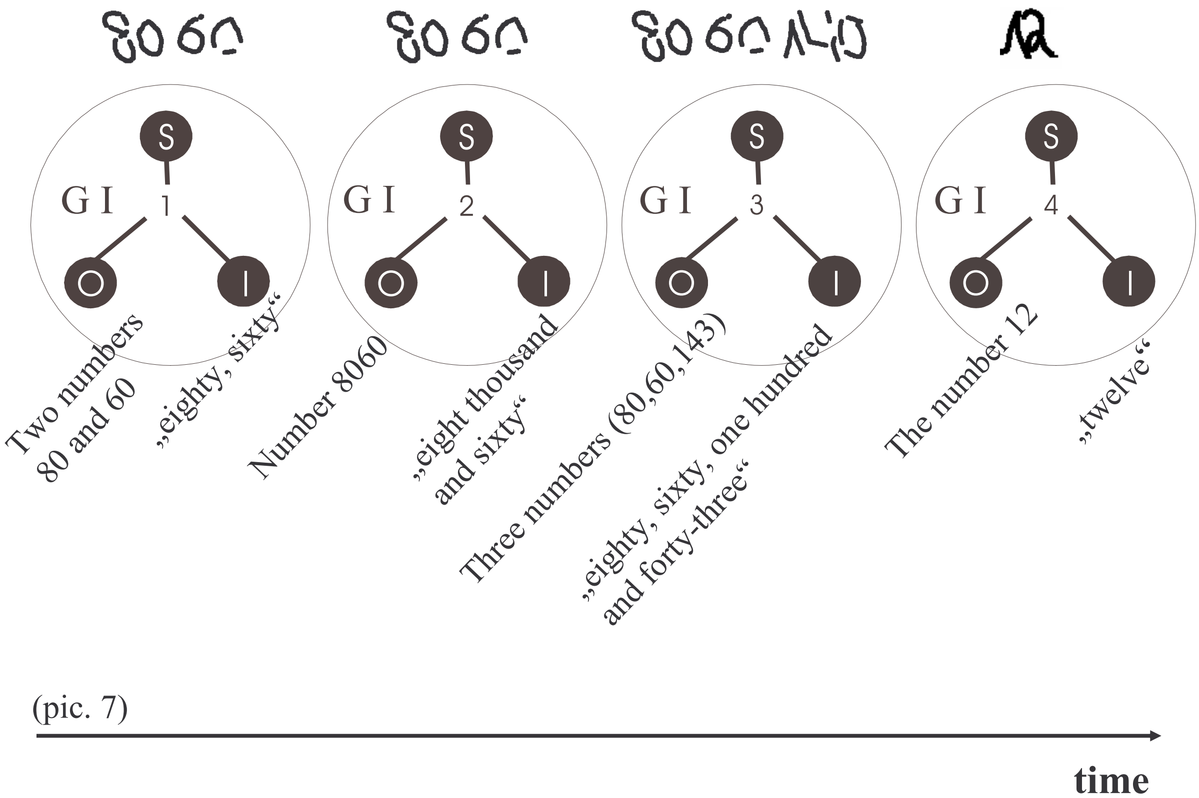
line/ time	Verbal utterances Sleepers	activities Sleepers	Whiteboard	activities Flippers	verbal utterances Flippers	time/ line
30:40						**30:40**
14	**S2**: this is [#5 s i x t e e n\ #5] 60	[#5 **30:41** writes 60 onto the white-board]			**F1**: 80 60\ 8060 yes\ (..) 14 80 60 143	6
15	**S2**: <plus 80 is					
16	**S1**: < 140\					
17	**S2**: [#6 140 #6]	[#6 **30:46** writes 140]			(..) yes\ 12 120\ (...)	**30:50**
30:50						
18	**S1**: (..) em\ (..)					
19	**S1**: <[#7 120\ (.)	[#7 **30:52** writes 120]				
20	**S2**: < 120\					
21	**S1**: #7] Plus 80\ is [#8 200\ #8] minus 20\	[#8 **30:58** writes 200]			200\ (5sec.)	**31:00**
31:00 22	**S2**: is [#9 180\ #9] plus 80\ is	[#9 **31:04** writes 180]			**F1**: 180\ (...) I see\ 80 is minus, so it is 60\ 60 plus 80\	**31:10** 7
31:10	[#10	[#10 **31:10** writes 260]				
23	**S2**: < 260\ #10]					
24	**S1**: < 60\					
25	**S2**: >[#11 240\	[#11 **31:15** writes 240]				
26	**S1**: > 240\					
27	**S2**: <#11] Plus 80					
	S1: < 80					

line/ time	Verbal utterances Sleepers	activities Sleepers	whiteboard	activities Flippers	verbal utterances Flippers	time/ line
31:20 29 30 31 32 33 34	 **S1**: > is two- **S2**: > is\\ **S1**: <hundred- **S2**: <hundred and **S2**: twenty **noo\\ threehundred-** twenty\\ 320\\ **S1**: *whispering* [#12 320\\ #12]	[#12 **31:26** writes 320]			is 140\\ (.) minus 20 120\\ **F1**: <120 plus 80 **F2**: < mmh\\ **F1**: is 200\\	**31:20** 8 9 10
31:30 35 36 37 38	 **S1**: **yes**\\ to say **S1**: < 1 2 3 4 **S2**: < 1 2 3 4 **S1**: **5** days\\				**F1**: plus (.) minus 20 is 80 260 240 320\\ how many/	**31:30** 11
39 **31:40** 40 41 42	**S2**: **Noo**\\ 4, 1 2 3 4 noo let us calculate \\ **noo**\\ wait **S2**: < 1 2 3 4 5 **S1**: < 1 2 3 4 5 **S1**: yes\\ it is 5 days\\		Whiteboard does not change		how many days does it take to crawl up / 1 2 3 4 fifth\\ (7 sec.)	**31:40**
43 **31:50** 44	**S2**: 5 days or/ 5, 5 **yes 5**\\ **S1**: 5 d a y s \\				**F2**: should we write right/	**31:50** 12

5.2 Second step: analysis on the basis of the developed sign-relation

Now I present an analysis by semiotic triads using the peircean sign-relation and Hoffmann's concept of the 'general'. It is the kind of illustration that can be given with the developed instrument. I will comment briefly on what is depicted (see pic. 7 to 9):

In this analysis, the sign or representamen in triad no. 1 is the beginning of the inscription shown in the transcript. It is a rhematic sinsign, creating the interpretant "eighty, sixty", and its object is the numbers 80 and 60. The same rhematic sinsign in triad no. 2 in reference to the same general, 'general I' (G I), evokes the interpretant "eight thousand and sixty", which stands for its object, the number 8060. When the inscription is continued, the sign in triad no. 3, also a rhematic sinsign, evokes in reference to the same general the interpretant "eighty, sixty, one hundred and forty-three", standing for the object 80,60,143 as three distinct numbers. Afterwards, Flippers do refer only to the part of the inscription shown in triad no. 4. This rhematic sinsign evokes the interpretant "twelve" in reference to the 'general I' (G I) standing for its object, the number 12.

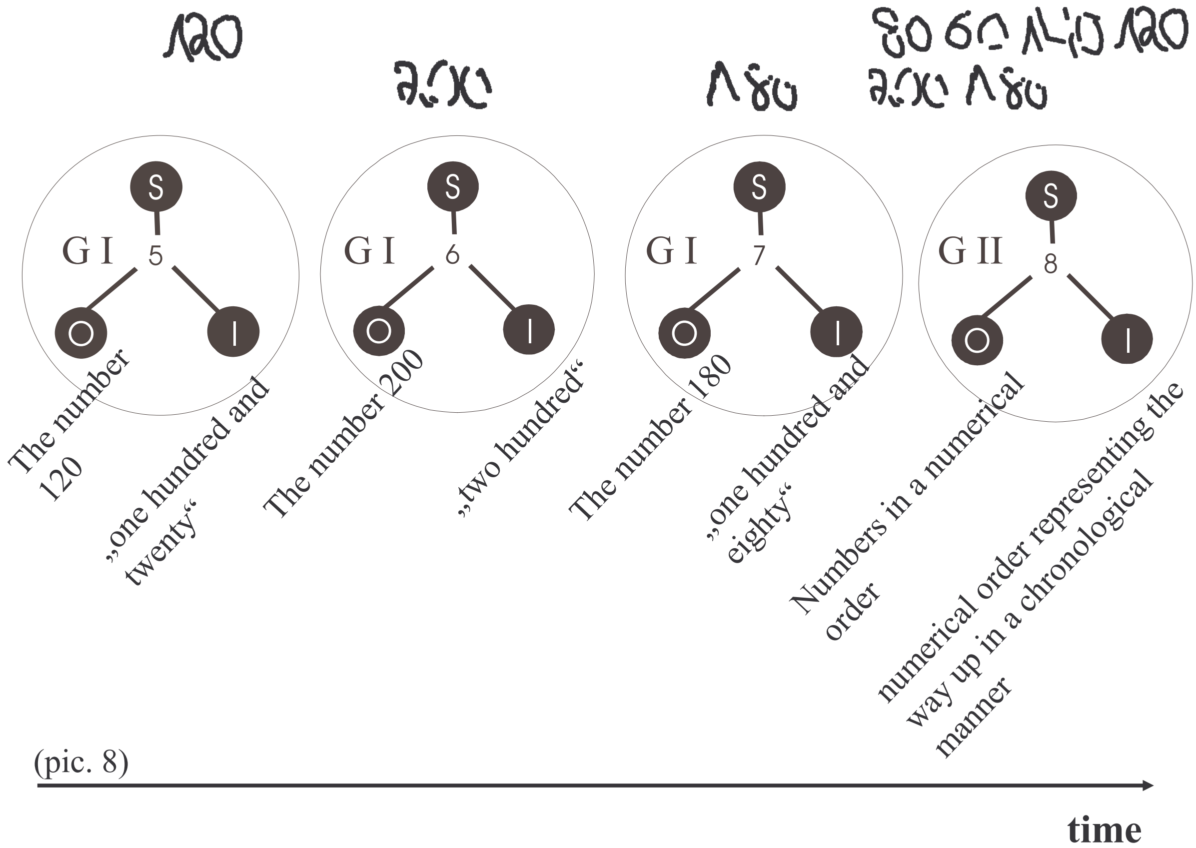


(pic. 7)

'General I' (GI): the knowledge of digits and numbers and the decimal system.

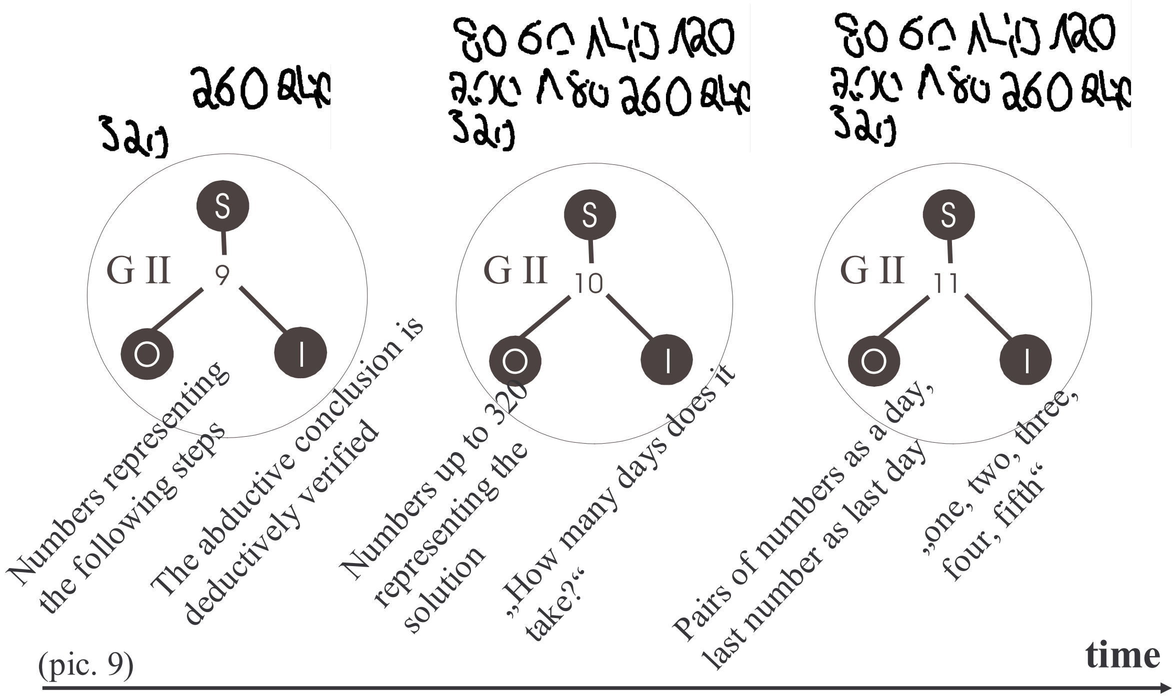
When a zero is added to the inscription, the sign in triad no. 5, also a rhematic sinsign, evokes the interpretant "one hundred and twenty" standing for its object, the number 120. Also in the triads no. 6 and 7, there are rhematic sinsigns evoking interpretants in reference to the 'general I' (G I), the knowledge of the decimal system and the implicit knowledge of the correlation of digits and numbers. In the following triad you can see the change described in section 5.2: Flippers refer to the whole up to now created inscription, respectively to the interpretants evoked by the inscription just before.

So the sign here is an argumentative legisign, because the relation between the numbers is satisfying a law. The 'general' (G II) is an idea of these numbers as a numerical order representing the task in a chronological manner. The evoked interpretant, numbers signifying steps on the way up, is an abductive conclusion. In triad no. 9, the representamen is the continuation of the argumentative legisign. The evoked interpretant is the deductive verification of the abductive inference. The 'general' is also the idea of these numbers as a numerical order and the idea that it will represent the task in a chronological manner (G II). In triad no. 10, the argumentative legisign, the whole inscription, evokes the rephrasing of the question in the task, referring to 'general II' (G II). Its object is the representation of the solution by the numbers up to 320. In triad no. 11, the same sign together with the question of the task evokes the interpretant "one, two, three, four, fifth" referring to 'general II' (G II). The represented object is the representation of days by pairs of numbers (80 and 60; 140 and 120; 200 and 180; 260 and 240) and the last day by one number (320) only.



(pic. 8)

'General I'	(GI):	the knowledge of digits and numbers and the decimal system.
'General II'	(GII):	the idea that the numbers are a numerical order representing the solving of the given problem in a chronological manner.



(pic. 9)

6 Conclusion

As described in section 5.2, the signs up to triad no. 7 are rhematic sinsigns. The signs in the triads no. 8 to no. 11 are argumentative legisigns. This change is linked with the change of the 'general'. While the 'general' is the differentiated knowledge of the correlation between digits and numbers and the decimal system (G I), the signs evoke possible numbers composed of the appearing digits. When Flippers are referring to the 'general II' (G II), numbers as a numerical order representing the solving of the given problem, they can recognize the relation between the numbers satisfying a law, and they can verify the abductive conclusion, when the inscription is completed.

The sinsigns occur all in a rhematic manner, whereas the legisigns occur in an argumentative manner. In either case, the object is related in a symbolic way.

In my example, the Sleepers are solving the given task. For them, the produced inscription can be a kind of tool to solve the problem as well as a tool to communicate their steps and their solution to the Flippers, or also something between. This aspect was not of interest here and can be analysed for another occasion. For the Flippers, the emerging inscription is a kind of a developing representation of a solving-attempt by pupils of their class. It is a possibility to compare the task with this representation and to recognize, if this representation is useful and perhaps advantageous, if it is readable and what one can see there. Without fostering their own solving-attempt they get to know a kind of useful representation. It is proved by themselves because they are able to read the several steps and the solution as well. Even they can read and predict the solution "five days", before this was written in the chat-box by the Sleepers.

As it is an example of a collective problem-solving process by chat communication, without any instruction or assistance of a teacher, it is a genuine representation of the pupils themselves. Such as in other examples of the mathematical internet-chat sessions, they use a very minimalist representation of the task-solving process not only as their own notes but also for their joint communication. Also the use of a symbolic representation in the sessions was preferred and proved to be advantageous for solving the task and for communicating the solving process to the chat-partners.

Primarily, the aim of this article is to demonstrate and to probe a tool for analysing inscription-based communication. As the process of development is not yet finished, it is an outline serving as basis for further improvements. Principally, classroom discourse in mathematics is a combination of vocal and written communication. Thus, its analysis needs a common theoretical basis for these two modes in math class interaction. Peirce's semiotic can be taken as such a ground. Here, the underlying project focuses by means of the experimental setting on the inscriptional aspect of these communication-processes, which is seen as the aspect needing more attention in theories about classroom interaction. Further research is envisioned to connect these two aspects in an integrated interactional theory of mathematics learning in classroom situations.

This kind of semiotic analysis enables us to name and to describe the varying signs and their effects. So the advantage is the possibility to depict graphically and to understand in detail the ongoing problem-solving- and learning-processes, particularly if these processes are inscription based. The development of this graphical depiction will be improved in further papers considering further aspects of peircean semiotics. It can be of interest for researchers of didactics of mathematics and other researchers interested in this kind of communication either between pupils as in my example or between teachers and pupils in more traditional teaching processes in classrooms.

The applied analysis seems to be appropriate, because of a sophisticated sign-model, enriched by focusing on the underlying 'general'. Precisely these concepts, theories, habits and competences are decisive for the emergent problem-solving and learning-process. The 'general' in the internet-chat examples can be recognized exactly by carrying out an interaction-analysis on the basis of the prepared transcriptions.

7 Appendix: Transcription Rules

1. column and 7. column
- line numbers and time

2. column and 6. column
- shortnames of interacting persons on the left hand side (Times New Roman 12 pt bold).
- verbal utterances on the right hand side (Times New Roman 12 pt); incomprehensible utterances are marked as (incomprehensible).
- paraverbal information, (special characters see below), for example *emphasizing, whispering* etc. (Times New Roman 12 pt italics)
- # refers to actions on the computer

3. column and 5. column
- actions marked with # are actions on the computer of each chat-participant.

4. column
- part of the screenshot with time information (here every 10 seconds)

Special characters:

,	short break in an utterance
(.)	break (1 sec.)
(..)	break (2 sec.)
(…)	break (3 sec.)
(4 sec.)	duration of a break longer than 3 sec.
/ - \	rising, even, falling pitch
yes	**bold**: accentuated word
s i x t e e n	s p a c e d: spoken slowly

(„ < “) and (“ > ”) two participants are talking both at the same time, for example:

8 S2: < plus 80 is 140/
9 S1: < 140\ ok

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