

## Application of CAS for teaching of integral-transforming theorems<sup>1</sup>

Ildikó Hámori Perjési, Pécs (Hungary)

**Abstract:** Teaching vector calculus (the integral-transforming theorems, in particular) to students of engineering has got many difficulties. On the one hand complex and deep knowledge of vector algebra (in three dimensions) and calculus (in one and many variables) is necessary for understanding, on the other hand it is difficult to illustrate the concepts of vector calculus exclusively by means of blackboard and chalk. In this paper we demonstrate how the line integral, the surface integral, the theorems of Gauss and Stokes are taught with the help of a MAPLE computer algebra system (CAS). Application of this pedagogical method supports the hypothesis that using of CAS makes it possible to teach those kinds of concepts and theorems which are often used in engineering practice but couldn't be presented because of lack of time and appropriate preliminary training.

**Zusammenfassung:** Der Unterricht der Vektoranalysis (und vorwiegend der Sätze, die von den Integraltransformationen handeln) für Ingenieurstudenten verursacht viele Mühe. Einerseits braucht es ein komplexes und gründliches Wissen in Vektoralgebra (in drei Dimensionen) und in Analysis (in ein und zwei Variablen), andererseits ist es schwierig, die Begriffe der Vektoralgebra mit Kreide an der Tafel zu veranschaulichen. Es wird in diesem Artikel demonstriert, wie wir das Linienintegral, das Oberflächeintegral und die Gauss'schen und Stokes'schen Integralsätze mit Hilfe des CAS Program MAPLE lehren. Die Anwendung dieser pädagogischen Methode unterstützt die Hypothese, dass solche - in Praxis oft benutzte aber vorher wegen Zeitmangels und mangelhafter Vorbildung nicht vorkommenden - Begriffe und Sätze mit Hilfe von CAS gelehrt werden können.

**ZDM classification:** D40, I10, M50, M60, M80, N80

### Introduction

Most fundamental changes in mathematics education are from usage of the computer in teaching and learning. Nowadays there is a strong connection between open learning and face-to-face learning, and the computer plays the fundamental role in strengthening this connection. [1] Because of these two positive reasons and some negative ones (poor pre-education, great number of students, decrease of number of lessons, etc) we worked out a new method to teach a special part of mathematics.

Teaching of vectorcalculus for students of engineering has got many difficulties:

- complex and deep knowledge of vectoralgebra (in three dimensions) and calculus (with one and many variables) is necessary for understanding,

- it is difficult to illustrate the concepts of vectorcalculus exclusively by means of blackboard and chalk,
- in some cases, unfortunately, the subjects of engineering apply the concepts of vectorcalculus before they are exactly introduced in mathematics,
- in some cases there is no time in mathematics to teach vectorcalculus.

In the following sections of this paper we try to summarise the advantages of using computer and the computer algebra system by means of our teaching-learning method.

### 1. Computer in mathematics education

Computer is a modern example of cognitive technology; it is a cognitive tool. In the modern pedagogical research it is shown that the basic goal in the teaching-learning process is to organise and reach the optional increase of knowledge with help of connecting the new information with the previous. The two basic steps of learning is the assimilation (integrating new phenomenon into existing mental structure) and the accommodation (adjusting the structures to fit the observed environment). To reach these goals (to accommodate and to assimilate) there are some specialities in teaching of mathematics for undergraduate students especially for students of engineering:

- Communication between mathematics teachers and the undergraduate students is poor in general, because many mathematical ideas are difficult (and lengthy) to verbalise.
- Mathematics is not a tree but like "scaffolding". Once the "scaffolding" is solidly in place, it is not hard to build it higher, but it is impossible to build a layer before previous layers are in place. (Thurston, 1990)
- Mathematics is the language of engineering science, and to learn the engineering subjects students should know this language (at least at middle level) at the beginning of their studies.

So, the „Guided discovery learning” process is necessary for understanding mathematical concepts. This is an essential internal activity in students' mind and represents synthesis of a new cognitive object. Two mental processes can be co-ordinated to form a new one. Interiorization is a process, when students are involved in dealing with externally observed phenomena, encapsulation, where a dynamic process is crystallised as a mental object (e.g., the concept of „integral” can be used on a higher plain, e.g., line- and surface integral).

With the help of these processes students can create schemas, which are coherent collection of mental objects and processes and they can apply them in a new context (we call it as generalisation).

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In this teaching-learning process the teachers' role is changing, but they are still as important persons as they were earlier. Teachers have the knowledge of the curriculum and choose the teaching strategies, are experts (they have to answer question from student), know how to present the material, have an idea of preliminary knowledge of students and can build the „scaffolding”.

How should we use the computer to fulfil the goal that our students acquire the best knowledge in the most effective way [2]?

The usage of computer in mathematics teaching:

- provides a stronger connection between the teacher and students (not only in distance learning) with the help of e-mail,
- makes the teaching materials developed by the teacher or other mathematicians all over the world (Internet) available for creating an interactive learning environment,
- can help to communicate by text, signs, pictures, animations, etc.

Visualisation becomes an integral part of mathematical concept [3], which does not replace the formal definitions and theorems, but rather complements them. With the help of visualisation there is a possibility to make multiple-linked representation between symbols, pictures and common language, the symbolical and graphical representation of concepts.

- with the help of special mathematical software (Computer Algebra Systems) it can develop modular thinking (hypertext, processes, sections links in CAS).

Each computer package is a black box at some level of use, but the question is: what the level should be? Naturally, it depends on the context [4].

- provides an internal language that helps to translate the common language into mathematics. Unfortunately this language is based on English, so it can be a problem for some of the Hungarian students.
- stimulates to learn new topics (CAS has internal processes, which are unknown for students)
- has to avoid a danger: press few buttons instead of thinking
- works as an amplifier, which means that students can understand difficult concepts without being masters of basic mathematical skills (Leave the routine calculations to the computer. Of course it is always relative, what routine means.)
- works as an organiser, it changes the nature of activity, reorganises the knowledge available to the learners.
- gives all students the possibility of not only learning but also doing mathematics.

## 2. Interactive learning environment in teaching

### vectorcalculus

The computer is one of the most important parts of our learning-teaching method.

The participants of this method are (Fig. 1.):

- a web-based syllabus, which contains 10 lessons, multiple choice tests, self-controlling questions, MAPLE worksheets in html format, downloadable MAPLE worksheets and information about the course [5]. It can be reached at the <http://matek.pmmf.hu> Internet address.
- a book, which contains much more physical and mechanical examples using vectorcalculus [6].
- the CAS applications that can be reached on a local network.
- students' own PCs, on which they can use the campus-licensed MAPLE software
- the organiser of the learning-teaching process, who is the teacher. The connection between students and the teacher (tutor) is realised through e-mail. In addition, there is the possibility of a face-to-face meeting every week in the framework of a three-hour long consultation.
- the students, who are in the middle of this learning environment.

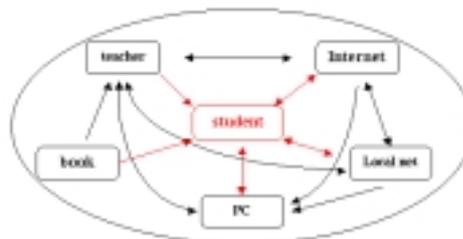


Figure 1: Interactive learning environment

It is hard to decide, which of the tools is the best to fulfil our goal, i.e., that the students accommodate and assimilate the necessary knowledge the most easily and effectively.

Our tools that the participants used in the learning-teaching process of vector-vector functions were (Tab. 1.):

- chalk and talk (CT)  
We used it, when we introduced the basic concepts (definitions), or when we first applied the theorems, processes on the exercises that are easily calculated.
- multimedia (MM)  
It was used to understand the physical meaning of the mathematical concepts. (Fortunately, there is lots of video files about floating of liquids.)
- hyperlink (H)  
We have not got enough time to speak about all definitions, theorems (and their proofs), so they are on the web-based syllabus.

- computer algebra system (CAS)
  - for visualisation (CAS-V)
  - for numerical calculation (CAS-N)
  - for symbolical calculation (CAS-S)

The next part of this paper contains the details of using CAS on the example of teaching Gauss' and Stokes' theorems.

	subject	tool	the main mental activity
1.	vectorfield	CAS-V, MM, H	conceptional understanding
2.a	surface integral	CT,H, CAS-V	assimilation
2.b	calculation of surface integral	CAS-N, H	accommodation
3.a	divergence	CT, MM, H	conceptional understanding
3.b	physical meaning of divergence	MM	assimilation
3.c	calculation of divergence	CT, CAS-N, H	accommodation
4.	Theorem of Gauss	CAS-V, CAS-N, H	generalisation, conceptional understanding
5.a	physical meaning of curl	MM	assimilation
5.b	curl and calculation of curl	CT, CAS-N, H	conceptional understanding
6.	successive application of $\nabla$	CAS-S	generalisation
7.a	line integral	CT, H, CAS-V	assimilation
7.b	calculation of line integral	CAS-N, H	accommodation
8.	potential	H, CAS-S	accommodation
9.	Theorem of Stokes	CAS-V, CAS-N, H	generalisation, conceptional understanding

Table 1: Applied methods in teaching vector-vector functions

### 3. CAS in Theorems of Gauss and Stokes

#### 3.1 Modules of teaching Gauss' theorem

There are four main parts of the MAPLE worksheet:

1. Visualisation (CAS-V) of
  - 1.1 the closed surface (Fig. 2, Pict. No.1),
  - 1.2 the vector- vector field (Fig. 2, Pict. No.2),
  - 1.3 the surface and the field together (Fig, 2, Pict. No. 3 and
  - 1.4 those vectors, which originate from the surface (Fig. 2, Pict. No. 4) . MAPLE has no built in procedure to do it. The following commands show, how we solved this problem, when the surface was a cylinder , and the field was  $field:=x,2y,z$ . The MAPLE commands are:

```
nyilak:=NULL:
for j from 0 to 5 do
for i from 0 to 5 do
pont:=evalf(subs({u=a+i*du,v=a+j*dv},cylinder):
irany:=subs(op(pont),field):
hossz:=linalg[norm](irany,2):
if hossz<>0 then
vegpoint:=[rhs(pont[1]),rhs(pont[2]),rhs(pont[3])]+ir
any:
nyil:=arrow([rhs(pont[1]),rhs(pont[2]),rhs(pont[3])],
vegpoint,1,2,0.1):
nyilak:=display([nyilak,nyil]):
fi:
od:
od:
surface:=plot3d(map(rhs,cylinder),u=-10..10,v=-
0..10, orientation=[-75,55], style=hidden):
display([nyilak,surface],orientation=[170,-
126],scaling=constrained);
```

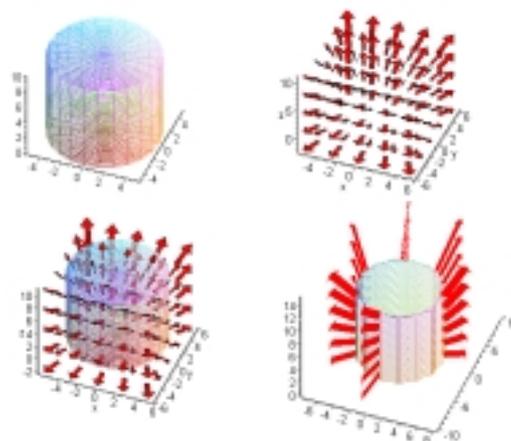


Figure 2. Vectorfield on a closed surface

## 2. Calculation of surface integral (CAS-N)

- 2.1 Partial derivation of the surface's two-dimensional dimensional vector-scalar function (in MAPLE the command is *diff*).
- 2.2 Calculation of the surface normal vector (*crossprod*).
- 2.3 Localization of field into the surface (*subs*). The visualization (Fig. 2, Pict. No. 4) helps to understand this process.
- 2.4 Scalar production of the localized field and the normal vector (*dotprod*).
- 2.5 Double integration scalar product (*Doubleint, value*).

In our example, when the surface is a cylinder, this calculation has to be done three times: (a) for the curved surface, (b) for the basic and (c) for the top plane of the cylinder. At the end we have to add the three surface integrals.

## 3. Calculation the volume integral of divergence of the field (CAS-N)

- 3.1 Determination the divergence of field (*diverge*)
- 3.2 Calculation of volume integral (*Tripleint*). In our example it is easy to do, because the divergence of field is constant.

## 4. Comparison of field's surface integral and divergence of field's volume integral.

After this well-worked-out example there are other examples to be done alone. It does not only mean to substitute an other field and an other surface. For example, if the surface is a cube and the divergence is not constant, the triple integral has to be calculated with the help of *Tripleint* and *value* commands.

These example are good for generalizing the statement, that in some conditions the vector-vector function's surface integral on a closed surface and the volume integral of divergence of this function are equal, when the surface is the boundary of the volume.

After these examples naturally it is necessary to proof the exact theorem, but not in the class. It is to be found on the Internet in hypertext form, or in the book. After studying the proof students have got the conceptual understanding.

### 3.2 Modules of teaching Stokes' theorem

There are four main part of the MAPLE worksheet:

1. Visualization (CAS-V) of
  - 1.1 the closed curve
  - 1.2 the vector field

### 1.3 the curve and the field together

(Fig. 3, Pict. No. 1), and

### 1.4 localised vectors on curve and curve together

(Fig. 3, Pict. No. 2). The procedure is similar as it was shown on Gauss' theorem.

### 1.5 surface, curve, which is the boundary of the surface, curl of field localised on surface

(Fig. 3, Pict. No. 4 and No. 6).

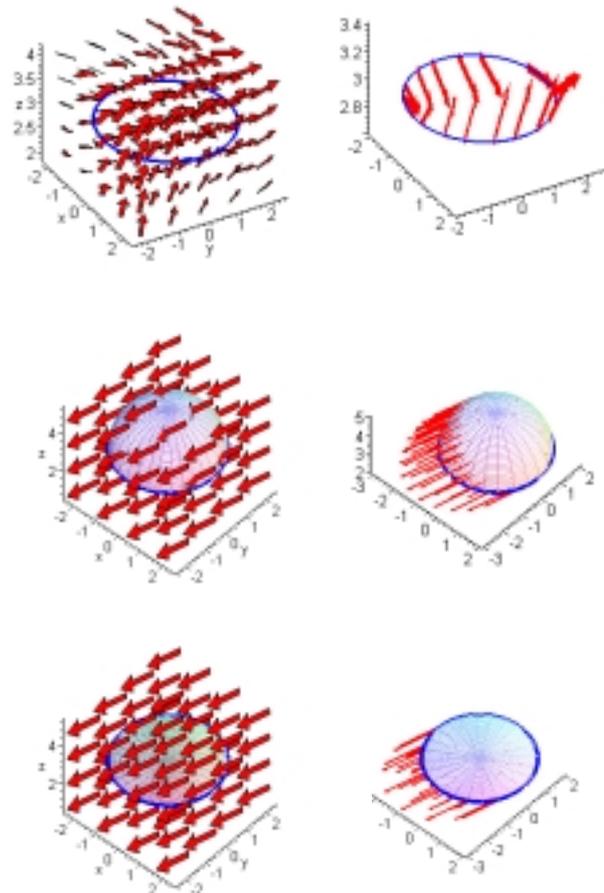


Figure 3: Visualization of a vectorfield on a curve, the curl of vectorfield on a surface, and curve, which is the boundary of the surface

## 2. Calculation of line integral (CAS-N)

- 2.1 Determination of tangent vector of the curve's one-dimensional vector-scalar function (in MAPLE the command is *diff*)
- 2.2 Localization of field into curve (*subs*). The visualization (Fig. 3, Pict. No. 2) helps to understand this process easier.
- 2.3 Scalar production of the localized field and the tangent vector (*dotprod*)
- 2.4 Integration of scalar product (*Int, value*)

3. Calculation of surface integral when the field is the curl of original vector field (CAS-N)
  - 3.1 Giving such a surface in form of two-dimensional vector-scalar function, for which the closed curve is the boundary.
  - 3.2 Determination the curl of the field (*curl*).
  - 3.3 Localization of the curl of field into curve (*subs*). The visualization (Fig. 3, Pict. No. 4 and No. 6) helps to understand this process.
  - 3.4 Partial derivation of the surface's two-dimensional vector-scalar function (*diff*)
  - 3.5 Calculation of surface normal vector (*crossprod*)
  - 3.6 Scalar production of the localized curl of field and the normal vector (*dotprod*)
  - 3.7 Double integration scalar product (*Doubleint, value*)

We made this process on two different surfaces, for which the curve is the boundary.

4. Comparison of curl of field's surface integral and the fields line integral.

After self-supporting exercises could come the generalization, that in some conditions the vector-vector function's line integral on a closed curve and the surface integral of curl of this vector-vector function are equal, when the curve is the boundary of the surface.

## 5. Conclusions

Application of the above mentioned pedagogical method supports the hypothesis that using of CAS makes it possible to teach those kinds of concepts and theorems which are often used in engineering practice but couldn't be presented because of lack of time and appropriate preliminary training.

On the other hand, we hope that we can reach our goal: the students can become active participants in the learning-teaching process. But we would like to underline, that the role and responsibility of the teacher do not decrease using computer in teaching of mathematics. He/she must be a very well prepared organizer and exporter of mathematical knowledge, who knows the input (the preliminary knowledge), the output (what kind of knowledge to reach is the goal) and the method well, which are the most useful to reach the goal the most effectively.

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### Autor

Perjési Hámori, I., University of Pécs, Pollack Mihály Engineering Faculty, Dept. of Mathematics, Boscorkány u. 2., H-7632 Pécs, Hungary,  
e-mail: perjesi@witch.pmmf.hu