CAS in general mathematics education

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Abstract: A rational discussion of the use of Computer algebra systems (CAS) in mathematics teaching in general education needs an explicit image of (general) mathematics education, an explication of global perspectives and goals on mathematics teaching focusing on general education (chapter 1). The conception of general education according to the „ability of communication with experts" described in chapter 2 can be such an orientation for analysing, considering, classifying and assessing the didactical possibilities of using CAS. CAS are materialised mathematics allowing for more or less exhaustive outsourcing of operative (also symbolically) knowledge and skills to the machine. This frees up space of time as well as mental space for the development of those competences being in our view relevant for general mathematics education. In chapter 3 the idea of outsourcing and the role of CAS for it is discussed more detailed as well as consequences being possible for the CAS-supported teaching of mathematics. Beyond, CAS can be didactically used and reflected as a model of communication between (mathematical) experts and lay-persons (chapter 4). Chapter 5 outlines some research perspectives.

Kurzreferat: *CAS in einem allgemeinbildenden Mathematikunterricht.* Eine rationale Argumentation zur Frage des CAS-Einsatzes in einem allgemeinbildenden Mathematikunterricht bedarf eines explizierten Bildes von mathematischer (Allgemein-)Bildung, einer Explizierung von globalen Sichtweisen und Zielsetzungen eines allgemeinbildenden Mathematikunterrichts (Kapitel 1). Einen solchen Orientierungsrahmen für die Auslotung, Einordnung und Bewertung der didaktischen Möglichkeiten des CAS-Einsatzes kann das in Kapitel 2 beschriebene Allgemeinbildungskonzept „Kommunikationsfähigkeit mit ExpertInnen" darstellen. CAS sind materialisierte Mathematik und ermöglichen eine mehr oder weniger weitgehende Auslagerung (auch symbolischen) operativen Wissens und Könnens an die Maschine. Dies führt zu zeitlichen wie mentalen Freiräumen für die Entwicklung jener Kompetenzen, die von uns als zentral für eine mathematische Allgemeinbildung angesehen werden. Kapitel 3 setzt sich mit der Idee von Auslagerung und der Rolle von CAS dabei sowie mit möglichen Auswirkungen auf einen CAS-unterstützten Mathematikunterricht auseinander. CAS können darüber hinaus auch als ein Modell einer Kommunikation zwischen (mathematischen) ExpertInnen und LaiInnen didaktisch genutzt und reflektiert werden (Kapitel 4). In Kapitel 5 wird auf Forschungsperspektiven verwiesen.

ZDM-Classification: D20, D30, E10, E20, U70

1 Introduction

"*Should students be able to*

- solve the equation $x^2 + 4x + 4 = 0$?
- determine the maximum value of the function f when $f(x) = x(1-x)$?
- calculate the integral $\int_0^1 (x - x^2)dx$?

Our answer is: of course, they should!

Should students be able to solve these problems without using CAS?
We have no answer to that question, rather we must pose a new question:
There are various possibilities for solving the problems mentioned above. Should it be forbidden for the students to use any number of other intelligent solution possibilities aside from CAS? If so, why? If not, why CAS?

Should we spend our teaching time extensively explaining to the students how to work out the solutions of such similar problems by hand?
Our answer is: That depends on the image of mathematics, the teaching thereof and mathematics general education which we ourselves have and it depends upon that image of mathematics which we want to convey to our students.
We ourselves are convinced that a mathematics classroom, in which the necessity of manually working out solutions and practicing them has taken the upper hand and has become so time consuming can hardly be rescued – and by that we mean the concrete teaching of mathematics as well as mathematics as a subject in general." (Peschek & Schneider 2001, p. 7-8)

The quotation above underlines our belief that in order to have a rational discussion about the question of using technology in mathematics classrooms one needs an explicit image of mathematics and (general) mathematics education. Only a developed view of mathematics and mathematics education offers a framework allowing for rational classification and assessment of arguments, and thus for rational discussion.

Thus, at first we would like to sketch our view of (higher) general mathematics education based on the conception of general education[1] of Fischer which is, in our opinion, well suited to classifying and to assessing considerations and the experience of using CAS in mathematics classrooms. A more detailed description of this framework can be found in (Schneider 2002, p. 13-38) as well as in the references there.

2 Communication with experts as a principle of orientation for higher general mathematics education

The functionality of our democratic society, built as it is on the division of labour, is essentially based on emancipated contact with highly specialized expert knowledge: as mature, responsible citizens we are permanently confronted with statements made by experts which we then must assess and judge in order to be able to make (our own) decisions. As a rule we will rely on the professional correctness of these expert statements yet do need to judge their importance for ourselves and for the good of the community. As lay-persons we must be in the position of being able to ask the experts the right questions, to assess their answers and to draw our own conclusions (cf. Fischer 2000, p. 36-37). That requires a highly developed communication between experts and lay-persons concerning the communicative-social dimension as well as the dimension of contents and concepts.

[1] The considerations of this paper focus on secondary schools of grade 9 – 12/13 (14-18/19 years old students).

The German educator and mathematics didactic Heymann perceives the “problem of communication between experts and lay-persons ... to be one of the key problems of a highly differentiated and structured democracy based upon the division of labour“ (Heymann 1996, p. 113). In this case, schools are playing an important role not only in terms of a professional education, and communication between teachers and students is a model for shaping communication between experts and lay-persons that society desires.

Fischer picks up these considerations whereby – in contrast to Heymann – he primarily focuses on the aspect of the contents and concepts of communication between experts and lay-persons: he proposes that those persons who have attended institutes of higher learning (high schools and vocational high schools), “’the more highly educated’“ in particular, should be able to explain the experts’ statements in an understandable fashion and judge their importance; he suggests that such an “ability to communicate with experts and with the general public“ is to be taken as a “principle of orientation“ for determining the curriculum at schools of higher education (Fischer n.d., p. 3).

Fischer identifies the following three fields of competence as those which are to be acquired: *basic knowledge* (notions, concepts, forms of representation), *operative knowledge and skills* (solving problems, proofs, in general: generating new knowledge), *reflection* (possibilities, limits and meaning of concepts and methods).

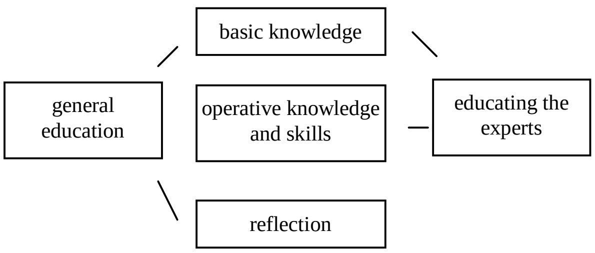

Figure 1: Fields of competence (acc. to Fischer n.d., p. 5)

While the experts in particular have to be competent in the first two fields, Fischer considers the fields of *basic knowledge* and *reflection* to be particularly important for the generally educated lay-person (cf. Figure 1): Basic knowledge “is a prerequisite for communicating with experts“, reflection “is necessary for judging their expertises“ (Fischer n.d., p. 5).

Fischer points out that this classification should not be taken as an absolute; neither should the experts be relieved of their responsibility of viewing that which they are doing in a self-critical manner, nor can doing operations be completely removed from the framework of a (general) mathematics education. However, the focus and profiles for experts and lay-persons do clearly differ.

The consequence for the teaching of mathematics is, in short: ”The reduction of expectations with regard to operations and an increase in the expectations with regard to reflection.“ (Fischer n.d., p. 6)

In the following chapters we will attempt to relate our considerations of using CAS in mathematics classrooms to this view of (general) mathematics education and to reflect upon it against this background.

3 CAS and outsourcing the operative

3.1 Principle of outsourcing

According to Fischer, the competences of a lay-person on which emphasis is to be placed are basic knowledge and reflection, while operative abilities and skills can largely be delegated (outsourced) to mathematical experts and/or be included in the communication by these experts.

Of course, there is however an essential difference between that knowledge and those skills which we cognitively have available and are directly able to use and those which have been “outsourced“, being available to us solely by external means (for example, by books, machines, experts). The demand to know all things, however, is not absurd just today; the narrowing of this demand to one specific field refers to a narrow specialisation which runs counter to any general education. What it can be interested in from a theory of education viewpoint is the conscious and understandable outsourcing of knowledge wherever this is possible in a meaningful way. An emancipated application of knowledge like that is of great importance not only in our daily lives but also in the sciences; this is valid in mathematics particularly because of scientific-theoretical and socio-philosophical reasons (cf. above all Fischer 1991; Peschek 1999a, 1999b):

In mathematics we constantly work with the method of *outsourcing*. This occurs not only in elementary procedures such as division algorithms, but also in the more complex notions and procedures up to and including proofs: One needs not know why the division algorithm being applied works in order to get the right answer when dividing; one needs not bother with the logical reasoning behind an equivalence transformation when using such a transformation to solve an equation; and one needs not recognize the basics of Set Theory of the concept of function in order to succeed in Calculus when calculating a derivative. One uses many of these mathematical concepts and procedures as comprised bits of knowledge (modules) within mathematics, of which one needs to know very well the effects and the “interfaces” to outside in order to be able to apply them correctly, but not to know their internal workings. (Such modules are frequently called “black boxes”.)

In a certain way outsourcing occurs in mathematics whenever one abstracts relationships from the (reference) context and presents them with symbols, thus outsourcing the problem in the formal-operative system of mathematics. This outsourcing allows operations to be carried out on a syntactic level, without having any correspondence to the reference context and not being bound to it (therefore, in a certain sense, “without understanding“); the results calculated within the formal system can be interpreted in the original context and result in the solution of the investigated problem. Such an approach reduces the complexity of the problem, it allows for economic thinking as well as for solutions and methods of solving which otherwise, without the possibility of outsourcing in the formal system, would either not be found or not be so simple to find.

Such an outsourcing is something genuine for mathematics; it is one of the characteristics of mathematics and it is an essential basis for its performance ability and efficiency. Computers and CAS are for the moment just the last step in the development; they are an extension (in regard to transformations by rules) and perfection of outsourcing made possible by materializing mathematically abstract situations in machines.

One can immediately establish analogies between these scientific-theoretical considerations and considerations on the *socio-philosophical level*: in our high-tech and high-economic society with its organisation by division of labour the constant use of "black boxes" has long become an indisputable necessity.

Mathematics is taking on a special role with increasing social importance:

> "Mathematics is relatively secure, socially accepted, codified knowledge which, notably allows for a separation between understanding and doing ... (it) owes its high social relevance to the fact that, in utilizing outsourcing, it even works when the user has no idea anymore as to why". (Peschek 1999b, p. 406)

(A more detailed and encompassing discussion on this matter can be found, for example, in Fischer 1991; Maaß & Schlöglmann 1988; Peschek 1999a.)

Thus, an elaborate image of mathematics, of its characteristics, its ways of thinking and working and its socio-cultural relevance would include outsourcing as an important scientific-theoretical and socio-philosophical aspect; CAS and computers in general, as notable and illustrative examples of such an outsourcing, could simplify reflection and discussion of this basic characteristic of mathematics.

Peschek has formulated these thoughts as a didactic principle, the principle of outsourcing (cf. Peschek 1999a, 1999b; Peschek & Schneider 2001a, 2001b).

This principle of outsourcing shows clearly that outsourcing does not at all mean renouncing mathematical understanding. The opposite is true: operative knowledge and operative skills should only be outsourced on CAS in as far as this outsourcing seems to make sense didactically (which needs to be considered by, concretised, reasoned and perhaps also negotiated with the students in each case). At any rate there are prerequisites, effects, ranges and limits of the operative modules to be reflected upon in order to guarantee an understanding and efficient use of the modules. And last but not least, outsourcing should also be experienced as a fundamental characteristic of mathematics and be understood as a constitutive aspect of its social relevance.

The computer and especially CAS can serve as very demonstrative and obvious examples and models for such an outsourcing to experts and like these be thematically reflected upon and used in the teaching of mathematics.

3.2 Using the space made available

In accordance with the conception of general education sketched in chapter 2, the principle of outsourcing aimed above all at reducing operative abilities and skills in favour of basic knowledge and reflection.

CAS are able to operate with symbolically presented mathematical objects in a way which is equivalent to operating by rules (in narrower sense) within mathematics. For these purposes they have been developed, in this sense they are a modern materialization of (constitutive parts of) mathematics and at the same time, for the moment, the latest step in the development of trivialising it. Through using CAS, operative knowledge and skills are being brought into the mathematics classroom. Operative knowledge and skills are to a great extent available to students under certain preconditions without them cognitively having to develop the knowledge and the skills themselves. In a CAS-supported mathematics classroom `solve(ax^2+bx+c=0, x)` is the solution of the equation $ax^2+bx+c=0$ as well as operating with the formula for solving quadratic equations in a "traditional" (no use of CAS) mathematics classroom. In this sense, the ability to use CAS in an adequate way could be considered as a modern form of operative mathematic knowledge and skills.

Through outsourcing free space arises for the teaching of mathematics which offers the possibility to concentrate the classroom communication on basic knowledge, basic concepts (incl. forms of representation) as well as on reflection and interpretation. Figure 2a and 2b illustrate this shift in focus.

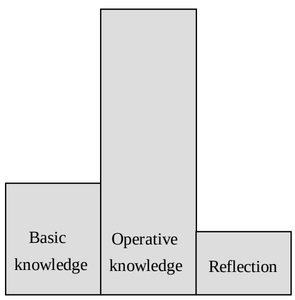

Figure 2a: Traditional teaching of mathematics

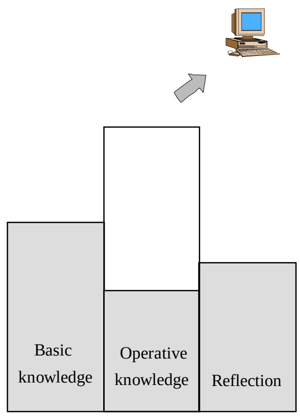

Figure 2b: CAS-supported teaching of mathematics

A communication focusing more intensely on basic concepts and basic knowledge as well as on reflection and interpretation of mathematical expressions and/or representations deals with just those fields of competence which in the sense of Fischer's conception of general education should be considered as particularly relevant for generally educated lay-persons and for their ability to communicate with mathematical experts.

At the same time CAS also make available "experts' knowledge" which can form the content of the communication (in the classroom) whether by asking for “answers from the experts” or by interpreting and assessing “experts' answers“.

In the following we will point out which consequences such a focus in teaching of mathematics can have for the consideration of mathematical notions and concepts. This description is exemplarily completed by concrete experiences made within the project “Use of TI-92 in teaching mathematics“ which we carried out in the period 1996-2000. In the course of this research and development project two mathematics teachers who wished to change their teaching from “computerless“ to CAS-supported teaching were given advise and support from us regarding questions of mathematical content, didactics issues, and (classroom) organisation. (A more detailed discussion of the project can be found, for example, in Schneider 2000 or 2002, p. 72-170.) One of the goals of the project has been the common development of conceptual ideas and of teaching materials for a continuously CAS-supported teaching of mathematics lasting several years, as well as testing (and evaluating) them by the teachers in the classes.

Different forms of representation

Doing mathematics assumes the mastery of representations used in mathematics. These are the means and object of each mathematical activity. Whoever wants to (or must) learn mathematics, cannot avoid learning the representations and forms of representation used there; thus, in this sense they are part of the renounceable basic knowledge of mathematics.

Representations have also to be assigned to basic knowledge with regard to communication between experts and lay-persons (cf. chapter 2), whereby it is immediately clear that the ability to communicate increases with the mastery of different forms of representations. In addition, especially graphical and tabular representations (of mathematical situations) are also of relevance outside of mathematics – both for the mutual communication and for the reflection and assessment of results.

Schematic representations (like graphical and tabular representations) mostly still shows particularly close connections to certain aspects of the reference context and (so) they can give rise to intuitive presumptions and to the development of general ideas. The generalisations done mentally then are expressed with appropriate symbolic representations. Because of the higher operative flexibility of symbolic representations they are suitable in a particular way for syntactic justification and operative interpretations. The reflection and adequate use of such relations between different forms of representation can give insights into essential ways of thinking and working in mathematics.

Different forms of representation focus on different aspects of mathematical situations, too. This seems to be relevant with regard to the interpretation, the reflection and the communication about mathematical situations but also with regard to the development of rich basic ideas about mathematical concepts, of linked basic knowledge and basic understanding, as well as of “translation qualifications“. Of particular importance here are the shift between different forms of representation, the transformations being necessary, and the relations between the representations.

Therefore, an intensive discussion of different representations should be of particular relevance in mathematics classrooms. This should be done also for the use of different representations as descriptions of relations in non-mathematical contexts and for appropriate interpretations of different mathematical representations in reference contexts. In this connection, the functionality and efficiency of representations in the same way as the limits and weaknesses should be reflected upon, decisions on that should be taken within the context, as well as the required qualifications of interpreting and translating being discussed.

CAS allows for a rapid construction of a few important representations (graphs, tables) and thus also for a rapid and uncomplicated shift between different representations. Modifications within one representation (for example modifying the part of the graph or table of a function represented on the computer screen) can easily and rapidly be realised, too.

This easy availability of various representations raises of course their efficiency and could be used didactically in the sense of the considerations mentioned above: The (graphical and tabular) forms of representation offered by CAS can be used much more frequently than would be possible manually. Thus, students can get to know these important mathematical representations (as basic knowledge of mathematics) much better; they can learn to interpret them and to use them, in order to represent mathematical situations and ideas and to be able to communicate about them. The easy and rapid change between various representations as well as their parallel use allow for multiple interpretations and “translations“; there are sufficient opportunities then for mathematical discourse (among students) about the purposes of the various forms of representation and their advantages and disadvantages as well as a lot of opportunities for experiencing the convenience of various forms of representation for the development of intuitive presumptions and ideas, for syntactic justification or for operative working with mathematical concepts.

Within our project we were able to observe obvious changes towards an increased use of different forms of representation in the teaching of the two teachers. Across almost the whole curriculum the mathematical representation of non-mathematical situations and the interpretation of mathematical representations with regard to the reference context were required, whereby different forms of representation had to be used. Changes were frequently made between different forms of representation (verbally, symbolically, tabular, graphically) and also within one representational form representations were often changed (algebraic and recursive descriptions, different parts of function graphs, different intervals of tables, etc.), whereby also functionality and effectiveness (strengths, weaknesses, limits) of different (forms of) representations were brought up for discussion. One example from the

CAS supported classrooms within our project is the representation of growth and decrease processes by verbal descriptions, by recursive and algebraic function equations, by function graphs as well as by tables, whereby in each case the use of these representations was decided within the context by the situation in question (problem) and by the information required (see in addition Schneider 2000; 2002 as well as Prugger & Rauniak & Schneider 2000).

One finds further encouraging examples supporting the increased use of different representations in the teaching of mathematics for instance with Heid et al. (e.g. Heid et al. 1999). However, we must not leave out of consideration that handling different representations and the required capacities of interpreting and "translating" for this have to be practised in a well directed and appropriately guided way - whereby neither the students' enthusiasm nor appropriate success appears automatically (see e.g. Yerushalmy 1991, Canet 1996, Goldenberg 1988, Hillel 1993).

Local meanings and meaningfulness of mathematical concepts

One needs basic elements of the technical language of mathematics in order to be able to communicate with mathematical experts. But it is not sufficient in mathematics only to learn the appropriate technical terms and thus to know the designations, in mathematics certain concepts are connected with the notions which must be understood in order for it to be used appropriately. That is, a mathematics teaching which orients itself on the conception of general education outlined in chapter 2 should aim primarily at the development of basic ideas and basic understandings of mathematical concepts with regard to an appropriate application of them, whereby the focus should be on an intuitive understanding of concepts. The emphasis should be on local meanings of mathematical concepts, on their assessment and on their relevance, as well as on their mathematical representation and on possibilities of their (non-mathematical) applications.

CAS can support this focus by outsourcing the operative to CAS. The possibility of outsourcing operative activities and thus also of time-intensive and complex calculations and/or constructions is directly connected with possibilities such as variations of values of variables and of parameters, repetition of calculations and constructions by means of any number of concretisations, constructions of different representations, modifications of and shifts between them. This can be used in a didactically meaningful way for instance by experimentally investigating aspects of mathematical concepts and of constitutive as well as typical characteristics of mathematical concepts.

In the teaching of our project the teachers' obvious re-orientations in favour of deeper and wider understanding of basic mathematical concepts and procedures could be observed. We were able to recognise for instance the emphasis on the constitutive characteristic of the concept of exponential functions and its meaning for the mathematisation of non-mathematical situations or also the analysis of algebraic and recursive function equations regarding their strengths and weaknesses as materialisations of exponential processes. A further example of this achievement, in Calculus, was the interpretation of the difference and differential quotient as a rate of change in different contexts of application, and the working out of relations between derivative and monotony or of constitutive characteristics of maxima, minima, inflection points, etc. (More details can be found e.g. in Schneider 2002.)

Within the project, the extent of the changes was similarly significant and comprehensive in this field as in the field of using different forms of representation. Thereby competences referring to basic knowledge were more intensely addressed, competences referring to reflection unfortunately more rarely. One of the reasons for this is from our point of view the (still given) strength of task orientation in the teaching of mathematics. Assessments of mathematical concepts, discussions of relevance, thus interpretations and reflections, were attainable mostly only if these were formulated in appropriate tasks - and there were simply too few such tasks in the teaching materials.

Fundamental ideas and global meanings of mathematical concepts

Regarding the ability of communication between experts and lay-persons it will be of interest to know the main links between mathematics and other parts of the whole culture, i. e. to be conscious of fundamental ideas of mathematics (also referring to its ways of thinking and working), and to be familiar with them. So, connections as well as differences between mathematical and non-mathematical ways of thinking, between fundamental (inner-)mathematical ideas and their correspondences in non-mathematical everyday life should (also) be discussed in the teaching of mathematics; the development of adequate knowledge in this matter, thus of a knowledge about possibilities of application, about effectiveness, ranges and limits of mathematical concepts and methods should be a fixed component of mathematics classrooms.

CAS can make available space for focusing on the concepts and methods itself by outsourcing the operative. Beyond that, in CAS the fundamental idea of the functional correspondence is particularly supported for instance by the possibility of the formation of modules (modules as multidimensional functions). In addition, CAS can also be seen as materialisations of another fundamental idea of mathematics – of the idea of modular outsourcing (and the use of black boxes - see section 3.1) – and be discussed and reflected as a particularly illustrative and "vivid" model of it in the classroom.

Efforts towards larger connections and towards fundamental ideas of mathematics could basically be recognised in the project classes (e.g. idea of the functional correspondence - not least also through the characteristics of CAS; investigation and description of non linear changes with the help of the rate of change). More far-reaching connections and fundamental ideas, ways of thinking and working in mathematics or the strained relation between mathematical thinking and thinking in everyday life however were rather rarely and in most cases only implicitly addressed in the project classes (see in addition e.g. Schneider 2002).

Applications

One needs special handling of mathematics in the classroom in order to make the (direct as well as indirect) connections of mathematics to the world clear to the students. Therein both the application of mathematical concepts to non-mathematical problems and mathematical modelling play a central role.

More applications in the teaching of mathematics do not result compellingly from using CAS in the classrooms but refers to a component of the use of CAS in the teaching of mathematics which is very often mentioned in experience reports and project reports and was also observed in our project (more details can be found e.g. in Schneider 2002).

CAS again offers a central contribution to such a focus in the teaching of mathematics by taking over operative activities: The share of the operative is often quite high just in mathematical models of non-mathematical situations and the operative is in such cases often also (from a school mathematics view) more complex and time-consuming so that the relief through the outsourcing of the operative can be noticed particularly clearly. (This is discussed more detailed by Berry in this issue.) The free space produced by this can be used for focusing on representations and interpretations, and thus on the formation and assessment of the mathematical model, as happened, for example, within our project. Beyond that the outsourcing of the operative activities can have a positive effect on the assessment and reflection of mathematical models and on the investigation of the models regarding their “adequacy” (“suitability”), since assumptions of the model can be varied rapidly and without large expenditure, different models can be calculated and checked, etc.

Teaching culture

As in many other (systematically evaluated) teaching experiments, within our project we also could observe obvious changes concerning the classroom interaction in the teachers’ CAS-supported classes (cf. e.g. Schneider 2002).

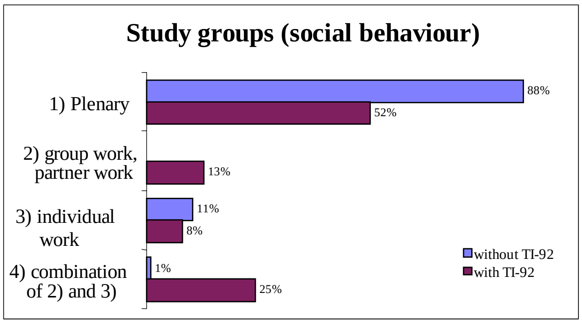

Figure 3: Social behaviour

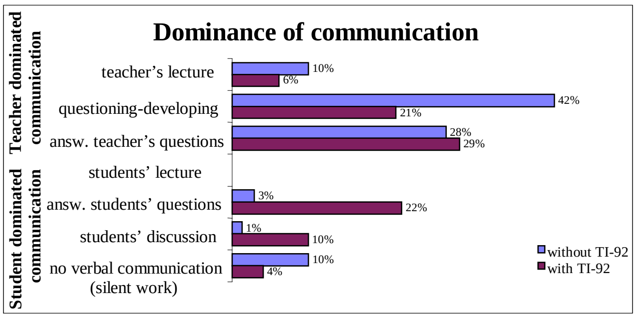

Figure 4: Forms of communication in the classroom

The results of classroom observations represented in Figure 3 and Figure 4 show that in the project classes the communication about mathematics was intensified among the students. The teaching design included to a large extent also forms of interaction, which made independent working of the students on the one hand and communication determined by the students on the other hand possible or even necessary and the organisation of the communication between teacher and students more balanced.

4 Using CAS is communication with experts

According to Fischer, we assigned the field of competence for operative knowledge and skills primarily to the experts. This turns out to be exactly the same field of competence which could be outsourced most completely to CAS. In this (narrow) sense we can perceive CAS as a simple electronic model of a mathematical expert.

Admittingly CAS does not cut a very good figure as a mathematical expert, because it is too limited and rigid in its communication with us users; its basic knowledge of mathematics, its abilities of representation and of interpretation are too insufficient. CAS can sometimes even disappoint us in the operative field. CAS can never become a substitute for human experts (and particularly not for teachers). But in using CAS, in the communication between human being and machine, elements can be seen that are quite significant, also for communication between lay persons and human experts, because:

A successful and profitable interaction with human as well as electronic mathematical experts requires

- wide basic knowledge of mathematics (especially also knowledge about important mathematical forms of representation);
- quite exact conceptions of the nature of the expert‘s knowledge, of the possibilities and limits of the mathematical expert‘s knowledge and skills, and of the range of validity of mathematical expressions;
- the willingness and ability to ask the “right“ questions, to be precise when formulating one‘s own questions and considerations and to present them in a form which can be interpreted by the expert;
- a verification as well as an appropriate interpretation and assessment of the answers given by the expert.

Whenever CAS users (students) are working in corresponding forms of interaction, something else is happening: There might arise the transmission of the answers provided by CAS to other lay persons, the discussion of these answers among the lay persons and the negotiation processes of their interpretation and justification as well as of any further questions to the expert. All in all, these are essential components of what we described in chapter 2 as communication with the general public.

For to the reasons briefly outlined here we see the use of CAS as a simple but very useful model for the communication between mathematical experts and lay persons. The reflection of it can and should be used didactically and pedagogically.

5 Concluding remark

The further development and concretisation of the aspects mentioned in this paper require further and deeper investigations and negotiations about what, for instance, should be considered as basic knowledge of mathematics and in particular how the area of reflection could be more concretised and realised in the teaching of mathematics, and how (which sequences of tasks, teaching culture) competences of reflection could be taught systematically in the classroom. Comprehensive consideration of such questions is intended to be done in our future research work.

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