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Beyond Constructivism, Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching

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A Source Book for Teachers and Teacher Education

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This book has 28 chapters, 31 authors, and runs to 609 pages. It's not the kind you would take with you on a summer holiday. Yet, once you get into it, it's rewarding - - at least if you are interested in improving education and in the constructivist orientation in particular.

Constructivism, as the label of a learning theory, derived from the work of Jean Piaget and entered the field of mathematics education about thirty years ago. The activity of conceptual construction was a corner stone in Piaget's Genetic Epistemology and the inherent model of knowledge acquisition. The term "constructivism" quickly became fashionable, was extended to include Vygotsky, generated fierce discussions, and was widely used and misused. It was also adopted as a qualification by individuals as well as schools of thought that had little or nothing in common with the original orientation. In spite of quite a few successful applications both in science- and mathematics education, there has recently been something of a backlash advocating a return to traditional teaching methods.

"Beyond Constructivism", the title the editors Lesh and Doerr (L&D in what follows) have chosen, will undoubtedly be welcomed by the reactionary quarters - a plus, perhaps, in the eyes of the publisher's marketing department. But if the book is read with the attention it deserves and critical remarks are kept apart from recommendations, most of what is actually advocated will not be to the liking of enemies of constructivism or friends of traditional methods.

However, to separate the critical from the positive is not as easy in this text as one might expect. The reason is that there are two levels of critique. On the one hand, amplifying the title, constructivism is presented as a useful phase, necessary at the time and not to be forgotten, but now in need of expansion. On the other hand, there is pervasive criticism of what is characterized as the traditional methods of teaching. In this second area the authors agree with constructivists and consider the

tradition prevalent in most school as the main obstacle to better educational practices.

The subtitle mentions "Models and Modeling Perspectives", an expression that functions as name for the advocated approach and is abbreviated in the text as "M&M", just like the bean-sized chocolates that are available in slot machines all over the United States. Unlike the sweets, the presented perspectives require some effort to be digested: they are presented for seasoned researchers rather than for beginning teachers.

L&D distinguish two meanings of the term "model". The first, where the term is "used as an adjective that is synonymous with the word 'ideal'", as in "a model classroom" (p.9). This is not the meaning intended in their book. Of the second meaning (which is the one they want) they say:

"Ever since mathematics educators first became aware of Piaget's Research on children's mathematical reasoning (1970), they have been developing models to describe the conceptual systems that they believe students use to make sense of certain kinds of mathematical experiences. Sometimes, these researcher's models are described as if they were the actual conceptual systems that children employ (e.g. Carpenter, Fennema, & Lamon, 1988). But for most of the cases considered in this book, it is important to emphasize the distinction between models that researchers develop to describe children's thinking, and models that children develop (explicitly, not just implicitly) to construct, describe, or explain mathematically significant systems that they encounter." (p.9)

I thoroughly agree with the exhortation that researcher's models of children's thinking must not be presented as descriptions of what is actually going on in children's heads. Indeed, more than a decade ago, Steffe and I wrote a paper about "Conceptual models in educational research and practice" (Glasersfeld & Steffe, 1991) in which we discussed this distinction. A second important distinction, we said, has to be made between models ascribed to children and models the researchers or teachers have of the conceptual systems they consider to be the targets of education.

To describe the most essential characteristics of the kind of models the authors present in this book, they write:

"Models are conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation systems, and that are used to construct, describe, or explain the behaviors of other system(s) - perhaps so that the other system can be manipulated or predicted intelligently.

A *mathematical* model focuses on structural characteristics (rather than, for example, physical or musical characteristics) of the relevant systems." (p.10)

The examples given to amplify these definitions do not show all distinctions that would be helpful. L&D explain that aeronautical engineers, for instance, "construct model airplanes to guide the design of real airplanes ... because it is too expensive or too dangerous to experiment using the real systems." In other fields,

scientists “may construct computer-based simulations (i.e. models) to investigate weather patterns or other complex phenomena that occur naturally” (p.10). I see a functional difference there. The model airplanes are constructed to test the *physical* features of the engineers’ theoretical model of a plane, but by themselves they offer no explanation. The climatologist’s model, in contrast, embodies a *theory* of weather patterns and *is* used as their explanation. Mathematical models would seem to be of the second kind.

Raising the question “Where do models reside? In the mind? In representational media?”, the authors answer:

“Both! First, because they are conceptual systems, they are partly internal and are similar to (but usually more situated and contextualized than) the conceptual systems that cognitive scientists refer to as cognitive structures (or schemas for interpreting experiences). Second, in mathematics and science education, the conceptual systems that are most powerful and useful seldom function in sophisticated ways (or in nontrivial situations) unless they are expressed using spoken language, written symbols, concrete materials, diagrams or pictures, computer programs, experience-based metaphors, or other representational media.” (p.11).¹

The notion that, because they can be expressed in all sorts of media, conceptual structures *reside* in these media is one point where the authors clearly disagree with constructivism. There is more to this than a manner of speaking. The assumption that conceptual structures could be carried from teachers (or books) to students by means of language or other representational media is an integral part of the *traditional* theory of teaching. Constructivism, however, has emphasized that this is a fallacy. Words, diagrams, pictures, and, I would add, mathematical formulae, do not *contain* meanings; they can only elicit them from a repertoire that is already present in the listener or reader.

It is a pity that the authors of this book are not clear about this fundamental characteristic of all means of communication, including models. In chapter 3, Lehrer and Schauble write:

Consider, for example, maps, in which the resemblance of the model to the world relies heavily on convention, rather than perception. Reading a typical street map depends on knowledge about conventions like the compass rose, the meaning of different line weights, the presence of a key, and often, other conventions as well. (p.63)

Indeed, models depend on conventions and have to be interpreted. But their most important feature is that they depend on the user’s knowledge of some basic operations. In road maps, for instance, certain lines are intended to represent roads. To “read” them, you have to follow them with your eyes knowing that the movements and changes of direction your focus of attention is executing on the map are intended to be analogous to those which you could make, on foot, in a car, or on a bicycle, on the “real” road. In other words, you have to be able to monitor the movements and turns you make with your eyes, before you can begin to interpret the lines of

the map. This is rarely said explicitly. It seems obvious in the case of road lines, but I mention it because it illustrates a feature that is common to didactic models: only if some basic operations are available to the interpreter, can models indicate a way of mentally operating that may be useful in the solution of certain problems.

For one who like myself is interested in educational research rather than teaching, chapter 4, “Piagetian conceptual systems and models for mathematizing everyday experiences” by Lesh & Carmona, is one of the most rewarding. It shows among other things that these two authors are constructivists where epistemology is concerned. About concepts such as *inertia* (in science) and *center of gravity* (in mathematics) they say:

Both of the preceding constructs are properties that are associated with the system that we use to make sense of experience. They are not simply properties of the objects themselves.

Now, the surprising fact that Piaget and his followers have emphasized is that virtually all mathematical ideas similar to inertia and centers of gravity in the sense that judgments about them depend on thinking in terms of systems of relations, operations, and transformations. Of course, this fact should come as no great surprise to mathematicians who long ago abandoned the notion that mathematics is about truth and reality, or properties of real objects.” (p.82)

My guess is that it *will* come as a surprise to many mathematics teachers and that it will not be easy to change their minds.

Piaget was primarily interested in establishing a general pattern of cognitive development, not in teaching. Mathematics educators, on the other hand, are interested in developing methods for the teaching of mathematics. This is the difference that chapter four brings out very clearly by discussing several “conservation tasks for adults” that are not only instructive but also entertaining.

Rather than considering mathematical constructs “as if they were specific manifestations of general/all-purpose cognitive structures,” L&C conclude “the local conceptual developments that we observe during model-eliciting activities suggest that the evolution of mathematical constructs tends to be more accurately characterized as gradual increases in local competence.” (p.96). For most Piagetian constructivists this fact was covered by the notion of “horizontal décalage”, but L&D are right in stressing a difference in this regard, because not enough research had been done on the accommodations required when a given structure was to be applied in new contexts. This book is a valid effort to fill the gap.

Chapter 5, “A semiotic look at modeling behavior” by Kehle & Lester, is less successful. The authors intend to introduce Charles Peirce’s notion of *abduction*. This is a good idea, but they proceed to do it in conjunction with the theory of signs, which makes it far more complicated than explaining it as a form of inference, which is what Peirce himself did (Peirce, 1931-35, vol.5, p.189)². In a footnote they say that Peirce “wrote in a style often as obscure as his ideas are stimulating” (p.100); their own

exposition, based exclusively on secondary sources, does little to clarify the concept of abduction and distorts it by suggesting that it is always based on the use of an analogy.

The next five chapters expound and exemplify the M&M approach to teacher development, which is here defined very well as “the creation and continued refinement of sophisticated models or ways of interpreting the situations of teaching, learning and problem solving” (p.126). The main targets proposed for teachers’ progress are three: making significant observations about students, identifying the strengths and weaknesses of students’ results that could be used in assessment, developing ways for students to assess, revise, and refine their own work (p.149). This clearly is a big advance from traditional notions such as that all elementary school teachers have to know is the multiplication table and the standard algorithms for the four basic arithmetical operations.

Chapters 11 to 16 extend the horizon of the book beyond school. They present detailed arguments for why and how the M&M approach to mathematics teaching could benefit professional abilities in areas such as engineering, technology in general, economics, and others. The effort to shift the focus of teachers’ attention from the traditional “behavioral objectives” to the development of cognitive skills is certainly worth making – even if it will take time to change attitudes in commercial establishments

Chapter 17, “Ends-in-View Problems” by English and Lesh, explains the term by giving the example of a physician who diagnoses what ails his patient. His goal is to cure the patient, i.e., he should strive to eliminate the illness, not just the symptoms (if only all physicians saw it like that!). Because End-in-View problems do not pre-determine a specific path of solution, they challenge students to explore possibilities and thus provide different learning opportunities. But as the authors admit, more research is needed to clarify just how such learning takes place.

The advantages of group learning are illustrated in chapters 18 and 19 (Zawojewski & Lesh, and Zawojewski, Lesh, & English) at the hand of extended protocols that show very clearly how much reticence and patience are required on the part of the teacher if students are really allowed to solve a problem by themselves.

Chapter 20 by Harel & Lesh has the title “Development of Proof Schemes” and deals, not with formal proof, but with “what convinces a student about the validity of a claim” (p.361). The authors examine the processes that lead to the relevant cognitive structures in the course of three students interaction in the context of the fascinating “Smart Shadows Problem”. They conclude that Piaget’s analysis of reflection and abstraction does provide a reasonable account of the construction of some of the most important mental operations.

A critique of metacognition, the influence of affective and motivational states of students, the role of individual differences, and an application of the “Lesh Translation Model” are discussed in chapters 21-24. All

four strive to make the M&M approach to learning and its constructivist roots accessible for teachers. It is to be hoped that teachers’ progress will indicate the extent to which these chapters succeed.

Chapter 25 reports research carried out by Carlson, Larsen, and Lesh on students’ reasoning about change and the covariation of two measurements in particular. Although specific results are not given, the authors claim that, owing to the application of the M&M method, the improvement of the students reasoning abilities exceeded expectations. In chapter 26 (Shternberg & Yerushalmy), students struggle with the problem of calculating the braking distance of a car, when it’s reduced speed is given at five points in time before the stop. The protocol shows what a demanding struggle it was and it reveals interesting details of the students’ thinking. It is a little embarrassing that “the symbolic model” (i.e., an equation), which the authors give at the beginning of the chapter, saying that it should be included in “the complete and accurate solution” of the problem, shows a breaking distance of 79.17 km for a car that was doing 72 km/h.

The history of research on problem solving that makes up two thirds of chapter 27 (Lester & Kehle) is focused exclusively on mathematics education and disregards the constructivist work on physics problems by Rosalind Driver, John Clement, Eleanor Duckworth, Whemby & Lochhead, Edith Ackermann, and others. In the rest of the chapter the authors consider the situation today and come to the conclusion that “the roles played by representations and patterns of inference in problem-solving activity and in responses to the model-eliciting tasks described in this volume need more careful study.” (P.511). Their very reasonable main point is that problem solving should not be treated as an independent art form but in conjunction with the other branches of successful thinking.

In the last chapter, L&D present an extensive tabular summary under the heading “In What Ways Do M&M Perspectives Advance Beyond Constructivism?”. It’s a determined effort to accentuate differences and it occasionally goes a little astray. On the second page of this section, they say:

“In contrast to constructivist claims that all knowledge must be constructed, M&M perspectives recognize that there are at least four qualitatively distinct types of instructional objectives that are important in mathematics education – and that not all of these need to be constructed independently by students. They are (a) *behavioral objectives* (BO’s) such as basic facts and skills, (b) *process objectives* (PO’s) such as habits of mind that are not connected to any particular mathematical constructs, (c) *affective objectives* (AO’s) such as attitudes, beliefs, feelings, and (d) *cognitive objectives* (CO’s) such as models and accompanying conceptual systems (contracts) for constructing, describing, explaining, manipulating, and controlling mathematically (or structurally) interesting systems.” (p.533)

The switch from “knowledge” to “instructional objectives that are important in mathematics education” is probably due to a quotation (p.532) from Kilpatrick, an old critic of constructivism. In Ch. 11, the only other place where the instructional objectives are briefly explained, Lesh, Zawojewski, and Carmona state that

(traditional) schools alternatively focus on the first three, but neglect the fourth (p.215). Indeed, this is the reason why constructivists distinguished the *teaching* of conceptual knowledge from the *training* of behaviors. On closer examination, of course, it becomes clear that basic facts and skills, habits of mind, attitudes, beliefs and feelings, cannot be sent as packages from teacher to student. They, too, have to be built up by the learning individual, albeit by means such as imitation, blind repetition, empathy, and others, means that are less demanding than conceptual construction.

On the whole, what distinguishes constructivism from the M&M approach is not quite as stark as L&D's final summary suggests. Their quotation from Davis, Maher, & Noddings (1990) makes the point very well. It states that Constructivism can prescribe guidelines for good teaching but not specific teaching methods (P.533). The book by L&D explicitly suggests teaching methods and ways in which they can be developed. In this, I think, it is successful.

Readers familiar with constructivism will notice that the suggested methods do not conflict with the radical orientation except in one point: the epistemological status of what they intend to teach. With reference to a quote from my exposition of constructivism (1984) L&D say:

Rather than begin with the claim that the reality that you think you see isn't really out there, M&M perspectives begin with much more simple and straight forward (sic) assumptions. (a) people interpret their experiences using models, (b) these models consist of conceptual systems that are expressed using a variety of interacting media (concrete materials, written symbols, spoken language) for constructing, describing, explaining, manipulating or controlling systems that occur in the world. (p.536)

The only difference from the constructivist perspective that I can find in this statement is the claim that the systems that are being constructed, described, explained, etc., "occur in the world". This gives the impression that they are "really out there" rather than in a constructed interpretation of experience.

This may not be a salient point in the teaching of mathematics, because serious students are likely to suspect sooner or later that the field is a human construction and that its rules are what mathematicians have decided them to be. But the situation is different where physics is concerned. At a time when physicists regard, for example, Bohr's model of the atom and the four-dimensional structure of Einstein's universe as limited models rather than as a representation of the real world, it is, I think, a teacher's duty to make this distinction clear. The scientific view of science as the source of Truth about reality is no less dangerous than any religious fundamentalism.

L&D do not share the scientific view. They "draw on many of the most basic ideals that were emphasized by *pragmatists*, while at the same time drawing on the work of modern descendants of Piaget and Vygotsky...." (p.529). They even conjecture that:

"William James surely would urge advocates of *models and modeling perspectives* to continue to contribute to (and draw upon) the wealth of Piaget-inspired and Vygotsky-inspired research that mathematics educators have produced during the past thirty years about the nature of students' developing mathematical constructs and conceptual systems -- and about social dimensions of development." (p.529)

In formulating this conjecture, RL&HD make it clear that their "models"-approach is firmly based on constructivism. It's a welcome reminder because most of the other authors do not emphasize the philosophical similarity of the two orientations. L&D are aware of it and mention it every now and then, but they disregard some important precedents. Thus there is no indication that the cognitive structures that Piaget called "schemes" (not schemas or schemata!) could have been called "models" and that the way they were thought to be constructed by means of iterative assimilation and accommodation is exemplified by some of the protocols in this book.

None of this detracts from the usefulness of such an extensive survey of educational gambits and materials and the readership in the United States, where a sense of the past has barely developed, is not likely to be put off by historically short focus.

The review copy of the book was softbound, which helped me in handling its considerable weight. But the publisher has apparently cut back on proof reading. There are too many typos and not only the kind spell-checkers ignore (Charles Peirce is misspelled, p.362, and also given as "B.Peirce" in the index); "phenomena" is used as a singular, p.495); there are sentences in which a word is missing; and there are other errors that are clearly not the typesetter's. Finally, a warning to the reader: If your attention strayed for a moment while reading the Preface, you will be searching endlessly through the book for "Appendix A, B, C, etc." that are mentioned here and there. Only at the end of p.X are you told that these appendices are "digital" and can be downloaded from a web site. It takes a lot of time to do this – but if you like protocols, it's worth it.

NOTES

- ¹ For Piaget, conceptual structures may be *schemes* but never "schemas" (or "schemata"). Unfortunately this confusion was originated by English translators.
- ² An elaboration of the connection between abduction and the development of mental operations can be found in my "Scheme theory as a key to the learning paradox", in A.Tryphon & J.Vonèche (Eds.), *Working with Piaget: Essays in honour of Bärbel Inhelder*, (139-146). London: Psychology Press, 2001.

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