Educational Philosophies and Their Influence on Mathematics Education – An Ethnographic Study in English and German Mathematics Classrooms

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Abstract*:* In the first part of the paper different educational philosophies, developed in England and Germany in the last centuries, are described. In the second part results of an ethnographical study in English and German mathematics classrooms are presented. The study indicates the influence of this different educational philosophies on the educational systems of both countries and on the understanding of mathematics teaching and the teaching practice developed there.

Kurzreferat: Im ersten Teil des Beitrags werden verschiedene Bildungskonzeptionen, wie sie in England und Deutschland in den letzten Jahrhunderten entwickelt wurden, dargestellt. Im zweiten Teil werden Ergebnisse einer ethnographischen Studie im englischen und deutschen Mathematikunterricht beschrieben, die aufzeigen, welchen Einfluss die unterschiedlichen Bildungskonzeptionen auf die Bildungssysteme beider Länder und die dort vertretenen Auffassungen von Mathematikunterricht und die Unterrichtspraxis haben.

ZDM-Classification: A30, A40, C70, D10, D30, D40

1. Introduction

The influence of cultural traditions on education and educational systems has long been recognised going already back to Michael Sadler, who at the beginning of the 20th century visited Germany with a British expert commission and compared the achievements of the Prussian with the British educational system. He wrote:
„In studying foreign systems of Education we should not forget that the things outside the schools matter even more than the things inside the schools, and govern and interpret the things inside. ... A national system of Education ... has in it some of the secret workings of national life. It reflects, while it seeks to remedy, the failings of the national character. By instinct, it often lays special emphasis on those parts of training which the national character particularly needs.“ (Sadler 1902 (1964), p. 310).

Empirical survey within the framework of large international comparative studies, such as the TIMSS accompanying „Survey of Mathematics and Science Opportunities“ (SMSO), are indicating a strong cultural determination of lessons, even in subjects like mathematics and science. Thus the authors of the SMSO are stating:

> “Mathematics and science, unlike culturally e„mbedded subjects such as history and language, are often thought to be a-cultural. For example, many believe ‚numeration is numeration’ – the concept is the same across all contexts. ... One can argue that if there is something universal about mathematics and science content, there should be something universal about the way this content is presented to students. Our results, of course, suggest that this second assumption needs re-evaluating ... Countries have developed their own ways of engaging students in the substance of mathematics and science. There appears to be strong cultural components, even national ideologies, in the teaching of these subjects.“ (Schmidt et al. 1996, p. 132)

Following their analyses French teaching of mathematics emphasises formal knowledge, because French mathematics teachers identify themselves strongly with their disciplinary counterparts at universities. US educators appear bent on prolonging childhood as long as possible, at least as evidenced by the tendency to exposure to more basic and early-introduced topics in mathematics and science well into lower secondary school level.

Pepin (1997) reveals in her analyses also a strong dependence on cultural traditions for mathematics teaching in England, Germany and France. She states that of course it is interesting and exciting to read descriptions of mathematics teaching in other countries, but the aim of comparative education must be to find explanations for the observed differences and similarities. Thus she concludes that comparative education in mathematics should answer the following questions:

> „How can we understand teachers’ practices in the light of what we see and experience. How can we understand teachers’ practices in the light of what we know about the different countries? If we believe that the teaching and learning of mathematics is ‚culturally embedded’, what are the cultural and intellectual underpinnings that influence the teaching and learning of mathematics? Where do the cultural and educational traditions stem from, and how do they feed into the classroom? These and more questions have to be posed and answered, if we want to benefit from comparing teaching and learning mathematics in different countries.“ (Pepin, 2002, p. 246)

In the following it will be shown exemplarily with an ethnographical study about English and German mathematics teaching, what are the educational philosophies found in these two countries that, from an economical and sociological view, have much in common, and how these philosophies influence mathematics teaching.

2. Philosophies of Education in England and Germany

In the field of comparative education there exists only a few comparative studies dealing with educational philosophies and their historical development. One of the first contributions to this was the approach of Lauwerys (1959), who, based on studies about national character and various conceptions to general education, developed historically in various countries, he tried to develop a framework for the classification of different educational philosophies. For this Lauwerys (1959, S. 288ff) distinguishes the attempts of the "Liberal Education", the French "culture générale", the German "Allgemein-bildung", the American "General education" and the Russian "Polytechnicalization".

Proceeding from this, McLean (1990) developed various attempts to explain the different traditions of school knowledge, which proved to be solid enough to demonstrate the cultural embedding of English and

German mathematics teaching.

McLean's study (1990) analyses various theories about curricula and school knowledge traditions from Europe, for which he distinguishes three European traditions, the so-called "encyclopaedic", "humanistic" and "naturalistic" tradition.

The encyclopaedic tradition, which following McLean (1990, p. 16ff, p. 39ff) is found predominantly in the French educational system - on which I refer here in brief because of its relevance in the general discussion – and which is historically rooted in the ideals of the French Revolution: McLean (1990, p. 16) characterises this attempt through the following three principles:

- universality
- rationality
- utility.

Following McLean the principle of universality means, that on the one hand teaching is aimed to transfer as much as possible knowledge from all important subjects to all learners. On the other hand a certain degree of standardisation and homogeneity of the transferred knowledge shall be reached. Rationality which, following McLean, is the highest objective of the encyclopaedic approach, is aimed to enable the learners to understand central ideas, structures, logical and ethic systems, for which the understanding of structures and systems created by reasoning, gains great importance. The principle of utility means that rational knowledge is not important only because of it itself, but because it can be used in social, economic and political organisations. For this, the principle of utility means that general academic education is related to professional studies through the same intellectual principles. However, here McLean observes problems in considering the principle of utility in the French system of education.

For McLean the French system of education is the mainland of encyclopaedism. Concerning its manifestation in French curricula, Mclean states that the principle of universality implies also the homogeneity of the learners' performances and of the quality of schools as central characteristics of the French curriculum. Therefore, in France continuous control of performance standards carried out by teachers, as well as the repetition of class, if learners have not yet achieved the requested standards, are typical. Even though there is specialisation to a certain degree in the upper secondary level - which is contradictory to the principle of universality - in French schools significant core subjects consisting of French, mathematics and natural sciences, one foreign language and civics are kept until the end of school. Certain fields of knowledge are defined as being more or less of value within the encyclopaedic tradition. Especially philosophy and mathematics originally were regarded as the subjects which suited the rational principle most closely. However, in recent times, natural sciences and – because of their logical structure – languages gained more importance. Following McLean, in the French school system the main emphasis is put on the analysis and classification of ideas and their embedding into a coherent all-embracing system. Therefore, in his opinion the aim of the curriculum is that pupils shall be enabled to master the relations between the concrete and the abstract, i.e. they shall comprehend, that the understanding of the concrete is based on insight into abstract theories, which are necessary for that. Thus French mathematics teaching emphasises the exact representation of theorems and rules (often through frontal teaching), combined with a fast progress in order to get insight into the structures and uniqueness of the whole domain of the relevant knowledge.

As a second important current of European school systems McLean(1990) defines the "humanist perspective in education"[1] (p. 25), meaning the development of human virtues, which does not only include the development of understanding, compassion and charity, but courage, intelligence and eloquence. He characterises it as follows:

"It had the aim of linking thought and action in ways that would encourage the grandest of human possibilities in the individual and would project the highest possibilities of the individual into the state at large. ... It focused on the individual rather than the social group. It was moral in its emphasis on the development of human virtue but this morality was extended to include aesthetic appreciation and sensibility." (p. 25)

McLean (1990. p. 25) describes where this "humanist perspective", is found, especially in the educational systems of England and Wales, by using three kinds of principles, which are:

- morality
- individualism
- specialism.

Below I will refer to this view more in detail, in connection with the description of educational conceptions that developed within the English system of education.

As a third European educational tradition McLean (1990) defines the "naturalist views" (p. 32), which unfortunately he is not able to describe precisely by using joint principles, while stating:

"Naturalism is shorthand for a variety of individual and community orientated views which have challenged the other two dominant traditions." (p. 15)

McLean (1990) subsumes under this view child-centred epistemologies, which have especially in the Anglo-Saxon countries great influence, as well as work- and community-centred approaches. He locates the community-centred approaches predominantly in Denmark, while in his opinion work-centred traditions of education are especially significant in the German-speaking regions. McLean refers especially to Kerschensteiner's approach about the relevance of practical profession-oriented knowledge (p. 36).

However, at a whole McLean has great problems to include the educational conceptions of the German

[1] McLean uses in this context the term "Humanism" which has not the same meaning as "Humanismus", the term developed in the German-language area. The English term "Humanism" is much more influenced by aristocratic culture, while the German "Humanismus" always deals with the forming of humankind (at least at a theoretical level). Therefore in the following, when Mc Lean's perception is meant, the term "humanism" will be put into brackets or the expression humanistic-aristocratic will be used.

educational system into his structure. He connects the tripartite school system of the secondary level and, depending on the educational branch, with the three European mainstreams of educational philosophy: Thus he classifies the *Gymnasium* (higher type secondary school) as being "humanistic", the *Realschule* (intermediate type secondary school) as "encyclopaedic" and the *Hauptschule* (lower type secondary school)[2] as being "naturalist", while in his opinion, over time these three forms of schools changed towards a kind of "uniform encyclopaedism". However, to some degree McLean delimites this classification concerning its suitability and distinguishes German conceptions of education from the French encyclopaedism as follows:

> "Yet German encyclopaedism is an inferior version of a French model. Rationalism is not so central to German conceptions of ultimate knowing and thinking as it is to the French. The universalist principle becomes predominant. The knowledge diet is expanded continuously." (p. 79) And later on: "German education has an individualist tradition which is stronger, for instance, than that of France. Teachers do habitually develop close and sympathetic relations with individual children. Practical experience is drawn upon wherever possible in teaching." (p. 83)

However in my opinion, the classification of the *Gymnasium* tradition as being "encylopaedic" does not meet Humboldt's - or say the German - conception of general education which is based essentially on the selection of eduacting contents and the avoidance of too many subjects. In the following I will not further discuss this, because McLean overestimates the aspect of the vocational orientation of the German system of education. Of course, this aspect of vocational training in connection with an educational claim has been represented in the context of a modern vocational didactics, but it does not play that central role McLean gives to it with reference to the approach of Kerschensteiner. For this reason below under section 2.2 an own attempt to describe different approaches for eductional philosophies underpinning German education will be developed[3].

Central characteristics of the various conceptions of education as identified by McLean are found among others in the brief descriptions of Kästner (1994) about different European ideals of education:

> "The French ideal of education is still devoted to the spirit of Descartes. Rationality und intellectuality are its predominant characteristics. The highest criterion is the mastering of the subject content including the perfect representation of it. ... The English ideal of education is completely different to the French indeed. Since Locke the building of character and the forming of personality are most important. Education towards a well-mannered behaviour ranks higher than the accumulation of knowledge. Among the four treasures of education, as there are virtue, wisdom, good way of living and knowledge (Locke), the latter has the lowest value. Understanding and judgements preferably are conveyed through experience and valued concerning their experiental potential. .... The classic German and English ideal of education share the image of human beings as a personality, but the German ideal emphasises the unity of intellectual strength and mental insight. With the French ideal of education the German ideal has in common the forming through content, which should be of educational importance. ... In the republican Germany of the last century, the ideal citizen in state and society who uses his rights, who fulfills his obligations and performs his profession, has become a model of education for personality." (p. 36f)

[2] In the following the German notations of the tripartite system will be used.

[3] I will not refer to the approach of Jahnke (1990), who discusses in his study the development of mathematical knowledge in the Humboldt era in general and does only briefly refer to the educational proposals of Humboldt.

2.1. *Educational Philosophies of the English System of Education*

McLean (1990) characterises the English ‚humanistic' educational philosophy as follows:
The first principle "morality" he defines more precisely by referring to the ideal of the Christian gentleman:

> "Nineteenth-century upper-class education set out to create 'Christian gentlemen'. The qualities to be needed by this élite were moral sensibility, a commitment to duty and a capacity for decision-making based on action informed and moderated by contemplation. The explicit model was the philosopher-ruler class of Plato's Republic." (McLean, 1990, S. 25)

Therefore, the formation of the elite was not exclusively based on academic education, but also on the school's organisation, relation to teachers and to class-mates, who were regarded as educators too. Educational purpose was originally only accepted for old languages and literature, while mathematics and natural sciences only had a lower status, because within this value system these subjects did not contribute much to the development of moral virtues. Technical vocational orientated subjects did not have any meaning, among others because of the separation of general education and vocational training. (named "Education vs. Training Dichotomy"). The problematic relation of general education and vocational training is caused by the historic fact that a close relation between general education and working environment always was rejected:

"But work-cultures in Britain have never had close contacts with formal education as in the German Federal Republic. School teachers believe that schooling provides something higher than the alienating world of employment." (McLean, 1990, p. 105)

In the course of the democratisation of education and the introduction of state schools moral education remained a dominant goal:

"The process of transfer was aided partly by a belief that democratization of the curriculum meant dissemination of élite knowledge to a mass school population." (p. 26f)

This description of the central characteristics of the English educational conceptions as developed by McLean applies with other attempts: Thus Carr & Hartnett (1996) point out the ideal of "Gentlemanliness" in contrast to the development of intellectual abilities.

As second characteristic of the English educational tradition McLean (1990) recognises the principle of

"Individualism"[4], which is of central significance for "Humanism", because: "Virtuous individuals create a moral world." (p. 27). Regarded from a pedagogical point of view and in connection with knowledge transfer this means:

"Humanism entails an individual methodology and pedagogy. The humanist methodology of learning, following Plato, is intuitive. ... Standardized, methodical, systematic learning are not reconcilable with this intuitive view of education. Even elite traditional academic knowledge was acquired at the learner's own pace, ..." (p. 27)

Both, the dominance of individual learning methods and the dominance of individualising attempts in the English pedagogy which, in accordance with its moral purpose, emphasise interaction between learners and teachers, McLean regards as consequences of the principle “individualism”.

"The moral purpose of education gave a great stress to the interaction between teacher and student. Teachers, traditionally in English education, had a pastoral as well as an intellectual function. They had to know their pupils sufficiently well to direct them individually towards total moral development. The tutorial system of Oxford and Cambridge was a model for the rest of the élite education system." (p. 27)

By this a "loco-parentis-role" of the teacher is formulated explicitly, because he or she perceives an educational function in place of the parents. This role or function is historically rooted in the great importance of Boarding school education for children of the upper-class and of the military during the colonial times.

The third principle of the English educational tradition - "Specialism" – comprises following McLean (1990) also "an early and intense concentration on a limited range of specific subjects and even on particular topics within subjects" (p. 28). Following McLean this is what at that time distinguished the English system of education from all other European and non-European educational systems. The humanistic conception of education does not consequently imply for specialisation, but is compatible with it:

"The attainment of moral sensibility does not depend in any way on width of knowledge but on depth of perception and understanding. The addition of other subjects may weaken the intensity of moral enlightenment achieved by student interaction with specialist topics." (p. 28)

Specialisation is also supported by the principle of "Individualism", i.e. learners shall be given – depending on their desires and abilities – the opportunity for specialisation.

> "Hence the maxim that students should be able to concentrate on the subjects for which they have a special aptitude to the neglect of those in which they have little interest or ability." (p. 28)

Following McLean (1990) two characteristics of the English epistemology are the reason for the aspect of specialisation is not found in this way within the humanistic attempts of other European systems:

> "First is the empirical approach to scientific inquiry which had its British roots in the epistemology of Francis Bacon and David Hume but which began to find expression in the development of research in universities in the nineteenth century. ... The inductive, empirical rationale for specialism is that knowledge is created cumulatively and incrementally out of small-scale inquiries from which eventually ... broad conclusions and patterns might emerge." (p. 28)

McLean (1990) points out that this kind of empiricism and induction could develop so far because a rational tradition was missing. He clearly differentiates the English understanding of "Humanism" from those that developed within other traditions:

> "The English version of humanism could be anti-rational. Intuition was more important than reason and intuition could be given scope in depth studies than in broad, schematic intellectual architecture. This empiricism also enriched the self-view of the English as having a bent for practical action." (p. 28)

This empirical pragmatic orientation is rooted in the English empirical-utilitarian philosophy, which to a great part is based on Locke and which at the end of the 18th and the beginning of the 19th century had been adapted in pedagogy by utilitarists and "Philosophic Radicals". In the analyses of Rössner (1984) and Carr & Hartnett (1996) there are found similar references (for further details see Kaiser, 1999, p. 290f).

There is largely consensus in the curriculum discussion, that the humanistic-aristocratic tradition of curriculum and the élite-oriented views in England were put into question during the last decades by children-focused approaches concerning primary school as well as by approaches with vocational references. Despite these two challenges, the humanistic-aristocratic perception is still widely accepted, within the fields of education, economy and society:

> "Yet the ... tradition has not been seriously challenged in planning curricula for the majority of pupils aged 14 and above in the mainstream system. The attitudes not only of teachers but also of university academics and, rather strangely, of employers in their recruitment practices suggest that the essentialist view is still very fully entrenched." (Holmes & McLean, 1989, p. 49)

2.2. Educational Philosophies of the German Educational System

In the following I will concentrate on developments of educational philosophies in the German system of education of the 17th to the 19th century, because the educational conceptions’ and traditions’ bases which are characteristic for the German school system until present, evolved in those times. I distinguish two educational traditions, the tradition of realistic education for the *Volksschulen and Realschulen* (lower and intermediate type secondary school) and the humanistic tradition for

[4] In the following I continue to use the English term "Individualism" with quote marks in order to show that it does not mean the same as for instance in German. The English meaning of "Individualism" includes aspects of social behaviour as virtuousness.

the *Gymnasium* (higher type secondary school)[5] (for details see Kaiser, 1999, p. 296ff).

2.2.1. First development line: Realistic Education

During the 17th century, within the framework of the Age of Enlightenment, the first significant development line of German educational conceptions emerged: the realistic education. "Didactic Reformers" around Johann Amos Comenius, who became important for the whole continental European development, expressed their critical views about the out-dated school system through the keyword "Realism", especially concerning the overestimation of the Graeco-Roman times and the verbalism of curricula. For them content-based knowledge was more important than languages for which was only an intermediate function recognised[6]:

> "Der Schulunterricht sollte realistischer werden, indem er, statt sich auf bloße Worte zu beschränken, die im Wort gemeinten Sachen umfassend und im Lichte einer enzyklopädischen Philosophie zur Geltung bringen möge."

> "School lessons should become more realistic that way that it should not be limited to pure words but extensively accentuate and include all things of the meaning of a word and enlighten them by means of the encyclopaedic philosophy." (Blankertz, 1982, p. 32)

Comenius claimed, that education has to form humankind as a whole. Therefore, schools should not only teach knowledge but also in an overall sense morality and goodliness.

> "Alle alles gründlich zu lehren, das war gesellschaftspolitisch ein unerhört kühner, ja revolutionärer Anspruch, pädagogisch gesehen der noch ungebrochene Optimismus der Aufbruchstunde."

> "To teach everything thoroughly to everybody, this was from a socio-politically point of view outrageously bold, even a revolutionary claim, and pedagogically seen the still unbroken optimism of the uprising." (Blankertz, 1982, p. 35)

The claim "to teach everything thoroughly to everybody" must also be seen in connection with the various age levels, by which Comenius replaced the content-related logic as organising principle in the old school-canon of the *Septem artes liberales* by comprehensibility. Everything must be taught with increasing differentiation, by which the strict succession of the subjects was replaced by simultaneity of contents. This made the introduction of classroom teaching possible, while before lessons were given predominantly to single persons. For this reason compulsory school visit could be introduced. Furthermore, Comenius writes that lessons should instruct pupils to become independent, rules and laws should be discovered by themselves.

Realism, which Comenius defined more methodically, changed also in its content during the 17th century, and therefore became increasingly important to the bourgeoisie. Thus curricula as proposed by the mercantilists, were more oriented towards a special realistic-vocational conception of education for young people of the bourgeoisie. Reform projects of the 18th century which were aimed to enlighten the ordinary folk and bourgeoisie, were characterised by an overall ideal of education, that of "Industriosity". Industriosity differentiates the bourgeois and the aristocratic education in the meaning of "those human qualities which promised to be able to produce gainful employments with enthusiasm and creativity." (Blankertz, 1982, p. 57)

Education towards Industriosity as social virtue and the consistent pushing through of the compulsory education program, which was based on the ideals of the Age of Enlightenment, became realised for the ordinary folk through the idea of industrial schools (*Industrieschule*). The attempt of realistic-vocational education became realised for the bourgeoisie through the *Realschule* (intermediate type secondary school) and *Fachschule* (vocational schools), where technical-economical knowledge was taught.

These different tendencies of the time of Enlightenment as shortly described above, were especially represented by the independent pedagogical movement of philanthropism during the second half of the 18th century. On base of their theory of utilitarian education they justify the practice of the mercantile state, namely the claim that people shall function at a predetermined place**,** which also implied the simultaneity of formal and material education with regard to professional requirements (vocational and class-oriented education).

These developments took place in the context of the pedagogy of Enlightenment, which on the one hand produced a pedagogical practice, where with the introduction of compulsory school visit the problem of a school-theoretical vocational education was solved. On the other hand, as result of this development, there were the "old" schools (Latin school, religious elementary school) together with the "old" non-school based vocational education (apprenticeship in craft and trade) and were set into an opposite position to the “new” schools (i.e. *Realschule, Fachschule, Industrieschule*).

2.2.2. Second line of development: Humanistic Allgemeinbildung

In the middle of the 18th century – at the peak of the time of Enlightenment – when the above described realistic traditions of education developed, a new kind of humanism came up, which challenged the principles of the age of Enlightenment’s pedagogy and led to a new specific German conception of education. This movement of a new humanism (“*Neuhumanismus”*), as it was named by Paulsen (1921) – and which was strongly influenced by Wilhelm von Humboldt – criticised the age of Enlightenment’s pedagogy for its strong focus on vocational training and social rank combined with a social occupation of the individual, and in contrast to this a theory of education of humankind was created.

Starting point of Humboldt’s theoretical approach was the defending of the individual against its social

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[5] A differentiation in relation to the different school branches of the tripartite system seems not to be adequate, because this tripartition of the general education system developed quite late, namely in the 19th century.

[6] Due to the difficulties to translate this verbatim expressions – which are very specific for the classical German pedagogy - into English, we reproduce the original German and the translated formulations.

occupation. Following Blankertz (1982) Humboldt defined education as:

> "Weg der Individualität zu sich selber, dieser Weg aufgefaßt als unendliche Aufgabe, so daß Bildung nicht abschließbar sei, vielmehr das ganze Leben über währe." (p. 101)

> "way of the individuality to one's self, while this way is understood as an endless task, so that education will never be finished, but rather lasting for a whole life." (p. 101)

In this Humboldt understood individuality as an inner power of forming by which a human being transforms his perceptions into one's own nature.

> "Individualität mußte unaustauschbare Einmaligkeit sein, mußte im Streben nach Einzigartigkeit das Ganze sein und war zugleich doch auch die Schranke in bezug auf die Idee des Menschen."

> "Individuality should be an irreplaceable entity and, while striving towards uniqueness one should be the entirety, but simultaneously one was also the barrier concerning the idea of human being" (Blankertz, 1982, p. 102)

Self-liberalisation from external fate should be done through the inwardness of one's own subjectivity, i.e. through (general) education "human being should be strengthened, purified and organised" (Humboldt, 1920, Vol. 13, p. 277), the "main mental powers" (Humboldt, loc. cit., p. 263) should be practised. Therefore, the objective meaning of the tasks a humang being was given with respect to one's education, was thrust into the background by the meaning of the way how to master a task.

> *"Jede Beschäftigung vermag den Menschen zu adeln, ihm eine bestimmte, seiner würdigen Form zu geben ... Nur auf die Art, wie sie betrieben wird, kommt es an" (Humboldt, 1903, vol. 1, p. 118); so soll "durch vollständige Einsicht der streng aufgezählten Gründe, oder durch Erhebung zu einer allgemeingültigen Anschauung (wie die mathematische und ästhetische) die Denk- und Einbildungskraft, und durch beide das Gemüth erhöht ..." werden (Humboldt, 1920, vol. 13, p. 277).*

> "Each activity may dignify a human being, may give one a specific honourable form... Only the way how it is practised is important," (Humboldt, 1903, vol. 1, p. 118); so "through perfect insight into the properly listed reasons, or in rising it onto a level of a generally shared meaning (i.e. concerning mathematics and aesthetics) the intellectual and imaginative capacity, and through both, the mind shall be… risen higher." (Humboldt, 1920, vol. 13, p. 277).

Furthermore, a person should not simply follow one's changing affections; the true purpose of human being is rather more the "highest and proportioned education of one's strengths towards an entirety" (Humboldt, 1903, vol. 1, p. 106). Therefore the "entirety" must be represented in the individual peculiarity. This is what is described as the principle of totality in Humboldt's theory of education.

Following the perception of this view each individual should receive a „complete formation of humankind“ ("*vollständige Menschenbildung*"), which should not be limited neither in time nor in a general sense of meaning.

For Humboldt this means that a pupil should be educated universally, i.e. in linguistics, in history and mathematics as well. He rejects a too early concentration on selected subjects, because very often it does not become clear over a long time which direction a child will chose for its life. However, Humboldt accepted to some degree that emphases are put, if becoming apparent. This attempt of his educational philosophy, which later converted to the base of the new order of the Prussian educational system, Humboldt put in concrete terms. The purpose of school education was for him the "general exercise of the main powers of the mind" ("höchste und proportionierlichste Bildung seiner Kräfte zu einem Ganzen") (Humboldt, loc. cit., p. 263) for which he distinguishes the two following functions:

> "die Uebung der Fähigkeiten, und die Erwerbung der Kenntnisse, ohne welche wissenschaftliche Einsicht und Kunstfertigkeit unmöglich ist. Beide sollen durch ihn vorbereitet; der junge Mensch soll in Stand gesetzt werden, den Stoff, an welchen sich alles eigne Schaffen immer anschliessen muss, theils schon jetzt wirklich zu sammeln, theils künftig nach Gefallen sammeln zu können, und die intellectuell-mechanischen Kräfte auszubilden. Er ist also auf doppelte Weise, einmal mit dem Lernen selbst, dann mit dem Lernen des Lernens beschäftigt."

> "Exercise of capabilities and the acquirement of knowledge without which scientific understanding and skills are impossible. Both shall be prepared by him; a young individual shall be enabled partly really to collect already now the content to which all one's creative doing must be connected, and partly to collect it later following one's own favour pleasure, and to develop the intellectual-mechanical abilities. Therefore one is occupied in a double sense, on the one hand with learning itself, and on the other hand with the learning to learn." (Humboldt, loc.cit., p. 260f)

For the curriculum for general education at the *Gymnasium* Humboldt established three kinds of subjects: the lessons of gymnastic, aesthetic and didactic. Dedicated to the forming of humankind, the didactic lesson should be a preparation for science and university study. For this Humboldt concentrated education on the teaching of philosophy and mathematics, because for him the special function of mathematics was to exercise one's intellectual capacity by gaining insight into the functioning of the methods. He emphasised that it is necessary to concentrate on the most essential and most central, in order not to drown in the fullness of details. He writes: school

> "only must ponder on a harmonic formation of a l l abilities of a pupil; for he or she practises his or her intellectual powers with a as low as possible number of objects from as many as possible sides, to plant all knowledge into the mind only that way that understanding, knowing and intellectual creating does not win its attraction and beauty through outer circumstances but by means of its inner precision. For this and as a preparing exercise of the mind for pure science, mathematics must be practised predominantly, that is to say from the very first exercises for intellectual ability."

> "muss nur auf harmonische Ausbildung a l l e r Fähigkeiten in ihren Zöglingen sinnen; nur seine Kraft in einer möglichst geringen Anzahl von Gegenständen an, so viel möglich, allen Seiten zu üben, und alle Kenntnisse dem Gemüth nur so einpflanzen, dass das Verstehen, Wissen und geistige Schaffen nicht durch äussere Umstände, sondern durch seine innere Präcision, Harmonie und Schönheit Reiz gewinnt. Dazu und zur Vorübung des Kopfes zur reinen Wissenschaft muss vorzüglich die Mathematik und zwar

> von den ersten Uebungen des Denkvermögens an gebraucht werden. " (1903, vol. 10, p. 256)

In this quotation the central role of mathematics in school education that Humboldt gives to it, is fixed explicitly by the word "predominantly (*vorzüglich*)".

Because for Humboldt philosophical education was realised at its best by language lessons basing on classical antiquity, for him the didactic education was realised in language and mathematics lessons, which were regarded as the two central main subjects. Humboldt emphasised that it is important that all subjects convey a general understanding to which one can return later. Therefore, the understanding of the connections was of central importance to Humboldt, and for this reason he refused the mechanical teaching of non-understood formulae without any deeper reasoning. He consequently regarded most people as being able to understand mathematics, even or especially if it is kept "pure" and not related too strongly to real life and if not reduced to formulae. He thought that at the end of school education teaching of mathematics and its application to real life might be possible, in case there are no „special schools" for special further education. He wrote:

> "Mathematische Strenge ist jeder an sich dazu geeignete Kopf, und die meisten sind es, auch ohne vielseitige Bildung fähig, und will man in Ermangelung von Specialschulen aus Noth mehr Anwendungen in den allgemeinen Unterricht mischen, so kann man es gegen das Ende besonders thun. Nur das Reine lasse man rein."

> "Every suitable head is able to practice mathematical strength, and that is the majority, even without a versatile education, and if one wishes to mix more applications into general lessons because of missing special schools, one may do so especially towards the end. However, the pure one shall leave pure." (loc. cit., p. 282)

For him this teaching of pure concepts had positive impact on people's moral development, because from this the necessity of a strict and liable procedure becomes obvious, which otherwise, if one does not teach concepts in their pure form, would appear as constraint. In this way mathematics teaching is mainly aimed to give formal insight into the meaning of logical reasoning or into the logical structure of mathematics. Material aimed purposes and real life application of mathematics is of subordinate meaning. Also for language teaching Humboldt formulated mainly formal aims of education for the formation of the intellect in general. Therefore, for teaching at the *Gymnasium* he determined four central subjects, in which pupils should be educated: Latin, Greek, German as mother tongue and Mathematics. And he accepted preferences of pupils for mathematical and natural sciences or languages. Blankertz (1982) named this "an outrageous concentration of contents" (p. 123). This curriculum has stamped the structure of the *Gymnasium* until the seventies of the twentieth century, until the reform of the upper secondary level education.

Concerning elementary education, Humboldt gave the same function to mathematics, in order to train the general intellectual abilities as a contribution to formation of humankind. Therefore, the pupil should not just calculate in mathematics lessons, but get an understanding of general connections. Pupils should not only learn things by heart, because they would not learn how to apply a method of calculation in different situations, but pupils should be enabled by reasoning to understand how the calculation is done. By this pupils should be empowered to apply what they have learned in various situations and not only in the special taught one.

Humboldt's approaches, as described above, were meant as theoretical outlines for the reorganisation of the educational system. Unfortunately, the following real changes of the Prussian school system rather turned Humboldt's ideas right in reverse: The elementary school conception was changed from a "basic education for all people" to folk education for the lower social classes, and "*Allgemeinbildung*" became the education for the bourgeois class in order to save their privileges and to prepare their children for a leading position in state administration and society. Equally, the introduction of the year class system by Johannes Schulze in 1831 contradicted Humboldt's intentions. Humboldt had suggested to allow pupils to attend lessons of selected subjects in different departments, to give them the possibility to progress faster than others in a subjectin which he or she is outstandingly talented. In the decree on the year class system it is said that „if possible a uniform progress in all subjects" shall be achieved (Paulsen, 1921, p. 334). However, it is also mentioned, that this year class system has the "tendency of equalisation" and „mediocrity".

Blankertz (1982) altogether points out, that, despite the first failure of Humboldt's school reform, the reform of Humboldt must be viewed as "starting point of the structures of our educational system" (p. 135).

3. Results of an Ethnographic Study in English and German Mathematics Teaching

3.1. Methodological reflections

The present study is an ethnographic study embedded in a qualitatively oriented paradigm of the social sciences. In the following features of the ethnographical approach are shortly described, before details of the concrete methodical procedure are given. Ethnographical methods have been developed from the method of participant observation, which has a long research tradition in social anthropology and ethnology. Participant observation has been understood as a flexible contextualised strategy, which comprises multiple methods. Ethnographical studies are aimed at a description of a social context, in which people live and work. The main (question) effort is to evaluate, how different social realities are constructed, i.e. how the situation-related means are used by the actors within a social situation, in order to construct social phenomena from a participating perspective. Hammersley (1990) stresses the need to observe actions within the general context of a culture, because events and processes must be explained in terms of their relationship to the context in which they occur. In addition, he emphasises that participants in a social context can interpret and understand their own actions, and construct their responses on their own reflections and their cultural environment. This influences the research methods, which can be used.

The ethnographical view concentrates on those aspects of reality, which are presumed by the actors as self-evident, and asks how the participants manage to create social reality for themselves and for the others. It is therefore unavoidable that ethnographical research views at largely familiar issues as if they would be alien, it takes a detached view on social life dissociated from the observer, Amann & Hirschauer therefore talk about the “Befremdung von der eigenen Kultur” (1997).

The following three aspects can be formulated as characteristics of ethnographical research: Central is the long-term presence in the research field in order to assume an inside perspective. The process of diving into the research field can be described as process of partial enculturation (Amann & Hirschauer 1997). The second characteristics is the flexible research strategy, i.e. the researcher has to adapt his or her methodical approach to the situation and has to find a balance between the research interest and the requirements of the situation. In order to study the culture of the participants under study before producing explanations for their behaviour, participant observation and relatively unstructured interviews are the main ethnographical research methods. Formalisations and standardizations of the research procedures are therefore not adequate, in contrast, the methods consider that researcher and research actions are part of the cultural environment that is examined (Hammersley & Atkinson 1983). The third characteristic is ethnographical writing, which is centrally based on detailed field-notes taken during the observations. Field-notes, which are either made on the spot or written up as soon as possible after leaving the field, have to be seen as interpreted reconstruction of the observations. The question how to evaluate ethnographical data, in order to fulfil necessary scientific standards, has therefore become the focus of interest in the discussion the last few years.

Due to its focus on descriptions of real life and the construction of social phenomena, the ethnographical research approach seems to be especially adequate for the evaluation of mathematics education in England and Germany, its constituents and its determining patterns. Especially bequeathed educational philosophies, which influence the actions of the participants in the educational field significantly, are well-known – sometimes even unconscious - to all actors and are therefore only seldom made explicit. The view into another culture gives us insight into our own teaching culture and the determining constituents. The method of participant observation with its detailed field-notes allows a diving into the field, which is not possible with technically more ambitious research methods. Central basis of the study described afterwards, is, apart from the field-notes of the classroom observations, discussions with teachers, in the staff room during lunch, or after the classroom visits, the participation in school assemblies and discussions with pupils after lesson. Only for the analyses of the teaching-and-learning process, which needed verbatim statements, audio-tape records were made.

In the following I will describe a few more technical details of the study: The study included 17 different schools in England, of which 14 were state-run comprehensive schools and 3 were private schools with selective character, two of the state schools were grant-maintained. The 14 state schools were except one Grammar School comprehensive schools, 4 of the 14 schools were single-sex schools. The schools were spread all over England. The study is limited to the English school system, as the school systems and the educational philosophies in Scotland and Ireland are quite different from the English ones. As already mentioned,_ the study relied heavily on classroom observations, apart from the participation into school life, especially in England. 242 lessons were observed from Year 6 to A-level, mainly restricted to Year 9 to 11. In Germany schools from the three-tier system were included as well as comprehensive schools of different types (using streaming or setting systems). 6 of the schools were situated in the Federal state of Hessen, the others from various regions spread over Germany. 102 lessons were observed from Year 8 to Year 10. The study focussed in both countries on age groups at the end of lower secondary level. Concerning the achievement level of the pupils included, the study puts its emphasis on the two higher tiers of the German school system or of the top sets in the English school system. The reason for this choice points to a principle problem, well known in comparative qualitative studies: Many teachers of both countries were hesitating to open the classroom with lower achievement students for observations by a visitor. The classroom observerations were mainly carried out from 1990-95, further research has shown that mathematics teaching in Germany has not changed in a significant manner afterwards despite the TIMSS shock and political claims of the necessity of change. The English mathematics teaching has undergone a relevant change since the beginning of the nineties of the last century, especially concerning teaching-and-learning methods and the relevance of the subject structure through the introduction of the National curriculum and the accompanying key stage tests. These changes became visible already during the study and are covered by the classroom observations. Newer change is mainly focussing on primary education, which this study does not deal with.

In general, the study aims - as already stated - at generating general knowledge, based on which pedagogical phenomena might be interpreted and partly explained. Under a narrower perspective, the study aims to generate qualitative hypotheses on the differences between teaching mathematics under the educational systems in England and in Germany. Due to the usage of ethnographical method, the study cannot make any 'lawlike' statements; in contrast, the study refers to the approach of the 'ideal typus' developed by Max Weber (*Webersche Idealtypen*), and describes idealised types of mathematics teaching reconstructed from the classroom observations in England and Germany. That means that typical aspects of mathematics teaching are reconstructed on the basis of the whole qualitative studies rather than on one existing empirical case. The ‘ideal typus’ does not really describe empirical phenomena, it is constructed by overemphasising typical issues of single phenomena observed and by a combination of different phenomena (for details see Hempel, 1971, Weber, 1904).

The idealised description of English and German mathematics teaching is formulated in the following study by contrasting descriptions in order to clarify the distinctions made. In the light of further research – especially on French mathematics teaching just finished by Knipping (2003) – these descriptions have to be qualified. In an overall description of European educational approaches in mathematics, France and England might be seen as diametrically opposed to each other with German conceptions having an intermediate position. This holds especially with the aspect of understanding mathematical theory. Until now there exist only first attempts for the development of such a frame (for details see Hino, Kaiser , Knipping 2002). The three-country-study of Pepin (1997) covering France, Germany and England limits itself to the perspective of the teacher and does therefore not provide such a frame, but may be taken as an empirical basis for further research.

3.2. Scientific Knowledge versus Pragmatic Understanding of Theory

From a higher and mathematics teaching entirely comprising perspective, two contrasting characteristics of **German** and **English teaching** can be reconstructed as contrasting poles relating to the understanding of mathematical theory. Thus German mathematics teaching can be described by the ideal type characteristic of a scientific understanding of theory, including its graduations within the framework of the tripartite school system which is prevailing in Germany. This means that theoretical mathematical considerations are of great importance. From a ideal type perspective, English mathematics teaching can be described by its pragmatic understanding of theory, which means a practical and purpose-depending handling of theory. Differences between the comprehensive school, the dominating kind of school, and the selective school system, which for the most part consists of private schools, could be recognised. However, these differences are not as strong as found in the German tripartite school system. These fundamentally different orientations of German and English mathematics teaching on a level of understanding of theory can be seen from various aspects, such as the introduction of new concepts, the meaning of proof, importance of rules or precise mathematical language, which will be described in the following.

3.3. Subject structure versus Spiral-type Teaching

The characteristic of **German mathematics teaching,** a subject-based understanding of theory, leads to a curriculum whose lessons go along with the subject structure of mathematics. In lessons mathematical concepts and methods are taught in a subject-scheduled order. Lessons start from general concepts and rules, and then proceed with special conclusions and applications. Bigger coherent topic areas are taught, e.g. fractions, percentages, the Pythagoras' theorem and others. These big thematic fields are taught independently of relations to other topic areas, and are later on not referred to again. The subject based-understanding of theory is given shape also by the great importance of mathematical theorems. For this the different school types must be taken into account. The great importance of theorems become more obvious from topic areas of geometry, where the relevance and structure of mathematical theory often shall be demonstrated exemplarily. Thus theorems taught in Year 8 of the *Gymnasium*, such as the theorem of Pythagoras strongly serve to demonstrate and proving mathematical theory scaffolding.

The characteristic of **English mathematics teaching,** which is a pragmatic understanding of theory is apparent from the spiral-shaped structure of mathematics lessons and curricula, which means that mathematical concepts and methods are introduced quite early but on a more elementary level. Later, in higher classes, they are picked up again. This spiral-shaped approach implies that smaller and easily comprehensible topic areas are discussed, which are not taught in a subject-oriented structure. A fast switching from one topic to another is typical for English mathematics lessons. Sometimes even several topics are dealt with at the same time. Altogether, English mathematics teaching is rarely based on a subject-based systematic. The subject structure of the National Curriculum, as it is obligatory since the beginning of the nineties of the twentieth century, did not lead to subject-structured lessons. As the curricular goals in the National Curriculum are strongly individual based, there does not exist any obligatory canon of knowledge, to which teachers could refer to for continuing a topic as scheduled in the spiral-shaped curriculum.

This pragmatic understanding of theory which does not put the subject structure to the foreground, corresponds with the minor meaning of mathematical formulae, rules and theorems, because the creation of mathematical tools is regarded as being more important than structural analyses. Therefore, theorems like Pythagoras' theorem, which play a central role in a subject-structured curriculum, are called *patterns* in English teaching or they are not taught at all. Theorems and and their meaning are not the focus of interest, but for instance the constructive aspect of geometrical contents and the algorithmic function of algebraic contents (formulated as rules and formulae) in connection with problem solving.

3.4. The Introduction of New Mathematical Concepts and Methods

The subject-based understanding of theory of **German mathematics teaching** leads to the high importance of the introduction of new mathematical concepts and the deduction of new methods. Normally this is planned carefully by the teachers or they refer to detailed introductions from the text book. Often mathematical concepts and methods are illustrated by real-world examples, although the real-world examples depend on their purpose, they often appear artificial and far from real life. Partly, the introduction of concepts refer to basic understandings as representatives of the mathematical „nucleus“ of a concept. The introduction of new mathematical concepts is usually done by class discussion, in which the whole learning group participates under the guidance of the teacher. There exist various kinds of teacher guidance. A characteristic of the course of a lesson is, that the newly introduced concepts or methods are formulated in detail by phrases notes on the blackboard, which then is followed by exercises.

Below I will give an example which demonstrates the high value of the introduction of concepts and the connection with exclusively formally meant basic imaginations and the application of introductory real-world examples. This sequence about the introduction af the concept function I observed in a *Realschule* of Year eight:

It starts with a graph about the development of temperature over 24 hours on a working sheet distributed by the teacher to the pupils. The teacher writes on the blackboard "function". Then he asks what the task on the working sheet means and gives the answer by himself by the fact that he writes:

The connection between time and temperature.

time $\rightarrow$ temperature

At first various temperatures for given times are determined together by the pupils in a discussion, then the times are noted in a table. After it has been cleared at what time there is the highest and the lowest temperature, the teacher asks: How many temperatures we could declare for one time? A girl answers: one. The teacher comments on this: This we keep in mind. He writes on the blackboard:

At each time there is only one declaration of temperature.

He states that for this there is a specific name in mathematics and writes:

The assignment time $\rightarrow$ temperature is defined exactly.

He continues that this is to be completed by the name which is already written as headline on the blackboard, and then he writes:

Exactly defined relations are called functions.

Then various contrary examples are discussed in detail. The teacher introduces the notation form of a function and the word equation of a function; then further examples are discussed with all pupils.

The pragmatic understanding of theory of the **English mathematics teaching** influences the importance given to the introduction of new mathematical concepts and methods: Thus English mathematics teaching is characterised by a low importance of the introduction of concepts and methods. This is generally done pragmatically, and often the concepts or methods are given by the teacher just as information or in the style of a recipe. Contents-related information is replaced by referring to calculators or mnemonics. Especially new mathematical methods often are explained and demonstrated through experiments, by drawing and measuring. New mathematical concepts normally are not introduced explicitly but implicitly, in connection with exercise sequences. This corresponds with the fact that the introduction of concepts and methods is done in short class discussion sequences or individually with the text book.

In the following I describe a sequence of a lesson from English mathematics teaching observed in Year 9 of the top set at a comprehensive school, which deals with the introduction of sine and cosine in right angled triangles. The lesson demonstrates the usage of calculators instead of a content-based understanding::

During the first lesson the tangent is introduced as follows: Pupils draw individually various right angled triangles with the same angle X, then the length of the two short sides of a rectangular triangle and their ratio are determined from the drawing. Then, in a discussion guided by the teacher, it is clarified that the ratio of the two short sides of each rectangular triangles with equal angle is always stable. The teacher instructs the pupils to enter the angle X into their calculator, then to press the tan key. The pupils recognise that the already calculated ratio is the same as the X value of tan. By This way the tangent of an angle is defined as ratio of the opposite and adjacent side. Subsequently the pupils worked individually on further exampled.

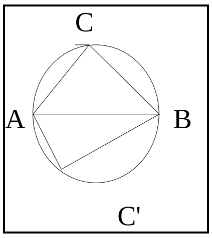
The next lesson continued with a methodically similar introduction of sine and cosine done by individual work. At the end of the lesson a wide variety among the pupils becomes obvious: Some of them are still busy with the exercises about the tangent, others already started with further exercises about sine and cosine.

3.5. The Position and Function of Proof

The understanding of theory in **German mathematics teaching** can be seen from the strong emphasis put on proof in German mathematics teaching, mainly at the *Gymnasium*. A formula-related understanding of proof prevails, while contents-related proofs are carried out quite seldom. There are great differences between the various school types, as proof is done less in the *Realschule* than at the *Gymnasium*, and at the *Hauptschule* they almost do not exist at all. Especially in geometry teaching at the *Gymnasium* great meaning is granted to the carrying out of proof. In this connection the importance of proof within the framework of the structures of mathematical theorems is made clear. For this the need of proof for theorems is of high relevance. Thus its meaning is to explain that experimental, practical proofs are not sufficient for the control of the validity of general statements and therefore formal proofs are necessary.

These characteristics – the great importance of explaining the need of proof for theorems – is demonstrated by the following example from Year 8 of a *Gymnasium*, which deals with the so-called theorem of Thales.

The lesson starts with the construction of the circle of Thales with different triangles, which each pupil does individually. One girl draws this figure on the blackboard.



The teacher asks what is special with it. Some pupils assume that these triangle are always right-angled, others express their doubts about this. The teacher then defines the circle of Thales as special circle and formulates the theorem of Thales as follows:

If we connect the points A, B with a point C on the circle of Thales, then we get a triangle with a right angle at point C (theorem of Thales).

The teacher asks, whether they may write down this phrase like this. A girl refers to the description of the construction. Then the teacher asks once more what the observation of Thales is and how they checked it. One girl says that she controlled the theorem with 3 or 4 examples. A boy states, that it is always

true with any triangle he draws, Further contributions to the discussions the teacher summarises by stating that trying does not help, and he asks what to do. A girl means that they must argue until everybody believes.
Then, quite suddenly - and without any further impuls from the teacher – the central ideas of the proof are given by two pupils, by drawing the connecting line between the centre of the circle and point C and looking at isosceles triangles.

At the end of the lesson the teacher resumes the discussion as follows: It is important to show, that triangles on the circle of Thales have a right angle. He announces the proof for the following lesson.

For a more profound and detailed analysis on proof in German mathematics teaching, I refer to the article of Christine Knipping in this volume.

Formal proof is of low importance **in English mathematics teaching**, both in the selective and in the non-selective school sector. Theorems are often developed experimentally with some examples Teachers do not make clear, that example-related explanations are not sufficient as proof, or that such considerations are not a proof in a formal sense. Very often teachers use the term "proof" for example-related explanations. Consequently, both, pupils and teachers as well do not make a difference between proof and example-related checking. This leads to the fact that many pupils in their own mathematical investigations end their work with example-based checking of formulae or solutions they found out, without trying to find out a general explanation. The low importance of proofs corresponds with the fact that mathematical theorems and methods are quite often announced by the teachers just without any attempt to give reasons for them.

The following sequence of a lesson observed in Year 10 at the Top Set of a comprehensive school shows exemplarily how proof and example-related checking are not distinguished.

In the lesson before this one I observed, various theorems on the size of angles of triangles inscribed in circles have been discovered by the pupils themselves through individual work.

At the beginning of the next lesson, after a repetition, the teacher asks the pupils to start with the practical check of the seize of the peripherical angle of a triangle inscribed into a circle (so-called Umfangswinkelsatz).
"Draw three diagrams with triangles in it, measure the angle and show that it is right, what we said yesterday." He points out that it is important to draw accurately. While the pupils are working individually and the teacher is walking around and helps some pupils he formulated several times:
"Yesterday we delivered the theory, today we will prove it."
After a couple of minutes most of the pupils have finished three drawings and recognised, that the angles in each triangle over a chord are nearly the same. However, as a part of the drawings were done inaccurately, there occurred quite great differences. The teacher asks the pupils what might have been the reason for he has let them done three drawings. Then the following discussion started:
Teacher: What do you think, why I have asked you to do three drawings?
Pupil: In order to have three checks.
Teacher: Do you think, I would be satisfied, if I would have a theory and just check it with three examples?
Several pupils: No, no.
Teacher: So, what do you think, how high is the probability that you all take the same triangle?
Some pupils: Unlikely.
Teacher: So we actually have 60 checks. We would be rather sure, that if our theory has worked for 60 cases, it will work on the whole.

After this exercises from the text are done, which are to be finished as homework. By this the efforts for a „proof" ended.

3.6. Focus on Rules versus Work with Examples

German mathematics teaching can be characterised as rule-oriented, in which rules are manifested by algorithms. Exact execution of arithmetic and algebraic algorithms is highly important. However, there are great differences between the different school types. Thus until Year 8 at the *Gymnasium*, the *Hauptschule* and *Realschule* mainly arithmetic algorithms for the calculation of percentages and interests play a central role. From Year 8 of the Gymnasium algebraic algorithms become increasingly dominant, for instance to transform terms and to solve equations. It is important to many teachers, that pupils are able to execute these algorithms with certainty by heart. Especially in the *Hauptschule* it is regarded important that pupils know by heart those algorithms which have a central meaning, which very often serve as substitute for a more profound understanding of algorithms, which in the opinion of many teachers would mean to overtax weak performing pupils. The transformation of one calculus, which is often noted by several formulae, while at the Gymnasium it is expected that pupils can exceed these formations if needed. However, even at the Gymnasium transformation of terms and binomial formulae are often reduced to their calculus and practised with plenty of exercises. Besides this orientation towards execution of algorithms by heart some teachers tend to emphasise a contents-related understanding of formula and the ability to develop such formulae by themselves.

A further characteristic of German mathematics teaching is the rigid predefined proceeding of problem solving and of working on tasks, which the teacher insists to be done. Thus when working on a mathematical problem, the order of steps is regarded as necessary, even if there is no obvious reason for this. This means, that a algorithms has to be performed in its predefined way, even if from a contents-based understanding or easy numbers another or easier kind of solution is obvious. This leads to rigid and ritual like solution processes. The teachers demands for an exact way to solve a problem– e.g. that arithmetic or algebraic tasks are to be solved with probe or a general formula, that real-world problems are to be answered by a phrase, and geometrical tasks with a description of the construction and a figure done before – and sometimes the teacher even gives exemplary solutions before.

The following sequence of a lesson observed in Year 9 at a *Hauptschule* shows the ritual like character in

executing central algorithms as typical especially for this school type, where own ways of solving are not desired.
The topic of the lesson is linear equations. The teacher asks two pupils to demonstrate their calculations on the blackboard, while the others calculate in their exercise books. After several solutions demonstrated in the way the teacher wanted it, he calls two boys and dictates the following equation: 5+13+2x=22. One of the boys writes on the blackboard:

5+13+2x=22
18+2x=22 |-18
2x=4
x=2

The second boys, who made all transformations mentally, noted the following steps:

5+13+2x=22
x=2

This is revised by the teacher. When the boy is not willing to do the transformation in the predefined order as he does not see the necessity to do so and insists on his solution, the teacher gets angry and tells both pupils to return to their seats. With the next task 27+3x+2x=3 he tells, that at first the factors must be summarised with x.

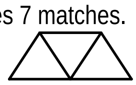
English mathematics teaching can be characterised by its focus on work with examples, with rules and standard algorithms being of minor importance. Central algebraic theorems, such as the binomial formulae, are labelled as patterns and are not identified as general statements. This minor meaning of rules and algorithms coincides with the minor importance of generalisation and general solving schemes in contrast to example-related explanations. Thus solutions are often formulated in connection with examples, and there often do not exist special names for general solution formulae like in German mathematics teaching.

The special meaning of teaching of strategies and methodical knowledge, as well as the emphasis on searching one's own way can be regarded as characteristic too. Teachers seldom teach meta-knowledge on how to solve problems 'economically' - meaning which kind of proceeding should be used -, but rather tend to remain on the level of working on a host of examples. Priority is given to the aspect of finding out one's own way instead of systematic proceedings that should be introduced by the teacher, a fact that underlines the orientation towards the individual of the English educational conceptions.

The following sequence from an English mathematics lesson in a mixed-ability class of Year 7 at a comprehensive school, shows the dominance of one's own way in contrast to teaching systematic knowledge.
The teacher starts the lesson with the following task she writes on the blackboard:

Match stick triangles

Match sticks can be used to make rows of triangles. A row of three triangles uses 7 matches.



Can you find a general rule, which will tell us how many matches we need to make n triangles?

At first it is decided together that many drawings shall be produced and then the results shall be noted in a table. Then the examination shall be continued with four-sided figures. The pupils start individually to work on the problem. It is striking that most of them chose a number of triangles or four-sided figures very arbitrarily. The matches are positioned that way, that for instance at first 8, then 3, then 5 triangles and then 1 triangle are produced. From this kind of proceeding it is impossible to make a connection to the process of production, from which one would easily get the formulae (number of matches with triangles = 1+2n or with squares = 1+3n).
At the end of the lesson almost all pupils have found a rule for the triangles and have started to work on squares.
On my question to the teacher why she did not react on the unsystematic way of the pupils, she answers that it is not intended by the teachers to guide pupils on **one way**, but they rather shall go their **own way**.

3.7. The Role of Mathematical Language

German mathematics teaching can be characterised in an ideal type sense by its great importance of a precise mathematical language in the context of an ‚official' classroom discussion, while the grade of usage differs in the tripartite school system. As of the high meaning of results from the class discussion, this part of communication dominates other forms of communication. Therefore, teachers at the *Gymnasium* generally strive for a mathematically precise and formally correct way of speaking, which they correspondingly also demand from their pupils. This often leads to the fact, that teachers interrupt the pupils' explanations and that they 'offer' correct and formally exact formulations, in order to enable them to formulate mathematically precisely. At the *Realschule*, in practice it is less uniform how important the usage of a mathematically correct language by the pupils is regarded, while at the *Hauptschule* it is generally less important. Especially at the *Gymnasium*, but at the *Realschule* too, mathematical expressions are taken as vocabulary which must be learned by heart, and sometimes this is explicitly the homework to be done.

Furthermore, a formal way of notation is important and characteristic for German mathematics teaching. Therefore, teachers often correct inaccurate notations, which also differ within the tripartite school system. Of course, at the *Gymnasium* it is more important than at the *Realschule* and the *Hauptschule*.

In the following sequence of a lesson in a class of Year 8 of the *Realschule* the difficulties of pupils with precise formulations and the interruptions by the teachers statements to correct the formulations become obvious.
The lesson's topic deals with linear functions with the equation $y = -\frac{1}{2}x$, for which among others function values shall be calculated. The teacher writes the following formula on an overhead transparency:

$$y(14) = -\tfrac{1}{2}(14) = -7$$

He asks, who would like to do this with x=3. He calls a pupil who rose his hand. The pupil starts: 'y at the point is equal'. The teacher interrupts him and claims a more precise way of speaking, which 'one learns like vocabulary'. The teacher says

the exact formulation for the previous value, that: 'y at this point is 14.'
The pupil starts again: 'y at point 3 is equal'. He flags again.
The teacher calls four other pupils, who do not succeed in a correct wording either.
A girl says: 'y at point 14 equals $-\frac{1}{2}$ times 14 equals -7.'
The teacher calls a girl who shall formulate the same for 3, but she says immediately that she did not pay attention.
The teacher calls a boy who says: 'y at point 3 is equal $-\frac{1}{2}$ times 3 equals -1,5.' The teacher writes this on the transparency:

$$y(3) = -\frac{1}{2}(3) = -1{,}5$$

After approximately 10 minutes the teacher continues with the next number. One pupil proposes -3. Then he starts to formulate: 'y at point -3 equals $-\frac{1}{2}$ times minus -3 equals $\frac{3}{2}$.'
The teacher asks the pupils to repeat the sentence once again. Then he states: 'Those of you who have not yet understood, just write it below. Next lesson everybody of you knows it. Learn it by heart!'

In **English mathematics teaching** the development of a collectively accepted terminology with reference to the language of mathematics in the context of 'official' communication is only of minor importance. This aspect is strongly connected with the minor relevance of phases of class discussion in contrast to individual teaching-and-learning styles.

Furthermore, for English mathematics teaching, the minor meaning of formal notation is typical. For many teachers correct and formal notations do not matter, and consequently, on the blackboard there occur quite often mathematical incorrect or negligent notations, such as not using an equals sign in algebraic transformations, and only few teachers correct wrong notations. Selective schools tend to care more for formally correct notations than other schools.

3.8. Role and Relevance of Real-World Examples

Typical for **German mathematics teaching** is a minor meaning of real-world and modelling examples. Real-world examples mainly function as introduction of new mathematical concepts and methods (see description in section 3.4) or are used for exercising mathematical methods. More extensive problems, that are meant to promote extra-mathematical goals, e.g. to develop abilities to master everyday life and to solve extra-mathematical problems by means of mathematics, are rather seldom in everyday school lessons. Normally such problems are only given within the framework of daily or weekly projects. Furthermore, it is typical for German mathematics teaching that real-world examples discussed in lessons are not authentic real-world problems, but made to illustrate mathematical contents. Therefore, these examples give a quite artificial and far from reality impression. Fairly modern mathematical areas, which widely conclude applications, such as graph theory, until now did not enter German mathematics lessons.

English mathematics teaching can be characterised by its fairly high importance of real-world examples, which have various educational functions. They serve to introduce and derive new mathematical concepts and methods, but also to impart abilities that enable the apply mathematics to solve extra-mathematical problems. These abilities are especially supported through coursework, and by projects within the framework of statistics lessons. These projects, integrated into statistics lessons or coursework, deal with real-world examples, and normally they are realistic examples.. However, besides this, wrapper examples are also used for the introduction of new mathematical concepts and methods. A further characteristic of real-world contents in English mathematics teaching is, that more recent mathematical topic areas, such as graph theory and network analysis from discrete mathematics with strong application references, are taught. Handling data, in German named *Stochastik*, is taught intensively and embedded into real-world contents in English mathematics lessons. These real-world examples are often taught through an activity-oriented method, with students doing research tasks they set and then evaluate themselves, often with the aid of computers. The teaching method when dealing with application, strongly depends on the example's function: In connection with the introduction of new mathematical concepts and methods there it is found both, class discussions and individual work. The prevailing method with application-oriented and more extensive projects, is individual work.. Generally speaking, many pupils are used to formulate and to solve problems independently – if necessary with their teacher's help.

In the following a sequence of a lesson is displayed with a class of Year 10, which I observed at a top set in Year 10 at comprehensive school, from which it can be seen how real-world examples are dealt with in the framework of a lesson on the topic handling data.

In the previous lesson, not observed, the pupils had started a statistical project as coursework, for which they should do their own interviews about a topic chosen by themselves. In that lesson most of the pupils had begun to develop a questionnaire. The teacher opens the lesson by explaining to the pupils once more, what he expects from their work, in the meaning that they shall clarify their aims and formulate the questions and that they shall decide which sample and which procedure they want to use. Then the pupils start to work, mainly in groups of two. The teacher sits behind his desk and asks pupils to come to him and discuss the questionnaires. Therefore there is always a short queue of pupils at his desk. The pupils are planning projects on travelling, sports, television attitudes, school fees, cars, magazines. After approximately half an hour most of the pupils have finished a preliminary version of their questionnaire and have already discussed it with the teacher; and then they start a pilot study with their classmates. The final interviews shall be carried out during the following week. Shortly before the lesson ends, the teacher asks the pupils to pack up all their things.

3.9. Teaching-and-Learning Styles

There can be recognised significant differences between the teaching-and-learning styles of German and English mathematics teaching, which are influenced by the different educational conceptions of both countries' educational systems.

In **German mathematics teaching** there can be reconstructed the class discussion in which ideas are developed collectively combined with shorter phases of individual work as dominant teaching-and-learning style. New mathematical concepts and methods are nearly exclusively introduced by the means of a teacher-guided discussion. Individual work is less important, in time and in contents as well. Essentially, individual work serves for working on exercises. Normally lessons are structured that way, that after the lesson has started together with all pupils, tasks are given to the whole class which are then worked out individually. Then, if there is still enough time, the results are compared together with all pupils. Longer phases of individual work occur in connection with drawing and practical and experimental activities. Besides this kind of interaction, class discussion where ideas are developed collectively also can be reconstructed as exclusive style of teaching within one lesson. Differences between the different school forms are obvious by the fact that class discussion as exclusive teaching style is more used at the *Gymnasium* – especially with higher classes – than at the other school types. This kind of lesson is strongly used to introduce new mathematical concepts and methods as well as complex theory-oriented activities like proving. Lessons where pupils only work individually without any class discussion do actually not occur. Teamwork is a teaching-and-learning style which is found very seldom in German mathematics teaching.

In connection with class discussion the discussion among pupils can be reconstructed as a further characteristic, which means that pupils refer to each other, at least for a limited time. Furthermore different grades of teacher guidance, from moderation to authoritarian steering of the discussion, can be described. These attitudes do not only depend on the teacher, but also on the teaching-and-learning situation and the pupils as well. Central part of the class discussion is, in an ideal type description, the work on the blackboard, on which usually the teacher does the notation. Notations on the blackboard done by the pupils occur in connection with homework, for which normally pupils are asked by the teacher, to write their solutions on the blackboard.

Inner differentiation does nearly not exist in German mathematics teaching. In contrast, it can be reconstructed as a central characteristic the joint progress in a learning process within a class discussion, in which ideas are developed together with all pupils. Therefore, within the classroom community pupils are not explicitly 'left behind' nor are they allowed to go on faster than the others. Generally, lessons start and end with class discussion, concerning contents and form as well, while normally is starts with a discussion about the homework.

In **English mathematics teaching** there can be reconstructed as dominant teaching-and-learning style, that the main emphasis is put on long phases with individual work. During theses phases pupils work independently and discover, with the help of individualised learning materials, new mathematical contents or they practice already introduced mathematical contents. Individual work changes– sometimes often – with short phases of a teacher focussed discussion, in which also new mathematical methods are introduced or the results of mathematical tasks are compared. These phases are less important than the individual work, even in time. Furthermore, in an ideal type sense, individual work can be described as exclusively often occurring style of teaching and learning, that is executed by applying individualised learning material and with mixed-ability groups. Preparation for examinations is also often done with individual work. As exclusively seldom occurring teaching-and-learning styles the teacher-centred discussion and the pupil-centred discussion can be reconstructed. In pupil-centred problem solving, diverse activities are carried out, for instance projects, coursework and investigations.

Discussions in English mathematics teaching can be characterised by the teacher's dominance, which means, that the total communication takes place through the teacher, and very seldom pupils refer to each other. Therefore, pupils generally formulate quite restricted questions, which are addressed to the teacher and not to classmates. Consequently, the introduction of new mathematical methods is not done together with the group. Often new mathematical contents are introduced by the teacher quasi with a lecture style. In an ideal type characterisation of English mathematics teaching, the work on the blackboard is of minor relevance. If done, the notation on the board is normally done by the teacher.

English teachers often make internal differentiations in their mathematics teaching. This is necessary for various reasons. In some schools ability-mixed groups are taught together over quite a long time. And individual work produces quite soon a diversification of performance within a group. Individual work also leads to the fact, that very often there does not exist a common start of a lesson, as the pupils continue where they stopped the lesson before.

4. Impact of the educational philosophies on mathematics teaching in England and Germany

4.1. General references

The most significant principle of the English philosophies of education is – as described in section 1 - its orientation towards individual, which strongly dominates the English understanding of education. This kind of individual related orientation is connected with the emphasis on the support of an élite. Therefore, special attempts of education for the masses had not been developed, but these attempts were transferred to education of the majority in a rather diluted form. In German educational conceptions is the meaning of the individual contradictory: in the realistic tradition the individual is subordinate to society and social welfare. The humanistic tradition going back to Humboldt, rejects such an incorporation of the individual into society, but gives special importance to the individual. However, these perceptions emphasise rather an extensive formation of humankind in contrast to the development of the individual, as meant in the English educational tradition. Both traditions – the realistic and the humanistic one –

share an intention to educate the masses, and besides that, an education of the élite. Even if in practice the humanistic *Allgemeinbildung* is reduced to an education of the élite, while in his theoretical conception Humboldt expressed a claim for an extensive education for all social classes.

It is interesting that both, the English and the German educational system justify their specialisation and the shared canon of knowledge with the individual or individuality. This is what distinguishes the German and the English conceptions from the encyclopaedic attempt of the French educational system.

Significant for the English tradition of education is its pragmatism, which led to the fact that rational and theoretical knowledge, which was put into concrete terms by subject based systematic theories, is less important and therefore not taught systematically. Also systematic knowledge of a meta-level which includes strategies and procedures, does not play a central role. German educational traditions are more scientific and therefore focus on sciences. However, this applies more to the humanistic *Allgemeinbildung* than to realistic education. Especially within the framework of humanistic *Allgemeinbildung* an orientation towards contents-based theories are quite important. Both, the realistic tradition of education and the humanistic conception of *Allgemeinbildung* demand the teaching of formal learning objectives, in which learning to learn or the formation of the intellect has a central meaning. The scientific orientation of German educational conceptions became even stronger within the framework of the educational reforms in the sixties and seventies of the twentieth century, by which scientific contents was increasingly incorporated into school lessons, and lessons were constructed in accordance with scientific structures.

The strong individual based orientation as found in the English educational system is accompanied by an individual based differentiation of different levels for each subject (normally done by setting systems). This gives pupils the opportunity, with regard to their abilities and interests, to get on variously fast. Consequently, within one year performances vary strongly, and this helps the forming of an elite. The principle of year classes is characteristic for the German educational system, but not intended by Humboldt's conceptions. It was introduced by Johannes Schulze. This year class principle means that all pupils are to learn the same and to perform equally and that all of one group progress together in the same speed, one could say in "*Gleichschritt*" in the same rhythm. Thus weakly performing pupils are carried along, while strongly performing pupils are blocked in using their capabilities. Altogether this leads to equalisation and mediocrity and prevents the development of an elite.

4.2. References to mathematics teaching

The different basic orientations of English and German educational conceptions with their pragmatism and scientific character, led to great differences of the underlying theoretical understandings in mathematics teaching, which in the second session I labelled as subject- related in contrast to pragmatic. Besides that, these different basic orientations led to the fact, that great emphasis was put on mathematical concepts and methods in German mathematics lessons and mathematical theory and rules are very important, while in English mathematics teaching these aspects are less relevant. Here I give some details on that:

The differing value of mathematical theories and rules become obvious in German mathematics teaching from its scientific systematics, and in English mathematics teaching by the spiral-like proceeding which is rather not dependent on the subject. Additionally, this is underlined by the importance given to proof in German mathematics teaching at the *Gymnasium*, which are associated with strongly formal targets, for instance through exercises for logical thinking. In contrast to that, proof is less important in English mathematics teaching, as mathematical facts are transmitted pragmatically, and then they are applied. The theoretical orientation of German mathematical teaching, consequently leads to a kind of lessons that are structured by mathematical rules with mathematical algorithms and universally valid statements. This applies especially for the *Hauptschule* and the *Realschule*, where lessons are predominantly based on a realistic educational conception. The pragmatic orientation of English mathematics teaching with its emphasis on the application of mathematical methods, produces lessons which are dominated by example-related procedures and mathematical facts that are hardly formulated in universally valid solving schemes. The dominance of a science-based understanding of mathematics is put into concrete terms in German mathematics teaching by the great importance of correct use of mathematical language and formal writing. Consequently, discussions about errors play a central rule and it is claimed to use predefined procedures, as individual procedures and solutions and notation of solutions are measured by scientific measuring and therefore, must coincide with it. The great importance of solving is also a consequent of another position of the individual in German mathematics teaching to which I will refer to later. English mathematics teaching handles correct mathematical language and formal writing in a more pragmatic way. For this reason these aspects are not given great attention to, nor to a classroom discussion about errors. Similarly, the emphasis on theoretical solving procedures in contrast to trial-and-error methods, which are generally less guided by theory, are also proving the scientific based understanding of mathematics. The usage of trial-and-error methods such as autonomous solving procedure, which are usually practiced in English mathematics teaching, is a characteristic of a more pragmatic method, where theoretical considerations are less important. These different perceptions of theory correspond with the importance given to the use of pocket calculators in lessons: In German mathematics lessons it is allowed only from the middle of the secondary level and does not play a significant role. In English mathematics teaching it is a dominant tool, and it is not only utilised as "calculating slave", but increasingly also for the introduction of mathematical conceptions. The dominance of pragmatic explanations that can be touched with hands corresponds with the great relevance given to real world examples. More actual mathematical topic areas, such as Operations Research and Statistics are much more important in English mathematics teaching, as these topics are often based on trial-and-error methods and apply complex algorithms in school lessons only pragmatically, because of the level of performance demand, and they are not deducted nor justified mathematically detailed. Also real-world applications basing on modelling processes are structured less theory-related, but demand trial-and-error based procedures. These kinds of procedures do not match well with German mathematics teaching with its characteristic scientific structure. In my opinion this is one main reason for the low value given to real world contents in German mathematics lessons. This stands in clear contrast to the claims and demands articulated in the discussion about mathematics teaching, that extra-mathematical applications at school must be considered.

The differences concerning the role of the individual, including the high regard for the establishment of an élite versus the formation of a broad masses, the possibility to specialise in a course system against the general extensive education based on the year class principle, as they became clear from the above described the educational conceptions, also can be explained through the ideal type reconstructions of English and German mathematics teaching. Thus these aspects have strong impact on the structure of curricula, which in England define a wide range of learning objectives for each age. In contrast to that are German mathematics curricula of each year quite tight, and none of the different school types allow individual priorities. These opposite tendencies, the differentiated education in the English system on the one hand and equalisation in German educational conceptions on the other hand keep on with the lessons: In German mathematics lessons all pupils go on at the same speed, while in English mathematics lessons well-performing pupils are allowed to go on faster and, consequently, weaker pupils are left behind. This could be seen in classroom observations and was supported by the dominance of individualised teaching-and-learning structures in England, and classroom discussions of pupils and teachers together as one type of teaching in Germany.

The described teaching-and-learning structures are not only caused by an élite-orientation and the possibility to specialise in English teaching versus a comprehensive progress for all in German teaching, but also through the different meanings given to the individual in the educational philosophies of both countries. Therefore, in England the high priority of the individual led to an individualised structure of teaching-and-learning, in which an individual can follow its own way in its own speed. This implies the great importance of individual ways of problem solving with individual kinds of notation and formulation in English mathematics teaching. The more extensive general education of the individual in German mathematics teaching – at least at the *Gymnasium*, for which this educational conception applies more than for other school types – leads to a dominance of a joint form of teaching and learning, in which all pupils make the same learning progress. Thus individual ways are subordinate to common ways, and a language comprehensible to all pupils and notation gains great importance.

To summarise, the study has shown the strong influence of educational philosophies on educational structures as well on the classroom situation. Considering this fact, it becomes obvious how difficult and time-demanding real innovations in mathematics teaching in each of the two countries will be.

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