ICMI Study 14: Applications and Modelling in Mathematics Education – Discussion Document

This paper is the *Discussion Document* for a forthcoming ICMI Study on *Applications and Modelling in Mathematics Education*. As will be well-known, from time to time ICMI (the International Commission on Mathematical Instruction) mounts specific studies in order to investigate, both in depth and in detail, particular fields of interest in mathematics education. The purpose of this Discussion Document is to raise some important issues related to the theory and practice of teaching and learning mathematical modelling and applications, and in particular to stimulate reactions and contributions to these issues and to the topic of applications and modelling as a whole (see chapter **4**). Based on these reactions and contributions, a limited number (approximately 75) of participants will be invited to a conference (the *Study Conference*) which is to take place in February 2004 in Dortmund (Germany). Finally, using the contributions to this conference, a book will be produced (the *Study Volume*) whose content will reflect the state-of-the-art in the topic of applications and modelling in mathematics education and suggest directions for future developments in research and practice.

The authors of this Discussion Document are the members of the *International Programme Committee* for this ICMI Study. The committee consists of 14 people from 12 countries, listed at the end of chapter **4**. The structure of the Document is as follows. In chapter **1**, we identify some reasons why it seems appropriate to hold a study on applications and modelling. Chapter **2** sets a conceptual framework for the theme of this Study, and chapter **3** contains a selection of important issues, challenges and questions related to this theme. In chapter **4** we describe possible modes and ways of reacting to the Discussion Document, and in the final chapter **5** we provide a short bibliography relevant to the theme of this Study.

1. Rationale for the Study

Among the themes that have been central to mathematics education during the last 30 years are relations between mathematics and the real world (or better, according to Pollak, 1979, the "rest of the world"). In section 2.1. we shall deal with terminology in more detail but, for the moment, we use the term "*applications and modelling*" to denote any relations whatsoever between the real world and mathematics.

That applications and modelling has been an important theme in mathematics education can be seen from the wealth of *literature* on the topic, as well as from material generated from a multitude of national and international *conferences*. Let us mention, in particular, firstly the ICMEs (the International Congresses on Mathematical Education) with their regular working or topic groups and lectures on applications and modelling, and secondly the series of ICTMAs (the International Conferences on the Teaching of Mathematical Modelling and Applications) which have been held biennially since 1983. Their Proceedings and Survey Lectures (see the bibliography in chapter **5**) indicate the state-of-the-art at the relevant time and contain many examples, studies, conceptual contributions and resources addressing the relation between the real world and mathematics, for all levels and all institutions of the educational system. In *curricula and textbooks* we find many more relations to real world phenomena and problems than, say, ten or twenty years ago. While applications and modelling also play a more important role in most countries' *classrooms* than in the past, there still exists a substantial gap between the ideals of educational debate and innovative curricula, on the one hand, and everyday teaching practice on the other hand. In particular, genuine modelling activities are still rather rare in mathematics lessons.

Altogether, during the last few decades there has been a lot of work in mathematics education which centres on applications and modelling. Many activities have had a primary focus on *practice*, e.g. construction and trial of mathematical modelling examples for teaching and examinations, writing of application-oriented textbooks, implementation of applications and modelling in existing curricula or development of innovative, modelling-oriented curricula. Several of these activities contain *research* components as well if (as according to Niss, 2001) we consider research as "the posing of genuine, non-rhetorical questions … to which no satisfactory answers are known as yet … and … the undertaking of non-trivial investigations of a systematic, reflective and 'methodologically conscious' nature" in order to obtain answers to those questions. In this sense, there are specific applications and modelling research activities, such as: clarification of relevant concepts; investigation of competencies and identification of difficulties and strategies activated by students when dealing with application problems; observation and analysis of teaching, and study of learning and communication processes in modelling-oriented lessons; evaluation of alternative approaches used to assess performance in applications and modelling. In particular during the last few years the number of genuine research contributions has increased as can be seen in recent ICTMA Proceedings.

That applications and modelling has been – and still is – a central theme in mathematics education is not surprising at all. Nearly all questions and problems in mathematics education, that is questions and problems concerning human learning and teaching of mathematics, affect and are affected by relations between mathematics and the real world. For instance, one essential answer (of course not the only one) to the question as to *why* all human beings ought to learn mathematics, is that it provides a means for understanding the world around us, for coping with everyday problems, or for preparing for future professions. When dealing with the question of *how* individuals acquire mathematical knowledge, we cannot get past the role of relations to reality, especially the relevance of situated learning (including the problem of the dependence on specific contexts). The general question as to what, after all, "*mathematics*" is, as a part of our culture and as a social phenomenon, of how mathematics has emerged and developed, points also to "applications" of mathematics in other disciplines, in nature and society. Today mathematical models and modelling have invaded a great variety of disciplines, leaving only a few fields where mathematical models do not play some role. This has been substantially supported and accelerated by the availability of powerful *electronic tools*, such as calculators and computers with their enormous communication capabilities.

In the current OECD (Organisation for Economic Co-operation and Development) Study PISA (Programme for International Student Assessment), relations between the real world and mathematics are particularly topical. What is being tested in PISA is "*mathematical literacy*", that is (see the PISA mathematics framework in OECD, 1999) "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to engage in mathematics, in ways that meet the needs of that individual's life as a constructive, concerned, and reflective citizen." That means the emphasis in PISA is "on mathematical knowledge put into functional use in a multitude of different situations and contexts". Therefore, mathematising real situations as well as interpreting, reflecting and validating mathematical results in "reality" are essential processes when solving literacy-oriented problems. Following the 2001 publication of results of the first PISA cycle (from 2000), an intense discussion has started, in several countries, about aims and design of mathematics instruction in schools, and especially about the role of mathematical modelling, applications of mathematics and relations to the real world.

In mounting this Study on "Applications and Modelling in Mathematics Education", ICMI takes into account the above-mentioned reasons for the importance of relations between mathematics and the real world as well as the contemporary state of the educational debate, of research and development in this field. This does not, of course, mean that we already know all answers to the essential questions in this area and that it is merely a matter of putting together these answers in the Study. Rather, it is an important aim of the Study to identify shortcomings and to stimulate further research and development activities. Nevertheless, it is time to map out the state-of-the-art in theory and practice, in research and development of applications and modelling in mathematics education, and to document these in this Study.

Documenting the state-of-the-art in a field and identifying deficiencies and needed research requires a *structuring framework*. This is particularly important in an area which is as complex and difficult to survey as the teaching and learning of mathematical modelling and applications. As we have seen, this topic not only deals with most of the essential aspects of the teaching and learning of mathematics at large, it also touches upon a wide variety of versions of the real world outside mathematics that one seeks to model. Perceived in that way, the topic of applications and modelling may appear to encompass all of mathematics education plus a lot more. It is evident, therefore, that we have to find a way to conceptualise the topic so as to reduce complexity to a meaningful and tractable level. In the following chapter **2** we offer our conceptualisation of the topic: in section 2.1 we clarify some of the basic concepts and notions of the field, and in section 2.2 we suggest a structure for the field. This serves as a basis for identifying important challenges and questions in chapter **3**, the core of this Discussion Document.

2. Framework for the Study

2.1. Concepts and Notions

Here we shall give, in a rather pragmatic way, some working definitions which will be useful for the following sections. This is not the place for a deeper epistemological analysis of these concepts. Rather, this can be done in the Study itself.

By *real world* we mean everything that has to do with nature, society or culture, including everyday life as well as school and university subjects or scientific and scholarly disciplines different from mathematics. For a description of the complex *interplay between the real world and mathematics* we use one of the well-known simple models developed for that purpose (see Blum/Niss, 1991, and the literature quoted there). The starting point is normally a certain *situation* in the real world. Simplifying it, structuring it and making it more precise – according to the problem solver's knowledge and interests – leads to the formulation of a *problem* and to a *real model* of the situation. Here we use the term problem in a broad sense, encompassing not only practical problems but also problems of a more intellectual nature aiming at describing, explaining, understanding or even designing parts of the world. If appropriate, real data are collected in order to provide more information on the situation at one's disposal. If possible and adequate, this real model – still a part of the real world in our sense – is *mathematised*, that is the objects, data, relations and conditions involved in it are translated into mathematics, resulting in a *mathematical model* of the original situation. Now mathematical methods come into play, and are used to derive *mathematical results*. These have to be re-translated into the real world, that is *interprete*d in relation to the original situation. At the same time the problem solver *validates* the model by checking whether the problem solution obtained by interpreting the mathematical results is appropriate and reasonable for his or her purposes. If need be (and more often than not this is the case in "really real" problem solving processes), the whole process has to be repeated with a modified or a totally different model. At the end, the obtained solution of the original real world problem is stated and communicated.

The process leading from a problem situation to a mathematical model is called *mathematical modelling*. However, it has become common to use that notion also for the entire process consisting of structuring, mathematising, working mathematically and interpreting/validating (perhaps several times round the loop) as just described.

Sometimes the given problem situation is already prestructured or is nothing more than a “dressing up” of a purely mathematical problem in the words of a segment of the real world. This is often the case with classical school *word problems*. In this case mathematising means merely “undressing” the problem, and the modelling process only consists of this undressing, the use of mathematics and a simple interpretation.

Using mathematics to solve real world problems is often called *applying* mathematics, and a real world situation which can be tackled by means of mathematics is called an *application* of mathematics. Sometimes the notion of “applying” is used for any kind of linking of the real world and mathematics.

During the last decade the term “*applications and modelling*” has been increasingly used to denote all kinds of relationships whatsoever between the real world and mathematics. The term “modelling”, on the one hand, focusses on the direction reality $\rightarrow$ mathematics and, on the other hand and more generally, emphasizes the processes involved. The term “application”, on the one hand, focusses on the opposite direction mathematics $\rightarrow$ reality and, on the other hand and more generally, emphasizes the objects involved – in particular those parts of the real world which are accessible to a mathematical treatment and to which corresponding mathematical models exist. In this comprehensive sense we understand the term “applications and modelling” as used in the title of this Study.

2.2. Structure of the Topic Applications and Modelling in Mathematics Education

Let us begin by addressing what one may refer to as “the reality“ of applications and modelling in mathematical education. We think of this reality as being constituted essentially by two dimensions: The significant “*domains”* within which mathematical applications and modelling are manifested, on the one hand, and the educational *levels* within which applications and modelling may be taught and learnt, on the other hand.

More specifically, in the first dimension we discern three different *domains*, each forming some sort of a continuum. The first domain consists of the very *notions of applications and modelling*, i.e. what we mean by an application of mathematics, and by mathematical modelling; what the most important components of applications and modelling are, in terms of concepts and processes; what the epistemological characteristics of applications and modelling are, vis-à-vis mathematics as a discipline and vis-à-vis other disciplines and areas of practice; who uses mathematics, and for what purposes, and with what sorts of outcomes; what is modelling competency, etc. The second domain is that of the *classroom*. We use this term as a broad indicator of the location of teaching and learning activities pertaining to applications and modelling. Of course, this includes the classroom in a literal sense, but it also includes the student doing his or her homework, individually or in groups, and the teacher’s planning of teaching activities or looking at students’ products, written or other, and so forth. The third and final domain is the *system* domain. The word system, here, refers to the whole institutional, political, structural, organisational, administrative, financial, social, and physical environment that exerts an influence on the teaching and learning of applications and modelling. It appears that we have chosen not to consider individuals, in particular students and teachers, as constituting separate domains. This does not imply, however, that individuals are not part of our conceptualisation. The individual student is a member of the classroom, as defined above, when engaging in learning activities in applications and modelling. The individual teacher can also be regarded as a member of the applications and modelling classroom, namely when he or she is engaged in teaching, supervising, advising or assessing students. From another perspective, however, the teacher is also a member of the system. This happens when he or she speaks or acts on behalf of the system (typically in the form of his or her institution) in matters concerning selection, placement, and examination of the individual student, or invokes rules, procedures or other boundary conditions in decisions on, say, curricular matters.

The *second* dimension is constituted by the educational *levels* at which applications and modelling are taught and learnt. We have decided to adopt a relatively crude division of levels, both in order to avoid excessive detail in the discussion and in order to obtain a division which is compatible with the state of affairs in most, maybe all, countries in the world. The levels adopted are the *primary*, the *secondary*, the *tertiary* levels, and the level of *teacher education*. Here we do not primarily refer to age levels but to intrinsic levels of the learners’ knowledge and competencies. It goes without saying that a much more fine-grained division might have been an alternative. Nevertheless, at least the present one does allow for the consideration of applications and modelling at all educational levels, albeit possibly within larger clusters of rather diverse kinds of education.

We might, of course, design a more subtle structure and regard, for instance, the system as an independent dimension. However, for our purposes it is quite appropriate to represent the *„reality“* of applications and modelling in mathematics education by the Cartesian product of those two dimensions, the domains and the levels. If we do so then a major point in this framework is to identify and consider a number of *crucial issues* concerning that „reality space“. An issue addresses a segment of reality, which can be represented as a certain subset of the reality space. That subset consists of those objects, phenomena or situations - drawn from combinations of applications and modelling domains and educational levels - that the issue concerns. To illustrate the point, let us give but one *example* of such an issue:

Issue 0:

One of the underlying reasons for attributing a prominent position to applications and modelling in the teaching and learning of mathematics is that it is assumed desirable for students to be able to engage, outside of the mathematics classroom, in applications and modelling concerning areas and contexts that are new to them. In other words, it is assumed that applications and modelling competency developed in and for some types of areas and contexts can be transferred to other such types having different properties and characteristics. However, many research studies suggest that for some categories of students this transfer is rather limited in scope and range.

To what extent is applications and modelling competency transferable across areas and contexts? What teaching/learning experiences are needed or suitable to foster such transferability?

As it stands here, this issue concerns the classroom domain and (at least) the primary, secondary, and tertiary levels. This means that the issue addresses the "rectangle" constituted by the entire classroom domain and the first three educational levels. If for some reason the focus were to be limited to address, say, the secondary level, then of course the rectangle would be reduced accordingly.

In this Discussion Document, a number of issues have been identified as particularly significant to the present study. Readers are invited to comment on these issues or to suggest further issues to be considered in the Study. Each of the issues addresses its own segment of reality, but of course those segments may intersect. The reality space and the issues addressing it may be mapped as in figure 1:

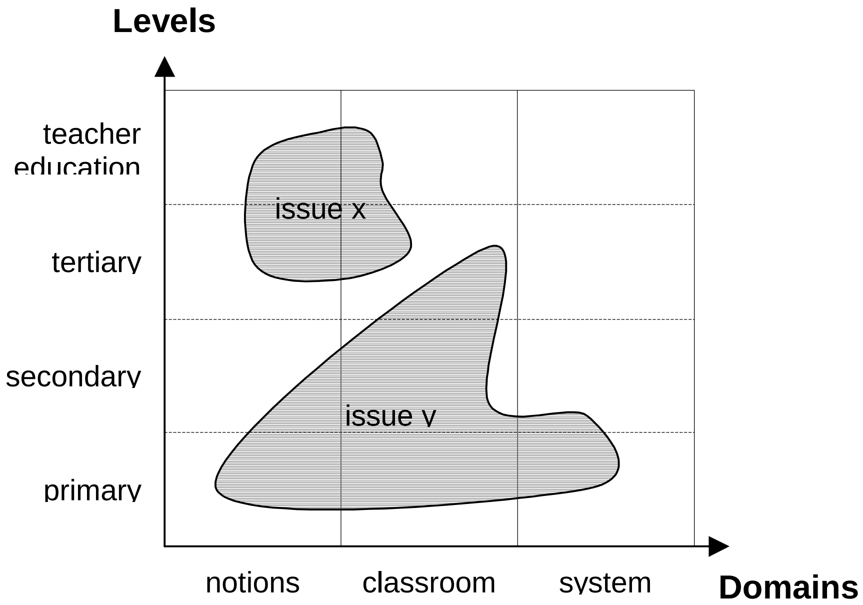

Figure 1: The "reality" of applications and modelling

The reader may have noticed that the formulation of the issue given above as an example consists of two parts. Firstly, a background part outlining a challenge, i.e. a dilemma or a problem which may be of a political, practical, or intellectual nature. For short let us call this part the *challenge* part of the issue. The second part consists of particular *questions* that serve the purpose of pinpointing some crucial aspects of the challenge that deserve to be dealt with in the Study. When the Study has been completed, a substantial portion of it will consist of analyses of a number of significant issues.

From the point of view of this Study, an issue concerning applications and modelling in mathematics education may be viewed and approached - depending on its nature - from a variety of different *perspectives*, each indicating the category of answers sought. The most basic of these perspectives is that of *doing*, i.e. actual teaching and learning practice as enacted and carried out in the classroom (in the sense defined above). Here, the focus is on what does (or should) take place in existing everyday classrooms at given educational levels. Another perspective is the *development and design* of curricula, teaching and learning materials or activities, and so forth. Here, the focus is on establishing short or long term plans and conditions for future teaching and learning. A third perspective is that of *research*, which focuses on the generation of answers to research questions as yet unanswered, while a fourth and final perspective is that of *policy* for which the focus is on the instruments, strategies and policies that are or ought to be adopted in order to place matters pertaining to applications and modelling on the agenda of practice or research in some desired way. Accordingly, a given issue may be addressed from one or some (perhaps all) of these four perspectives. To avoid a possible misinterpretation let us stress that the order in which we have presented these four perspectives does not imply a hierarchy. It appears that each of these perspectives can be perceived as representing a particular professional role: The role of *teacher* or *student*, the role of *curriculum developer*, the role of *researcher*, and the role of *lobbyist* or *decision maker*. An individual can assume all of these roles, but usually not at the same time.

In the above-mentioned example, the issue may be approached from the perspective of 'doing', provided the interest and emphasis is on the actual construction of learning environments and the carrying through of specific teaching activities meant to underpin transferability of application and modelling competencies cultivated within certain areas and contexts to other such areas or contexts. The 'development and design' perspective is adopted if the emphasis is on finding or devising ways to orchestrate teaching and learning activities that are hoped to generate improved transferability. If, on the other hand, the emphasis is on getting to know and understand the nature and extent of transferability of such competencies between areas and contexts, or the effect of an implemented design, then the 'research' perspective is being invoked. Finally, the 'policy' perspective is on the agenda if the focus is on pleading or lobbying for, say, making room and time in the curriculum for activities that are seen as necessary or desirable for allowing students to gain multi-faceted and rich first-hand experiences with a large variety of applications and modelling activities drawn from different areas, contexts, and situations.

We may depict, metaphorically, the different perspectives by a quadri-focal looking glass, in which each of the four segments represents a characteristic focus and a corresponding focal length, both defined by the perspective; see figure 2:

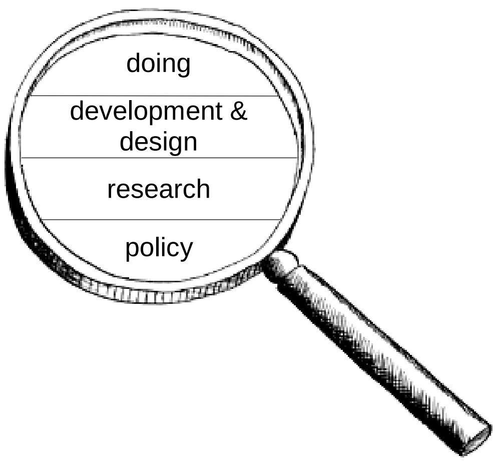

Figure 2: Perspectives on "reality"

So far, we have described the nature of the reality and the issues which are going to be considered in this ICMI Study, and the perspectives through which they can be addressed. One final component in our attempt to structure the Study remains to be identified: the *recording* of the deliberations and investigations that are going to be conducted in the Study. This Study will be conducted by employing the four perspectives to look at the reality and the issues, and the outcomes of analysing the reality and the issues. Reflecting upon these from various perspectives will provide the substance for what will be recorded in the resulting ICMI Study Volume. Accordingly, that Volume will consist of sections in which the "reality" is described in detail, together with the issues eventually identified, sections in which the different segments of the looking glass are polished, and sections in which conclusions concerning the issues raised and dealt with by means of the four perspectives are presented and discussed.

3. Examples of important issues

In this chapter a number of selected *issues* – consisting of *challenges* and *questions* – are raised. Although they have been grouped by certain inherent features, there is much overlap between them and different groupings are certainly possible. They are intended as a guide to the kinds of issues that the present Study intends to address. Readers are invited to come up with additional relevant issues.

3.1. Epistemology

Issue 1:

There are a number of different elements and characterisations of modelling and applications; some of these are posing and solving open-ended questions, creating, refining and validating models, mathematising situations, designing and conducting simulations, solving word problems and engaging in applied problem solving. All of these link the field of mathematics and the world. If the goal of knowledge is to assist us to ensure the sustainability of health, education and environmental well-being and improve its quality, then individuals must engage in applications and modelling to do so.

What is the description/representation of the components within applications and modelling relevant to this issue? How is the relationship between applications and modelling and mathematics including its domains, concepts, representations, skills, methods and forms of evidence best described? What is the relationship between applications and modelling and the world we live in?

Examples of specific questions that could be addressed here are:

- What are the process components of modelling? What is meant by or involved in each?
- How does our knowledge of applications and modelling accumulate, evolve and change over time?
- What parts of mathematics, if any, are less likely to be represented in applications and modelling?
- What parts of applications and modelling, if any, are less likely to be represented in mathematics?
- What is the meaning and role of abstraction, formalisation and generalisation in applications and modelling?
- What is the meaning and role of proof and proving in applications and modelling? Are there common features of proving and modelling?
- What are the various meanings of "authenticity" in modelling?
- How much extra-mathematical context must be familiar and understood to undertake applications and modelling?
- What is generalisability and transfer when working across contexts?

3.2. Application Problems

There exists a wealth of applications and modelling problems and materials for use in mathematics classrooms at various educational levels. These materials range from mere "dressed up" mathematical problems to "really real", authentic problem situations.

Issue 2:
An important aspect in the research and practices of applications and modelling is the notion and role of authenticity of the problems and situations dealt with in applications and modelling activities.

What does research have to tell us about the significance of authenticity to students´ acquisition and development of modelling competency?

Examples of specific questions:
- What authentic applications and modelling materials are available worldwide?
- Taking account of teaching objectives and students´ personal situations (experience, competence), how can teachers set up authentic applications and modelling tasks?
- How does the authenticity of problems and materials affect on students' ability to transfer acquired knowledge and competencies to other contexts and situations?

3.3. Modelling Abilities and Competencies
With the teaching and learning of mathematical modelling and applications, a great variety of goals and expectations are combined.

Issue 3a:
One of the most important goals is for students to acquire modelling ability and competency.

How can modelling ability and modelling competency be characterised, and how can it be developed over time?

Examples of specific questions:
- Are modelling ability and modelling competency different concepts?
- Can specific subskills and subcompetencies of "modelling competency" be identified?
- How can modelling ability be distinguished from general problem solving abilities?
- Are there identifiable stages in the development of modelling ability?
- What are the characteristic differences between expert modellers and novice modellers? What are characteristic features of the activity of students who have little experience of modelling?
- What is the role of pure mathematics in developing modelling ability?
- What are common features, and what are differences between students' individual ability and interactive ability in applications and modelling?

An especially important problem is the context dependence of acquired competencies. This holds for modelling competency as well. See **Issue 0** in section 2.2!

The development of applications and modelling abilities and competencies is of particular relevance for teachers.

Issue 3b:
It appears rare that mathematics teacher education programmes include orientations to modelling, and the use of the modelling process in mathematics courses.

How can modelling in teacher pre-service and in-service education courses be promoted?

Examples of specific questions:
- What is essential in a teacher education programme to ensure that prospective teachers will acquire modelling competencies and be able to teach applications and modelling in their professional future?
- Considering both the limited mathematics background of primary school student teachers and the limited time available for mathematics in their education, how can they experience real, non-trivial modelling situations?
- Which training strategies can help teachers develop security with respect to using applications and modelling in their teaching?

3.4. Beliefs, Attitudes, and Emotions
Beliefs, attitudes and emotions play important roles in the development of critical and creative senses in mathematics.

Issue 4:
Modelling aims, among other things, at providing students with a better apprehension of mathematical concepts, teaching them to formulate and to solve specific situation-problems, awaking their critical and creative senses, and shaping their attitude towards mathematics and their picture of it.

To what extent does applications and modelling have the potential to provide an environment to support both students and teachers in their development of appropriate beliefs about and attitudes towards mathematics?

Examples of specific questions:
- Taking into account available research on the role of beliefs, attitudes and emotions in learning applications and modelling, what are the implications of this research for changing teaching practice and classroom cultures with respect to applications and modelling?
- Can modelling effectively contribute towards promoting views of mathematics that extend beyond transmissive techniques to its role as a tool for structuring other areas of knowledge?
- What strategies are feasible for in-service teacher education that will address the fear experienced by some teachers when faced with applications and modelling?

3.5. Curriculum and Goals
Applications and modelling can make fundamental contributions to the development of students' competencies. This is why it ought to be present in all mathematics curricula.

Issue 5a:
Serious applications and modelling activities are always time consuming because attention has to be paid to several crucial phases of the modelling process including work on extra-mathematical matters. This implies that applications and modelling components of general mathematical curricula (at whichever level) will have to "compete" with other components of the mathematics curriculum, in particular work on pure mathematics.

What would be an appropriate balance – in terms of attention, time and effort – between applications and modelling activities and other mathematical activities in mathematics classrooms at different educational levels?

Examples of specific questions:

- What is the actual role of applications and modelling in curricula in different countries?
- Is it possible – or desirable – to identify a core curriculum in applications and modelling within the general mathematical curriculum?
- Which applications, models and modelling processes should be included in the curriculum? Does the answer depend on each teacher or should there be some minimal indications in national and state curricula?
- Is it beneficial to generate specific courses or programs on applications and modelling or is it better to mix them in the standard mathematical courses?
- Is it possible to treat applications and modelling in the curricula as an interdisciplinary activity?
- When applications and modelling are included at different places in mathematics curricula, how can it be guaranteed that basic modelling skills and competencies are acquired systematically and coherently?

The university level represents a particularly problematic case:

Issue 5b:
Although there are major differences between different places and countries, university graduates in mathematics (even the most specialised and advanced ones) embark on a large variety of different professional careers, many of which will have links to matters pertaining to applications and modelling. It can be argued that even future research mathematicians are likely to come in touch with applications and modelling in some way or another, perhaps because their research activities will be informed by application problems or because they will be teaching students with application careers in front of them.

Should a plea be made for all university graduates in mathematics to acquire some applications and modelling experiences as part of their studies? If so, what kinds of experiences should they be?

Concerning general education at the school level, some special questions arise.

Issue 5c:
Mathematics accounts for a large proportion of time in school. This is only justified if mathematics can contribute to general education for life after school.

How and to what extent can applications and modelling contribute to building up fundamental competencies and to enriching a student's general education?

Examples of specific questions:

- What meanings can be given to "general education", and what is the role of mathematical modelling therein? Is applications and modelling really an indispensable part of "general education" for students?
- What pictures do teachers have in their mind about the contribution of mathematics – and in particular of mathematical modelling – to general education, and how can these be influenced?
- What is a suitable balance within general education of the following emphases: to create one's own models of real situations and problems, or to make judgements about models made by others?

3.6. Modelling Pedagogy

The pedagogy of applications and modelling intersects the pedagogy of pure mathematics in a multitude of ways and requires at the same time a variety of practices that are not part of the traditional mathematics classroom.

Issue 6:
While examples of successful applications and modelling initiatives have been documented in a variety of countries, and contexts, the extent of such programmes remains less than desired. Furthermore approaches to teaching applications and modelling vary from the use of traditional methods and course structures, to those that include a variety of innovative teaching practices including an emphasis on group activity.

What are appropriate pedagogical principles and strategies for the development of applications and modelling courses and their teaching? Are there different principles and strategies for different educational levels?

Examples of specific questions:

- What research evidence is available to inform and support the pedagogical design and implementation of teaching strategies for courses with an applications and modelling focus?
- What are the areas of greatest need in supporting the design and implementation of courses with an applications and modelling focus?
- To what extent do teaching practices within applications and modelling courses draw on general theories of human development and/or learning?
- What criteria are most helpful in selecting methods and approaches suggested by such theories?
- What obstacles appear to inhibit changes in classroom culture e.g. the introduction of interactive group work in applications and modelling?
- What criteria can be used to choose (e.g. between individual and group activity) the most desirable option at a particular point within an applications and modelling teaching segment?
- What documentations of successful group learning practices exist?

3.7. Sustained Implementation

To change an educational system is a major challenge as it involves and impacts upon many different parties. Implementing new mathematical modelling curricula involves specific factors, such as in-service teacher training, technological requirements, etc. Sustaining this implementation requires changes in pre-service education, and more general agreement between mathematics faculty members at the post secondary level.

Issue 7:

With the increasing interest in and argument for mathematical modelling both inside and outside the mathematical community, there is a need to ensure that mathematical modelling is implemented in a sustained fashion at all levels of mathematics education.

In spite of a variety of existing materials, textbooks, etc., and of many arguments for the inclusion of modelling in mathematics education, why is it that the actual role of applications and mathematical modelling in everyday teaching practice is still rather marginal, for all levels of education? How can this trend be reversed to ensure that applications and mathematical modelling is integrated and preserved at all levels of mathematics education?

Examples of specific questions:

- What are the major impediments and obstacles that have existed to prevent the introduction of applications and mathematical modelling, and how can these be changed?
- What documented evidence of success in overcoming impediments to the introduction of applications and modelling courses exist?
- What are the requirements for developing a mathematical modelling environment in traditional courses at school or university?
- How does one ensure that the mathematical modelling philosophy in curriculum documents is mirrored in classroom practice?
- What continuing education experiences (and education support for teachers, teaching assistants, mathematics faculty, etc.) need to be provided?

3.8. Assessment and Evaluation

The teaching and learning of mathematics at all levels is naturally closely related to assessment of student achievement. Nevertheless, to assess mathematical modelling is not easy to accomplish. The more complicated and open a problem is, the more complicated it is to assess the solution, and if one adds the component of available technology, assessment becomes even more complicated. Similar problems are inherent at the course level for the evaluation of programmes with application and modelling components.

Issue 8a:

There seem to be many indications that the assessment modes traditionally used in mathematics education are not fully appropriate to assess students´ modelling competency.

What alternative assessment modes are available to teachers, institutions and educational systems that can capture the essential components of modelling competency, and what are the obstacles to their implementation?

Examples of specific questions:

- What are the possibilities or obstacles when assessing mathematical modelling as a process (instead of a product)? What can be learnt from assessment in the arts, music, etc.?
- If there is a change in the mathematics conception of students after experiencing and learning mathematical modelling, how do we assess that change?
- In teacher education, what techniques can be used to assess a future teacher’s ability to teach and assess mathematical modelling?
- When mathematical modelling is introduced into traditional courses at school or university, how should assessment procedures be adapted?
- When centralised testing of students is implemented, how do we ensure that mathematical modelling is assessed validly?
- How does one reliably assess individual contributions and achievement within group activities and projects?

Issue 8b:

There seems to be a need to develop specific means of evaluating programmes with an applications and modelling content.

What evaluation modes are available that can capture the essential features of applications and modelling, especially of integrated courses, programmes and curricula, and what are the obstacles to their implementation?

Examples of specific questions:

- In what way do usual evaluation procedures for mathematical programmes carry over to programmes that combine mathematics with applications and modelling?
- What counts as success when evaluating outcomes from a modelling programme? For example, what do biologists, economists, industrial and financial planners, medical practitioners, etc., look for in a student’s mathematical modelling abilities? How does one establish whether a student has achieved these capabilities?

3.9. Technological Impacts

Many technological devices are available today and many of them are highly relevant for applications and modelling. In a broad sense these technologies include calculators, computers, Internet and all computational or graphical software as well as all kind of instruments for measuring, for performing experiments, for solving all kind of daily life problems, etc. These devices provide not only increased computational power but broaden the range of possibilities for approaches to teaching, learning and assessment. Moreover, the use of technology is in itself a key knowledge in today's society. On the other hand, the use of calculators and computers may also bring inherent problems and risks.

Issue 9:

Technology can obviously provide support for well-structured mathematical problems that students will meet in their mathematical studies from the lower secondary level on.

How should technology be used at different educational levels to effectively develop students' modelling abilities and to enrich the students' experience of open-ended mathematical situations in applications and modelling?

Examples of specific questions:

- What implications does technology have for the range of applications and modelling problems that can be introduced?
- What important aspects of applications and modelling are touched (or not touched) upon by the technological environment?
- How is the culture of the classroom influenced by the presence of technological devices? Will button pressing compromise thinking and reflection or can these be enhanced by technology?
- What evidence of successful or failed practice in teaching and learning applications and modelling has been documented as a direct consequence of the introduction of technology?
- In what cases does technology facilitate the learning of applications and modelling? When may technology kidnap learning possibilities, e.g. by rendering a task trivial, when can it enrich them?
- In which cases is technology a crucial need in modelling in the classroom? Are there circumstances (if any) where modelling processes can't be developed without technology?
- With respect to non-affluent countries: can applications and modelling be successfully done without any technology?
- What are the implications of the availability of technology for the selection of assessment items and practices for use in contexts involving applications and modelling?

4. Call for Contributions to the Study

The ICMI Study on *Applications and Modelling in Mathematics Education* will consist of three components: an invited *Study Conference,* a *Study Volume* and a *Study Website*.

The Study Conference will be held in Dortmund (Germany) around mid-February, 2004. The conference will be a working one where every participant will be expected to be active. As is the normal practice for ICMI studies, participation in the study conference is by invitation only, given on the basis of a submitted contribution, and is limited to approximately 75 people. The Study Volume, to be published after the conference in the ICMI Study Series, will be based on selected contributions and reports prepared for the conference, as well as on the outcomes of the conference. The Study Website, accessible also after the conference, will contain selected examples of good practice in applications and modelling. A report on the Study and its outcomes will be presented at the 10th International Congress on Mathematical Education to be held in Copenhagen in July 2004.

The International Programme Committee (IPC) for the Study invites submission of contributions on specific questions, problems or issues related to this Discussion Document. Contributions, in the form of synopses of research papers, discussion papers or shorter responses, may address questions raised above, or questions that arise in response, or further issues relating to the theme of the Study. Submissions should not exceed 6 pages in length and should reach the Programme Chair at the address below (preferably by e-mail) no later than June 15, 2003, but earlier if possible. All submissions must be in English, the language of the conference.

The contributions of those invited to the conference will be made available to other participants among the conference materials or on the conference website. However an invitation to the conference does not imply that a formal presentation of the submitted contribution will be made during the conference.

It is hoped that the conference will attract not only "experts" but also some "newcomers" to the field with interesting and refreshing ideas or promising work in progress. Unfortunately an invitation to participate in the conference does not imply a financial support from the organisers, and participants should finance their own attendance at the conference. Funds are being sought to provide partial support to enable participants from non-affluent countries to attend the conference, but it is unlikely than more that a few such grants will be available.

The members of the International Programme Committee for this Study are:

- Werner BLUM (University of Kassel, Germany), *Chair of the IPC*
- Claudi ALSINA (University of Technology, Barcelona, Spain)
- Maria Salett BIEMBENGUT (University of Blumenau, Brazil)
- Nicolas BOULEAU (Ecolé Nationale des Ponts et Chaussées, Marne-la-Vallée, France)
- Jere CONFREY (University of Texas-Austin, USA)
- Peter GALBRAITH (University of Queensland, Brisbane, Australia)
- Toshikazu IKEDA (Yokohama National University, Japan)
- Thomas LINGEFJÄRD (Gothenburg University, Sweden)
- Eric MULLER (Brock University, St. Catharines, Canada)
- Mogens NISS (Roskilde University, Denmark)
- Lieven VERSCHAFFEL (University of Leuven, Belgium)
- Shangzhi WANG (Capital Normal University, Beijing, China)
- Bernard R. HODGSON (Université Laval, Québec, Canada), *ex officio, representing the ICMI Executive Committee*
- Hans-Wolfgang HENN (University of Dortmund,

Germany), *Chair of the Local Organising Committee.*

For further information and submission of contributions, please contact the Chair of the IPC:

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5. Bibliography

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