Computer Algebra Systems in Mathematics Education
Teacher Training Programs, Challenges and New Aims

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Abstract: The paper gives an overview of the increasing influence of Computer Algebra Systems in Mathematics Education. On one hand the new aims when teaching this new media will be shown. On the other hand examples will illustrate the principles behind the contents chosen for the lectures. Basic ideas of mathematics and computerscience will appear just as well as basic concepts of Mathematics Education.

Kurzreferat: Das Papier diskutiert die stetig anwachsende Bedeutung von Computer Algebra Systemen als Werkzeug für den Mathematikunterricht. Wesentliche Ziele, die beim Unterrichten mit diesem Neuen Medium in den Mittelpunkt rücken, werden vorgestellt und anhand von Beispielen aus verschiedenen Themengebieten illustriert. Fundamentale Ideen von Mathematik und Informatik werden dabei ebenso angesprochen wie grundlegende Prinzipien der Didaktik.

ZDM-Classification: D30, R20, B50

0. Prolog
Computer technologies have been pressing forward in Mathematics Education throughout the passed few years. This process has been mainly influenced by the introduction of Computer Algebra Systems[1]. More and more Mathematicians have been forced to provide models for the use of these new tools.

1. Teacher Training at Universities
First of all Mathematics educators have been addressed to react upon these challenges. They have been invited

to analyse traditional aims in Mathematics and
to evolve practicable courses for teacher students.

Since 1992 I have held several courses on the use of the new computertechnology at Salzburg University. Whereas Computer Algebra Systems were only one aspect among programming, working with spread-sheets or geometry programs in the beginning, the tool Computer Algebra System has been claiming more and more space in courses such as *Informatics for Mathematics Teachers* [FUCHS, 1997]. In this period I was working on my habilitation which I finished in 1998 with the thesis *Computer Algebra – New Perspectives in Teaching Mathematics* [FUCHS 1998].

One of the main outcomes of this work is that if we want to qualify teachers and teacher students to use this new tool we will not only have to train them in the technical use of these systems but to demonstrate them the decisive character of didactical principles and fundamental ideas to reach new specific aims.

Meanwhile I succeeded to convincing teacher students not only at Paris-Lodron- University Salzburg but also at the Leopold-Franzens-University in Innsbruck where I have been guest professor for Mathematics Education for two years and at the Karl-Franzens-University in Graz.

In ambitious theses for gaining a Masters degree which I had the pleasure to attend to [FINK 2001, HOHENWARTER 2002, SILLER 2002, STOCKHAMMER 2001] the students discussed models for the use of Computer Algebra Systems in Mathematics Education.

Additionally a PhD thesis on a special use of MATHEMATICA at Salzburg University has enabled the grammar school teacher Alfred Dominik to co-operate with colleagues from Toyota National College of Technology [YOSHIOKA, FUCHS, NISHIZAWA, DOMINIK 2000].

So let’s focus on some of these new specific aims in teaching Mathematics with Computer Algebra Systems.

2. New Aims

2.1 Modelling
The students’ abilities to describe real world problems and real world processes through Mathematics has been required of Mathematics Education for many years. We have got powerful tools for stepwise modelling with Computer Algebra Systems now.

We can use them for visualization of dynamic processes or make them ‘Calculation Servants’ for numerical and symbolic routines such as generating the values of sequences or symbolic differentiation of complex expressions, activities which are seen to be susceptible to mistakes. If the students change the values of parameters during their process of modelling the system will correct the output list or the shape of the plotted graph immediately. So the students have the chance to concentrate on the influence of the different parameters to the output critically.

Example: Modelling the consumption of fuel of cars with polynomial functions

Step 1: Deriving pairs (velocity / consumption) from a given value – table

tab. 1: Corresponding values

type of car		consumption depending on velocity				
volume	effect	50 km/h	70 km/h	90 km/h	120 km/h	140 km/h
1,1 l	40 kW	4,6 l	5,4 l	6,1 l	8,0 l	12, 0 l
1,1 l	43 kW	4,7 l	5,3 l	6,4 l	8,1 l	10,3 l
1,3 l	51 kW	4,8 l	5,5 l	6,0 l	7,7 l	10, 3 l
1,6 l	58 kW	4,8 l	5,3 l	6,4 l	8,2 l	9,7 l
1,6 l	71 kW	4,9 l	6,0 l	6,9 l	8,9 l	11,2 l

[1] The notion Computer Algebra Systems stands for a wide range of computersystems which can be charakterized by functions such as symbolic differentation [DAVENPORT 1994] or integration of rational functions [KUTZLER, LICHTENBERGER, WINKLER 1990].

data ={{50,4.8},{70,5.5},{90,6},{120,7.7},{140,10.3}}

Step 2: Fitting polynomial functions to the given points

Step 2.1 Quadratic $a_0 + a_1 x + a_2 x^2$

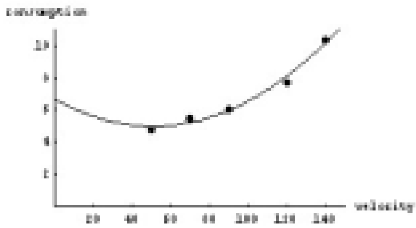

fig. 1: Polynomial fit (2nd degree)

with

obj01=ListPlot[data,PlotRange>{{0,150},
{0,11},Prolog->AbsolutePointSize[5]]
obj02=Plot[quadratic
expression,{x,0,150},PlotRange->{0,11}]

Step 2.2 4th degree polynomial function

$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$

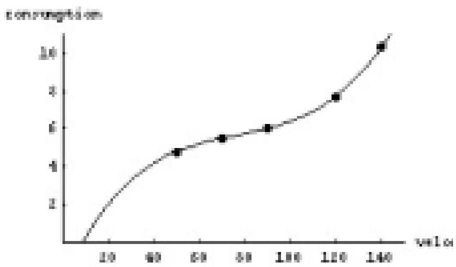

fig. 2: Polynomial fit (4th degree)

with

obj01=ListPlot[data,PlotRange>{{0,150},
{0,11},Prolog->AbsolutePointSize[5]]
obj02=Plot[4th degree polynomial expression,
{x,0,150},PlotRange->{0,11}]

Step 2.3: Interpreting the graphs and improving the model by fitting two quadratics to the given points

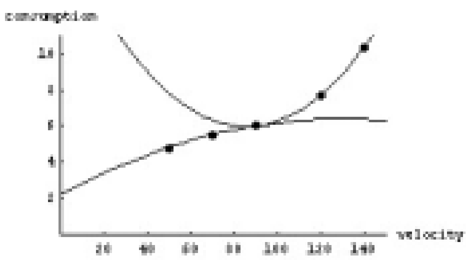

fig. 3: Dividing into parts

with

obj01=ListPlot[data,PlotRange>{{0,150},
{0,11},Prolog->AbsolutePointSize[5]]

and

obj02=Plot[quadratic expression,{x,0,150},PlotRange->{0,11}] fitted to splitdata01

obj03=Plot[quadratic expression,{x,0,150},PlotRange->{0,11}] fitted to splitdata02

2.2 Experimenting, Argumenting and Proving

Additionally Computer Algebra Systems have made an experimental approach possible using its graphical, numerical and symbolical capacities.

The students can experience Mathematical contents when

acting - graphically,
acting - numerically,
acting - symbolically.

First let's focus on **acting - graphically**. Willibald Dörfler [DÖRFLER 1991] points out that a central aspect of doing Mathematics is to find out general ideas which he called prototypes. Computer Algebra Systems help the students to trace out these prototypes. **Acting - graphically** the students can do transformations of graphs (of real functions) very easily on one hand and on the other hand they can concentrate on the central attributes of the Mathematical objects.

Example: Animated films generated with MAPLE

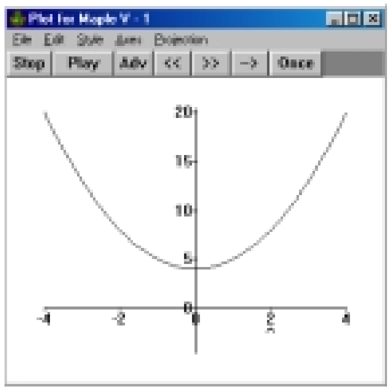

fig. 4: MAPLE animator screen

*obj: f:=x->a*x^2+b*x+c;*

with

subs({a=1,b=0},f(x));

and *animate(x^2+c,x=-4..4,c=-4..4);*

Pressing the Play - button which the students know very well from their walkman they tell the Computer Algebra System to create a film with the parameters x and c in the defined range.

The process of modifying the graphical representation by changing the parameters in our example dynamically will finally lead to a basic insight into the general characteristics of quadratics. Even if the graphs change they always have a parabolian shape.

It is very useful to repeat these experiments with different prototypes of real functions such as trigonometric functions, exponential - and logarithmic functions.

Acting - numerically means that the students can make first insights into Mathematical ideas when operating with numbers.

Example: Heron's Iteration with DERIVE (idea of the convergence)

Iterating the function $f(x)=\frac{1}{2}\left(x+\frac{5}{x}\right)$ with x_0 = 5 10 times will yield the following numerical list:

[5, 3, 2.333333333, 2.238095238, 2.236068895, 2.236067977, 2.236067977, 2.236067977, 2.236067977, 2.236067977, 2.236067977]

which is an approximation for $\sqrt{5}\approx 2.236067977$.

Iterating the function $f(x)=\frac{1}{2}\left(x+\frac{5}{x^2}\right)$ with x_0 = 5 20 times will yield another numerical list

[5, 2.6, 1.669822485, 1.731512614, 1.699607758, 1.715255107, 1.707360713, 1.711289575, 1.709320644, 1.710303974, 1.709812027, 1.710057930, 1.709934960, 1.709996441, 1.709965699, 1.709981070, 1.709973384, 1.709977227, 1.709975306, 1.709976266, 1.709975786]

which is approximately $\sqrt[3]{5}\approx 1.709975946$.

So when iterating the function $f(x)=\frac{1}{2}\left(x+\frac{5}{x^9}\right)$ the students may expect that this process will also stabilize around $\sqrt[10]{5}\approx 1.174618943$.

But iterating the function even 100 times will lead to the following numerical list

[5, 2.50000128, 1.250655996, .., 0.8800075394, 8.338828402, 4.169414214, 2.084713672, 1.045718252, 2.194749560, .., 0.9192503715, 5.793412353, 2.896706516, 1.448527358]

where no stability can be seen. Interested students may be provoked to find out the formula $f(x)=\frac{1}{10}\left(9x+\frac{5}{x^9}\right)$. It should be stable with our parameters x_0 = 5 and n = 10.

Acting - symbolically is based on the most formal Mathematical representation.

Example: Computer Algebra Systems as experts - First insights into differentiation rules (coded with DERIVE)

One of the central attributes of Computer Algebra Systems is the ability of symbolic differentiation [DAVENPORT 1994]. We want to use this function to reinforce the students' motivation to get more insights into differentiation rules.

Entering the sum f(x) + g(x) of two differentiable functions [f(x) :=, g(x) :=] and applying the Calculus option Differentiate to f(x) + g(x) will yield f '(x) + g '(x). So most of the students will suppose that f(x)·g(x) will lead to f '(x)·g '(x). But the answer for the first derivative of f(x)·g(x) will be g(x)·f '(x) + f(x)·g '(x). This can be seen as an initial signal for further proving because either the Computer Algebra System yields wrong expressions or there is something peculiar but most interesting going on [FUCHS 1999].

2.3 The esthetics of graphical representations

On one hand I have pointed out the importance of graphical - representations with Computer Algebra Systems in Experimenting, Argumenting and Proving and for stepwise modelling. But on the other hand the esthetics of plotting threedimensional objects with these systems is worth to be mentioned. The user can produce plots of objects from wireframe-models to hiddenline-models with shading. The aspect that basic knowledge of methods of constructive geometry is necessary when the students are programming the object generating functions reminds us of the important role of constructive geometry in the Mathematics curriculum [FUCHS, VASARHELYI 1998]

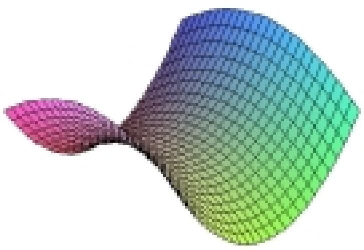

fig: 5: hyperbolic paraboloid

2.4 Aims which influence the contents of Mathematics teaching - Joyful Mathematics

Introducing Computer Algebra Systems will also influence the weight of Mathematical contents. On one hand logic and elements of computer science (such as basic algorithms) will become important, the simulation of complex processes with discrete models (systems of difference equations) will be attainable even for younger students.

Stochastics will get more influence in grammar schools when combining the abilities of spreadsheets with those of Computer Algebra Systems. These are only a few aspects I want to point out concerning Mathematical contents.

The traditional teacher-concentrated lectures in Mathematics cannot be upheld under the influence of Computer Algebra Systems any more. Projects, investigations in groups and solving problems together with partners will take over the traditional role. Teachers will slip into the role of supervisors. New examination models will become unavoidable [see WURNIG 1996, SCHNEIDER 1997]

3. Epilog

New qualifications and standards for Mathematics are discussed in many European countries. The integration of the computer-technologies has accelerated this process because increasing students' activities, new specific aims will make teachers' professionality indispensable. As being a teacher trainer I am actively looking forward with big interest on one hand and exertion on the other hand.

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