

## Euklides dynamic geometry

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**Abstract:** The spreading of computers is becoming more and more inevitable in education as well. Promising pathfinders are the computerised algebraic systems (CAS) and the dynamic geometrical systems (DGS). These tools are very important for students to be able to play, experiment, recognise the problems and ask the questions themselves. If we would give way to more experimenting activities, we could bring those number of students already derailed by the educational system back to the joy of gaining knowledge on their own.

This article introduces the Euklides Dynamic Geometry software, comparing to concurrent softwares and suggesting conceptional questions of dynamic geometry. Important special abilities are animation, track of a point (locus), and tools for examination of structure.

**Kurzreferat:** Die Computer werden in der Forschung und in dem Unterricht immer mehr verwendet. Unter den neuen Anwendungen bilden –neben CAS – die dynamischen geometrischen Systeme (DGS) eine bedeutende Richtung. Diese Mittel geben neue Veranlassung für experimentelle und entdeckende Studium.

Diese Möglichkeiten werden im Artikel mit Vorstellung des geometrischen Konstruktionsprogramms Euklides vorgeführt. Die Fähigkeiten von Euklides werden mit anderen DGS verglichen und aus konzeptioneller Hinsicht untersucht.

Als bedeutende Gegebenheiten sollen die Animationsmöglichkeiten, die Spurlinienbildung und der Apparat für die Analyse der Aufbau von der Konstruktion betont werden.

**ZDM -Classification:** B20, B50, G40, U60, N80

### 1. The genre

#### 1.1 Computerisation everywhere

Just like in so many different areas of life, the spread of computers is becoming increasingly important in education as well. The hotbed of computerised development is research at universities. However, the new approach and the new instruments are not as generally widespread in education, particularly in public educational, as they are, for example, in the business world, which is naturally quicker to respond for changes. The main reason for the educational use of computers, apart from the expense factor, is that a new generation of teachers and a larger number of computer users with no aversion to new instruments are just about to grow up. At the same time software-development is also in search of new ways. Under the label of educational applications there were generally equivocal software published in the first round, and they mainly included static lexical material, resembling computerised books rather than multimedia programs.

There are also software, partly pioneers, partly genre-makers, that apply those new instruments in a creative way, sometimes even reacting upon the original branch of study with new perspectives. Such promising pathfinders in mathematics are the computer algebra systems (e.g.

Maple, Mathematica) and the dynamic geometrical programs software? (e.g. Cabri, Geometer's SketchPad, Cinderella, and Euklides).

#### 1.2 Dynamic Geometry

The dynamic geometry systems are special graphic applications, supporting the mathematical-geometrical approach in order to help teaching, and particularly the study of geometry. Basically they belong to the genre of vector-graphics, but they have to be definitely distinguished from both the illustrating-editing (CorelDraw, Flash), and the engineering-designer systems (CAD) on the basis of their typical.

Dynamic geometry is a kind of virtual construction kit, a computerised realisation of geometric constructions. Some of the advantages of computerisation are the increased comfort level of users and its great accuracy. But that is not the point. A basic feature of this approach is that we can build on geometric objects (a point, straight line, circle, point conic etc.), with a well-defined link of derivation between them. The construction itself is equivalent with a structure and it is not only a static figure. The point of intersection of certain objects can be adopted; several further objects can be built upon them, etc. Specific parts of the constructions can be moved while the derivatives of the constructed objects are also moving but the structure remains stable defined. We can study structures, not only static graphs; we can make conjectures, solve exercises, analyze and make diagrams. Several arrangements or even the best settings of a given construction can be made extremely quickly to a simple static diagram.

In a didactic point of view it is very important for students to be able to play and experiment with such software. It has been the problem of educational settings, that there is usually no time to wait for students to perceive problems and ask questions by themselves. However, answers that would really add to our knowledge are the ones given to adequate questions proposed on our own. I believe that the 'grasp-all-thing'-method is ineffective and it spoils students' relationship with their studies. If we could promote experimentations that would allow discoveries of dead-ends that could achieve greater interest and motivation of students. We could bring those who have already derailed by the educational system back to the joy of gaining knowledge on their own. The time spent for experimentations seems to be absolutely wasted at first glance, but later it becomes worthwhile. For students geometry was always one of the worst parts of mathematics, and constructions were considered as only executions of algorithms. For example, take a compass in hand, open it, strike an arc, etc. It is not stated the the the use of compass and rules becomes unnecessary, but such computerised interactive demonstrations may help students see connections clearer.

The accurate structure and movability required by the software itself enforce students upon thinking; therefore it comprehends a didactic quality. Moreover, constraint of definition forces not only student users to use logic as appropriate, but also it gives the program developers further tasks of research. Questions that were not

available until now would appear, just as: “Let us take the points of intersection of two circles. How could we distinguish or identify them?” “If the point of intersection of two circles disappears when it is moved, what would happen to them? And what would happen to the objects derived from them?” “What would happen if the intersection comes about again, how should they get back?” “Should the theoretical borderline case of osculation exist, or should not?” And those were just some of the questions... Is there an only blissful solution? There may be several correct answers and conceptions available.

Among the widely known pioneers of this genre we have to mention the software Cabri Geometry and Geometer’s SketchPad that are still under development. We should also mention Cinderella and Euklides from the new generation, the latter of which is also the subject of the present article. There are, of course several programs besides the aforementioned, but from among the ones I know those seem to be the most considerable. The present article mostly reviews Euklides, making an effort to emphasise its advantages. However, it is absolutely necessary to state that all of the aforementioned software contain brilliant ideas and advantages compared to the others (as we are speaking about unbeaten paths and making new genres). They also have a high technical standard and they are generally consistent in their own conceptions. The differences between them are conceptual rather than quality categories that may be either combined or justified to coexist in future. I will do my best to be correct during the occasional comparing views on other software is described in this article. However, I must admit that my opinion is subjective, since I may not know the other competitors perfectly. Thereby, for the simple reason to be concerned is my somewhat questionable impartiality.

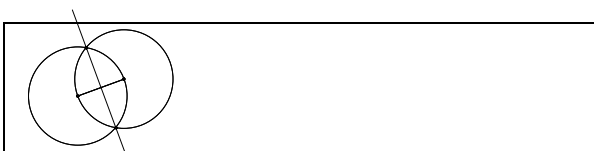
**2. Euklides Dynamic Geometry**

Euklides is a dynamic geometry software developed in Hungary, and it can be a worthy rival for its companions by its abilities, although it is not as well known as the others. In this paper we would like to discuss the availability of the program just to develop a taste to use it.

**2.1. Review**

**2.1.1. Building up a construction**

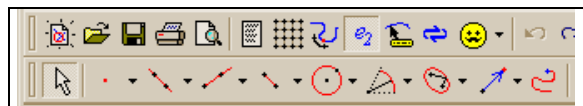
Just to make clear what it is all about, let us present a few simple examples of construction:



*Steps of construction for a perpendicular bisector of a segment*

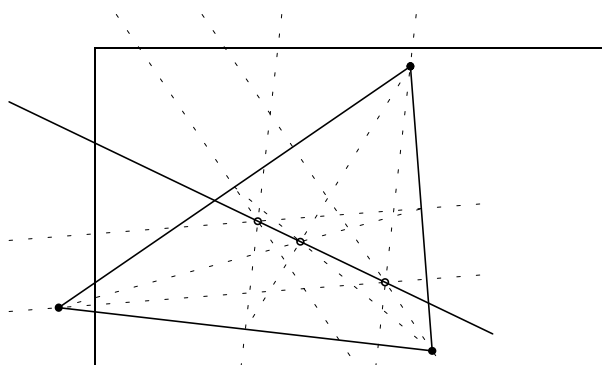
We adopt a segment to two given end-points. To construct its perpendicular bisector, at first we draw two circles from the end-points, practically with a radius as long as the segment, then we set a straight line to the intersection points of the circles. The object we have got

now is a simple construction defined by two base-points.



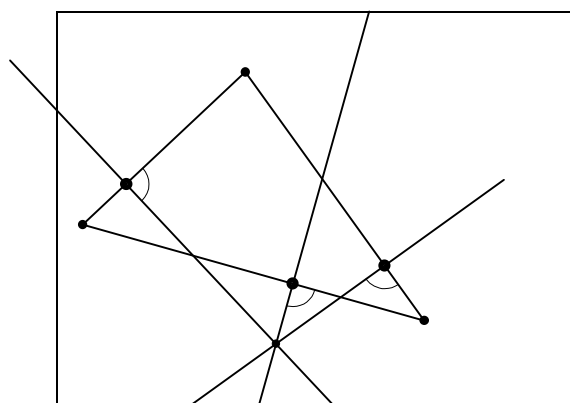
*Part of the toolbar optimised for geometric constructions*

We can build quite complicated constructions on three base-points, such as all the constructions built upon a triangle. Let us see Euler’s line as an example. It can be seen superbly even in a static diagram, that the orthocentre and weightpoint of the triangle and the intersection of the medians fall onto a straight line. It can be continuously seen on the screen when the triangle is moving. When students construct remarkable points then move the triangle; even less able students would come to realise this non-trivial relation.



*Euler’s line*

Let us take another example: we lay off perpendicular lines from a point to the sides of a triangle. How would they be settled? They usually do not fall onto a straight line, but sometimes they do.

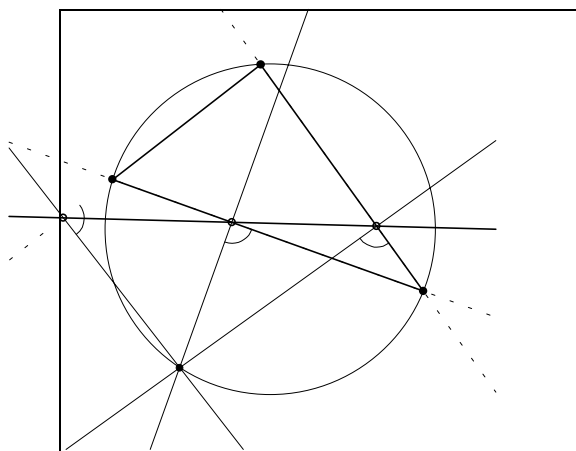


*The base-points of the perpendiculars do not always fall on one line*

If we ask the question students will start experimenting, playing with the point, pulling the object to and fro, and they will realise soon after that they would only get exactly onto one straight line when the initial point is right on the circumscribed circle. With the help of this tool students will discover that problem by themselves.

If we draw a circumscribed circle afterwards, our conjecture is proven. With the help of the gravity function the point can also be “stuck” to the circle in free motion. It can also be moved on the circle, but we can use

animation as well. Thereby students face their own discovery instead of an intruding, maybe dry and uninteresting, forced information, they consider the problem on their own and could be motivated to further examinations.



*Simson's line: a straight line of base points*

### 2.1.2. Object types and the generalisation of derivation

The Euklides version 2.4 can handle the following objects: points (base-point, point of intersection, transformed point), straight line, closed half-line, segment, circle, circle arc, conic sections, and trace. Let us call those *main types*.

In any case, the basic elements of the construction structure are the base-points. These are freely adopted points. The only ones that have no defining objects. Any other object is built directly or indirectly upon the base-points.

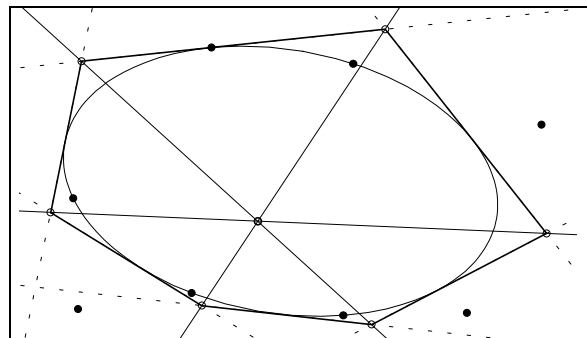
To form other objects we have to appoint the appropriate primary objects distinctively. Details may differ by subtypes within a main type. For example a circle (as a main type) can be built upon two points (centre point, peripheral point), or one point and a radius, or three points and so on. Those are the subtypes of the circle as a main type.

Actually, we do not really need a complete object as object input data, just some kind of parameters, that Euklides can handle generalised. For example a circle given by a centre point and a radius can be derived, say from two different circles, thus, from the first one we get the trivially given position parameter (centre point), and from the other we can derive the default size parameter (radius).

Euklides is also familiar with the basic point-transformations, projection on axis, projection on centre point, translation, rotation, projection, inversion. The features of transformations are evident. If we study them on dynamic constructions effects of a series of transformations can be studied easily.

In addition, the program is also able to handle conic sections. Besides several ellipses, hyperbolas and parabolas there is a possibility to define a general conic section by five points. Polar point of straight lines and polar line of points can be constructed to a given conic section with a built-in tools. Tangency constructions of line will not cause any problem. The construction of

tangents drawn to a general conic segment is a built-in option in Euklides.



*Brianchon's thesis*

### 2.1.3. Visual tools

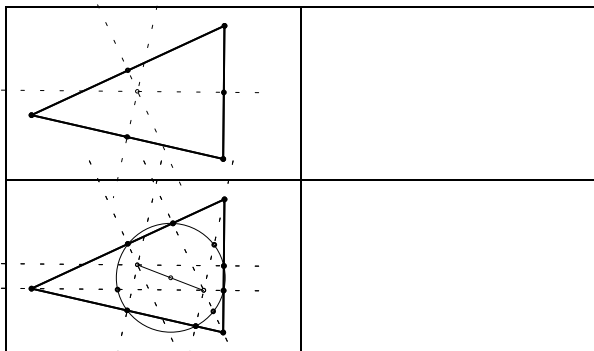
The traditional graphic options known from graphic applications can also be used in Euklides. Because of perspicuousness and aesthetic views, the objects of a diagram can be drawn in different colours, line-styles, line width, and there is also an option to hide non-essential objects.

To promote better understanding, we can also label different objects (cross-points, lateral sides of polygons), and even longer remarks can be displayed. We can use dynamic parameter specification in the labels such as coordinates of points, length of segments, and sizes of angles.

We can zoom into the details of constructions, or in the opposite, in order to review general layout, zoom out as well. Another peculiarity of Euklides is that we can magnify objects proportionally, with or without balanced increase and decrease of labels and line width. With the latter the richness in details of the diagram would increase, and we can study it under a magnifying glass. For example, whether curves actually cross each other in a point to that degree as they seemed in larger resolution.

### 2.1.4. Layers

What the dynamic geometry software generally miss – although it is necessary to make more difficult constructions – is the layer-technique, well known from other graphical applications. The essence of that is to classify objects on imaginary non-transparent layers by functional relations or other aspects and name those groups. The selection and parameter setting of such groups will be quicker and more comfortable. We can shift colours and line-styles or even hide and show the complete content of a layer with a single move.



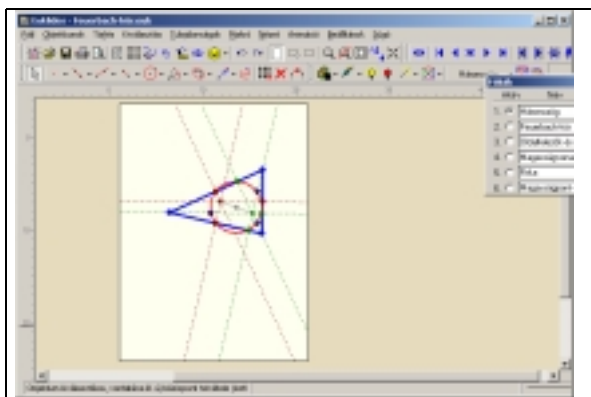
*Feuerbach's circle: the objects are situated on separate attachable layers.*

Thus, for example we can separate the initial data from the ones arising from the final construction. Difficult constructions can be disassembled to several steps and objects related in some point of view can be set off, which is didactically advantageous. It is always advised to build construction on several layers that is also reasonable in simple cases. The option *Making macro*, which will be introduced later is also built upon the logical separation of objects by layers.

**2.1.5. Printing, exporting**

We can make remarkably accurate diagrams with the program. A paper can be displayed on the screen, and arbitrary parts of the construction can be selected for printing.

It is also possible to transfer diagrams to other applications (word processor, graphics program, etc.). With the option WMF-export an arbitrarily selected part of the construction can be saved as standard Windows vectorial format.



*Selecting printer output area on the bench*

**2.1.6. Tools of structure examination**

The review of complicated construction may help to resolve serious difficulties. It can appear if it is your own, and it is especially true when you have to study an unfamiliar construction received from someone else. To resolve the problem, Euklides includes a wide-ranging set of tools to study the structure of constructions. We can ask about an arbitrary object's primevals and derivants. In a complex diagram it can be revealed within minutes that, say the straight line in the middle of the chaos is the perpendicular bisector of which segment, because the

program will display it.

Additionally/Besides that there is a set of tools suitable for moving back and forth manually or automatically, to study the setup of a construction. Up to a point there are certain levels of complexity, when even the most cunning ones may need such service, not to mention students, for whom it may be a great help indeed.

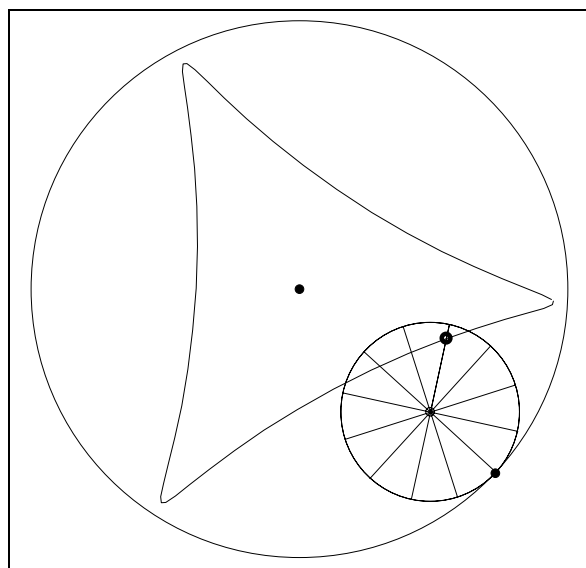
**2.1.7. Macros**

Euklides includes dozens of built-in objects. Some of them are complex such as the inscribed circle of a triangle or lines of tangency directly constructed to a conic section. Users can also define new objects theoretically in any complex resolution. The series of steps stated by the user are called a macro. Practically, a new object is born, it has also initial objects similarly to the built-in objects, and there is a displayable result, which can be a group of several basic object as well. In addition a macro is a totally conventional construction, in which the separation of the objects by function (input, output, hidden partial constructions) is helped by the layers. We can also make a macro directly former constructions.

A correctly constructed macro acts equivalently to ready-made objects and can be used in further constructions. Partial constructions get to a layer especially for a purpose, which can also be displayed if needed. In this way, tremendously difficult constructions can be made very conveniently.

**2.1.8. Track, geometric locus**

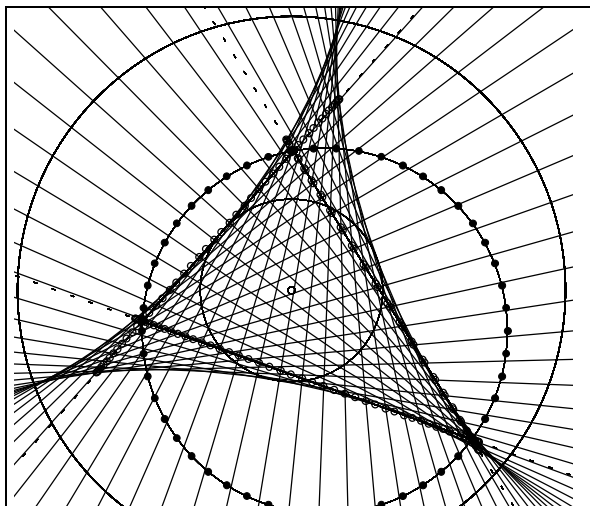
There are very frequent, so-called geometric locus-questions in geometrys such as "what is the geometric locus of those points..." or "how does the point move, if..." In those cases the following option appear to be handy. We run any of the base-points of a construction theoretically along an object (circle, straight line etc.), and the program, building the several phases of moving, displays the movement of any chosen derivative point with a line, just as it were drawn in ink.



*Traces of a marked point of circle third the size of the bigger one in which it is rolling. This diagram can be might as well viewed in motion on the screen.*

### 2.1.9. Animation

The computer offers the opportunity to display physical motion on the screen, so we can transform our constructions into moving animation. For example, in the case of a geometric locus exercise the imaginary movement can be worked out. The parameters of animation (velocity, number of phases) can be set arbitrarily. The real speciality of the program is the option that displays the phases simultaneously. We can see not only the movements but also the totality of the phases' snapshots.



*The multitude of Simson's lines of a given triangle – the simultaneously displayed phases of animation*

### 3. Theoretical and didactical pivots, dilemmas

In the conceptional development of Euklides logical and didactical aspects are emphatically important basic principles. In my opinion Euklides is the most coherent, user-friendly and foolproof among DGSs. So (or I would say even so) we take ourselves on the principle that no comfort function can impair didactically.

#### 3.1 Point adapted to an object

One of the basic principles of Euklides that differentiate it from all the rival software is that the notion of a *point adopted to an object simply does not exist*. So we cannot adopt a point to a straight line or circle, simply because its behaviour throughout movement is undefinable. However, every concurrent program plays it off without a word and moves it by an undefined, unsaid but an eventually educible inner logic. We considered that problem in a conceptionally unacceptable and totally inaccurate step, but we provide an option of forming projections to an object. We can adopt a point to an object by a base-point as a handle of its own projection. This behaviour is well-defined, understandable, calculable, and didactic.

#### 3.2 The question of intersection points

Intersection points are important objects these objects are essential for further objects that are built upon them. One of the central questions of dynamic geometry is the identification of intersection points. The study of their existence and the determination of their behaviour during movement.

These problems do not always exist. For example a circle and a straight line or two circles are not always in intersecting position. So there is not always a real, Euclidean intersection point, although there are generalisations to work with. There may be, for example, points in infinity (ideal points) by which the parallelism of straight lines can be integrated to the system. It is also true that, for example, there are analytical solutions for an equation giving intersection points of circles, even if there is no intersection. Well, Euklides considers all these cases and their derivatives temporarily non-existent. On intermediate level, which is scientifically more than that can be didactically less.

There is another question. Does the derivation system have to include the whole construction in an unequivocally derivable way, or does it have to be a function of the antecedent (movement) of the construction? For example, if we move two intersecting circles (or we bring them into an intersecting position), what should happen? Should they alternate or not? Cinderella, unlike Euklides (and all the others) gives us a very distinct answer. It says that it always depends on the movement, while the others do not. We can have efficient and malfunctioning examples for both conceptions. The absolutely perfect solution may be the optional behaviour of intersection points, but as far as I know, none of the programs capable to provide it.

### 3.3 The model's sphere of operation

The program Euklides can be adapted for Euclidean geometry. But it tries to be as consistently as possible. We are also scheming to do hyperbolic or elliptic operations, but it has not been developed until now. The software can handle only concrete points and coordinates. In that effect the present version tackles far less than Cinderella, which provides virtual points and different views (ViewPort) in one organic geometric system. For example if we set a straight line to the intersection point of two circles, and then we bring them into non-intersecting position, in Cinderella the intersection points will be virtual, but the straight line will be still existing and visible (radical axis). It is a wonderful generalisation fit into higher geometry but it is a problematic question whether it is didactically favourable for primary or secondary school students who have not yet heard of complex numbers and homogenous coordinates.

Euclidean geometry is only one of the several possible geometry systems. It is still the most important system I think that there must be a program, that takes it on and does it well. We can improve the features of the program, but it has to know that kind of behaviour – at least as an optional case, otherwise, even supposing that it contains the most mathematically correct solutions, it may also contain didactical errors as well.

### 4. Contact, information

Euklides geometric construction program is available on the Internet at [www.euklides.hu](http://www.euklides.hu), e-mail: [euklides@euklides.hu](mailto:euklides@euklides.hu).

This page contains all the necessary information about the program in Hungarian and English. The latest version

is a so-called shareware program with certain restrictions. It can be only used in test-mode, can be downloaded from the website. After testing it one can also purchase the complete version. At the moment the program is available in Hungarian and English. It has a detailed Help, which helps to use of the program. Besides there is also available, unfortunately up to now only in Hungarian, an even more detailed 100-page manual, which can be also downloaded in an electronic form. The stationary program also includes several examples of construction.

The former version of the program (v2.02 at present) is available for free, so we can say that Euklides is the highest-standard dynamic program, which is free of charge at the moment. There is also a free supplementary software called Euklides Reader, which is appropriate for opening and studying the newest formats. For example, students in the classroom can study and move the constructions made by their teachers, only that they cannot construct new objects, and they cannot save or print them.

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