Mason, John H.:
Mathematics Teaching Practice
A Guide for University and College Lecturers
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Teaching mathematics at university and college level is rapidly changing. Fewer and fewer students opt for exclusively mathematical studies. At least in the UK, recruitment of good mathematics graduates to mathematics teaching is at an all-time low. Given the substantial gap between secondary and tertiary mathematics teaching approaches, students feel increasingly alienated from the traditionalism of university-level teaching. Moreover universities are more than ever accountable to society regarding the quality of their teaching. By the late 90s most responses to these changes were in terms of modifying the tertiary syllabus. Soon it became evident that reform should be focusing on teaching both in terms of underlying principles and practices. Hence the arrival of this book is highly topical.

It is vital to maintain people's positive experiences of mathematics. This implies understanding mathematics beyond mere manipulation of techniques. This book aims at assisting university and college lecturers in this task. The perspective of the book is that students need to actively engage with mathematics hence the proposed tactics require student participation. Moreover the book sets out from the observation that creativity in mathematics teaching at university level requires the development of faculties that are distinctly different to those of mathematical creativity. This book aims to help with the former but also stress the potential benefits for the latter.

The teacher-student interaction in this book is described in terms of six modes: *expounding; explaining; exploring; examining; exercising; expressing*. Fully-blown use of these modes leads to effective learning, that is to effective use of the powers of mathematics: *imagining and expressing; specialising and generalising; conjecturing and convincing; ordering, classifying* and *characterising*. The mathematical themes mostly addressed via the use of these powers are: *doing and undoing; invariance amid change; freedom and constraint, extending meaning.*

The discussion and suggestions in this book mostly takes place in terms of *tactics*, a word preferred for its nuances of short term goal than long term aim; for it sounds a bit like *tact*; for its evoking of *tacking* in sailing, which is decision-making when *specific* conditions are considered. The interweaving material for the suggested tactics is the belief in active student engagement.

In the opening pages the reader is asked to consider their assumptions about effective learning and teaching by looking at two tables and by ticking issues according to personal priority (and, at the end, to also consider major current concerns with regard to teaching as well as examples that illustrate or contradict the beliefs in the two tables). The author preempts some typical responses (colleagues have been asked by him) and briefly rehearses some of them. The commentary concludes with the claim that we ought to be mathematical actors to enact mathematical behaviour in our students: *not* by mere exposition but by demonstrating the ampleness of mathematical perspectives *and* by striking a contract with them that this teacher-student interaction will not be a typical ride of a transmissive nature and that their part of the deal, engagement, is a crucial one.

Following these general opening remarks the book, in seven chapters and two appendices, focuses on the following themes: Classic Student Difficulties; Lecturing; Tutoring; Task Construction; Marking; Role of History in the Teaching of Mathematics; Synthesis of Issues – Appendices: Examples of Exploratory Tasks; Worked example of teaching a particular topic (convergence of a series). Each chapter starts with an introduction of the theme and concludes with a reflective task for the reader (mostly along the lines of considering and reflecting upon using the proposed tactics). In the following I comment briefly on each chapter.

Classic Student Difficulties.
The chapter kicks off with some pertinent observations: Have you noticed your students making extraordinary errors in routine calculations? Been unimpressed by their weak long-term memory? Their inability to transfer techniques across contexts? Their success in tests but lack of deep understanding…? However well and clearly one thinks one presents material to the students, the results vary according to how they themselves process this exposure. Students' perceptions of what a definition, a lemma, an example etc. is unsurprisingly differ from those of a proficient user of mathematics. Responses to what causes these student difficulties need to go beyond the usual 'tip of the iceberg' approach (e.g. lack of understanding, insufficient practice etc.). Here the complex map of student difficulties is ordered in terms of difficulties with Techniques, Concepts, Logic, Studying, Non-routine Problems and Applications. Within each category descriptions and partial diagnoses (well-grounded in the considerable literature in the area which is discreetly mentioned in the main text and constitutes the main part of the helpfully thematised bibliography at the end of the book) are followed by suggestions (tactics) for helping the students cope with these difficulties. Crucially the above reader-friendly grouping of difficulties, diagnoses and tactics is far from simplistic and compartmentalised. Take, for example, Difficulty C(oncepts)3: Turning compound objects and processes into single objects, a well-trodden focus of numerous research studies, and the corresponding Tactic: Inner Moves. Here the teacher is alerted to the students' possibly limited awareness of the mathematically potent

phrase Let … be a… and to the subtle shifts one performs when dealing with mathematical processes and objects. The recommendation here is that, in addressing the students, these shifts need to take place explicitly. Examples and cross-referencing to at least half a dozen other tactics proposed in the book aim to make this recommendation even more concrete.

Lecturing

To say that lectures seem not to fulfil their purpose in terms of facilitating students' acquisition and retention of an understanding of mathematical topics is an often made point. The chapter engages in detail with how they could. Starting from citing common lecturing formats (such as lectures that resemble or are drafts of textbooks; lectures based on students' queries on material distributed for study at the end of the previous lecture; lectures that present new mathematical topics as emergent from the need to resolve a problem) Mason recommends that, whatever the format, introduction to any topic must vary between going from the specific to the general and vice versa in order to increase students' adaptability and flexibility and enact their participation in their own learning. Justified versatility seems to be a theme running across the chapter. In employing screens (a rich repertory of how to make multiple use of blackboards, overheard projectors, epidiascopes, fixed video cameras, computer screens and smart boards is on offer here) and in introducing diagrams and symbols the emphasis needs to be on the ways in which these resources allow student thinking to focus and mathematical understanding to emerge. E.g. is a diagram transparent in terms of highlighting the various dependencies amongst its elements? Is it generic? Have you engaged with decoding the meaning assigned to a string of symbols on a board? Have you called upon your students' use of their *mental* screens before resorting to a physical one? No less than sixteen tactics are proposed here for doing so. Most relate to enacting student participation. Take, for example, *Tactic: Muddiest and Most Important*, where the students are regularly asked to write down their views on the least clear concept, definition, example etc. or, at the end of a lecture, to write down their views on the salient points made in the lecture. The chapter closes with addressing a host of other lecturing issues and related tactics such as *Punctuation; Being interested and stimulating; Keeping fresh; Handouts; 'Mixed ability'; Encouraging exploration and mathematical thinking;* and, *Providing additional support.*

Tutoring

Tutoring, claims Mason, is the teacher's opportunity to enter the student's world (as opposed to lecturing during which the student most often enters the lecturer's world). Most of the tactics proposed here – the comprehensive coverage of issues includes: *Conjecturing Atmosphere; Scientific Debate; Asking Students Questions; Getting Students to Ask Questions; Worked Examples; Assent – Assert; Collaboration Between Students; General Tactics; Advising Students How to Study;* and, *Structuring* Tutorials – aim at helping students with transgressing a common perception of tutorials as the place where the tutor demonstrates correct solutions and towards a perception of tutorials where they are actively engaged with learning. *Tactic: Anti-funnelling*, aimed to tackle the overall rather unsuccessful tendency of tutors to be 'drawn into a sequence of ever-increasingly specific and explicit questions, searching for something that the student can actually answer' (p75) is a good example of this.

Task Construction

Teaching is mostly about constructing tasks. Moreover tasks and tests are often perceived by students as the locus of consolidation (tests contain the essence of a course). The central distinction in this chapter is amongst tasks for diagnosis, for teaching and for assessing as well as in terms of tasks for the six modes of interaction between teacher and student this book is structured around (see introductory remarks). *Diagnostic, Start Up, Mid-topic, Extension, Mathematical Literacy, Revision and Assessment Tasks* are extensively exemplified here in what is one of the most satisfying parts of the book. Dyrszlag's list of mathematical forms (definitions, examples, facts etc.) and mathematical thinking activities (exemplifying, generalising, etc.) is proposed as an underlying rationale for task construction (with regard to focusing the thought on what type of form or activity each task is serving). This rationale is arguably helpful towards tackling the current vogue, at least in the UK, to specify objectives and expected learning outcomes for every course to such an excruciating degree that the inherent paradox of that act is being ignored: the *didactic tension* that the stronger this specification is, the more likely it is that learners will be able to emulate signs of the to-be-learnt behaviour without understanding. An *alternative formulation* is possible, claims Mason: ways in which the opportunities offered in a course can be used so that certain goals can be expected to be met (he splits these in: encounters, experiences and competencies). The above suggestion, complemented by two tactics, capitalises on the fact that learning tasks have outer, visible, explicitly expressed aims but also have inner aspects which – if revealed in the beginning – will never be discovered by the learner who will not benefit from the process of this discovery (so in this sense these inner aims are cancelled through this exposure). Moreover mathematical tasks can serve the purpose of identifying one's own propensities – learning habits – and thus focusing on modifying the less effective ones. As teachers we tend to label students according to these propensities but in fact it is the behaviour at certain times, not the individuals we ought to be labelling. Doing so allows us to foster multiple behaviours that the students show shortage of (e.g. *Tactic: Boundary Examples* where students are asked to construct examples with an increasing sequence of constraints and then asked to go backwards by constructing examples that *do not* satisfy the constraints) and also go beyond the natural tendency of a teacher to pay more attention and praise to students who think like him or her. And let us not forget, the author concludes, that evidence of absence (e.g. not responding to a question) does not necessarily imply absence of evidence (e.g. that the student knows an answer)…

Marking

Providing feedback is a non-trivial exercise. In this chapter the focus is on allocating marks and providing constructive feedback to students. Poignantly the author advises: *never* allocate uncommented marks - it is not useful at all! Consulting with the other markers about the clarity of the marking scheme, establishing an atmosphere of mutual support and exchanging scripts are proposed here as ways to cope with the potential isolation of the marking role. Moreover by this stage into the book it is of no surprise to the reader that the author recommends engaging students in the marking process too (e.g. *Tactics: Using group convenors* and *Collaborative work*) for the sake of their own (*and* the teacher's) learning. At the heart of the chapter however lies Mason's list of proposed tactics with regard to providing feedback to students in ways that reflect as accurately as possible the student's needs: *Focusing on what is mathematical; Developing a language; Finding something positive to say; Selecting what to mark; Summarising your observations;* and, *Providing a list of common errors or a 'corrected' sample of student argument*.

Role of History in the Teaching of Mathematics

Beyond the use of entertaining but not organically-embedded anecdotes, the author here proposes a rationale for the use of history through revisiting some well-rehearsed arguments in the relevant literature: mathematics as part of our cultural heritage, mathematics as evolving under the influence of cultural and social forces, ontogeny (the individual's learning trajectory) reflecting phylogeny (the community's learning trajectory). Five tactics, supported by examples, complete the chapter (e.g. *Tactic: Exploring controversy* where the development of mathematical ideas is explored in the context of political, philosophical controversies in which these ideas emerged).

Issues and Concerns in Teaching Mathematics

In this last chapter the aim seems to be to draw together the ideas proposed in the previous chapters and let certain theoretical foundations of the propositions be more visible. Mason introduces this shift towards general statement-making with a discussion of whether *there are no theorems in mathematics education*, given the diversity of contingencies in a teaching-learning situation and the idiosyncrasies of human nature (a comment that brings back to mind the *tacking* metaphor from the beginning pages of the book). Perhaps not then in the strictly contemporary sense of the word (expressing a generality etc.) but in its original sense, the Greek *theorein* (a looking, a way of seeing) theorems in mathematics education do exist. They are discussed here in terms of three tensions: *Agenda and expectations* (negotiating expectations; varying pace of coverage; alternating between serialist and holist views of a topic; varying the degree of challenge)*; Doing, Construing and Wanting* (steering a course between knowing and understanding while nurturing all three key perspectives on learning: affective, cognitive and behavioural)*;* and, *Being Subtle and Being Explicit* (considering the paradoxical didactical tension between these two - briefly mentioned above in *Task Construction*). Mason highlights and reinforces the points made across the book on a number of issues including: *the place of definitions, theorems and examples in mathematical exposition* with a particular emphasis on *what is exemplary about an example*, an area his work has extensively focused across a number of years; *the benefits of a conjecturing atmosphere*; *the Vygotskian 'scaffold and fade' model*; and, *motivation* (in particular with a focus on sensitive responses to 'why are we doing this?', on locating and exploiting the surprises and intrigue that are inherent in mathematics and on embroidering exercises that aim to develop technical facility in your teaching without reducing learning to a mechanistic procedure). Significantly the mathematical powers the teacher is recommended to call upon here lie within the student. Apart from the distinct trail of constructivism within this proposition, other frameworks for informing teaching proposed here include: *Michener's distinction* of mathematical understanding in terms of example spaces (start up, reference, model and counter-examples), result spaces (basic, key, culminating, transitional results) and concept spaces (definitions, heuristics, mega principles, counter-principles); and, *Tall and Vinner's Concept Image* framework. The chapter concludes with essence-capturing examples of the Mathematical Themes as promised in the introductory pages.

The book is not to be read cover to cover, warns the author, but to be (re) visited according to need and occasion. As a matter of fact I did exactly the opposite for the purpose of delivering this review within a set deadline and it is fair to say that sometimes I felt overwhelmed by the compactness of tactics and issues raised. However, perhaps more significantly, I increasingly also felt the urge to experience the tactics and their outcomes beyond the virtual context of the book's reflective tasks - with which each chapter ends - but *in vivo*. Which brings me to what I believe to be one of the most poignant contributions of the book: by raising awareness of issues and by proposing relevant tactics, it whets the appetite of the undergraduate mathematics teacher whose concerns the structure and content of the book addresses *directly* (unlike the majority of research-based literature in the area which either stays, safely and often understandably, clear of direct propositions for practice or makes *ad-hoc* propositions). A truly deserving posterity of the book would be an *action research*-based critical evaluation of its proposed tactics generated by undergraduate mathematics teachers themselves!

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