"Locus" and "Trace" in Cabri-géomètre: relationships between geometric and functional aspects in a study of transformations

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Abstract: The present text describes and characterises the tools "Locus" and "Trace" of Cabri-géomètre II, in relations to a study of geometric transformation, more precisely, the passage from the notion of transformation of figures to the notion of applications[1] that map points on the plane onto the plane itself. In particular it discusses how the conception of image of a figure under a transformation can evolve – through interaction in a "*milieu*" organised around Cabri-géomètre – such that students move from views of figure-images as undecomposible entities to see them as sets of image-points. Moreover, the study allowed the identification that the notion of trajectory (in a dynamic interpretation) has an important role in this conceptually difficult passage and that dynamic geometry environments renovate this notion.

Kurzreferat: Der Text beschreibt die Werkzeuge "Ortskurve" und "Spur" der Software Cabri-géomètre-II und deren Rolle beim Studium geometrischer Abbildungen. Genauer wird der Übergang von den Abbildungen einer Figur zu den Abbildungen aller Punkte der Ebene untersucht. Im Einzelnen studieren wir, wie sich der Begriff des Bildes einer Figur unter einer Abbildung entwickelt, wenn sich diese Entwicklung in einer Umgebung ("milieu") vollzieht, welche durch die Nutzung von Cabri-géomètre gekennzeichnet ist. Die Lernenden gehen dabei von einer Sichtweise der Bildfigur als unzerlegbare Einheit über zu einer Sichtweise als Menge von Bildpunkten. Außerdem erlaubt die Untersuchung die Feststellung, daß der Begriff der Spur einer Bewegung (in einer dynamischen Deutung) eine wichtige Rolle in diesem begrifflich schwierigen Übergang spielt und daß Dynamische Geometrie-Software (DGS) dieser Vorstellung neues Leben einhaucht.

ZDM-Classifikation: C30, C70, G50, N80, U70

0 Introduction and context

The notion of geometric transformation occupies an important place in mathematics teaching in various countries and, in particular, in France, which is the context for this research. A number of Mathematics Education research studies have been devoted to the understanding of transformations (see, for example, Grenier, 1990; Healy, 2002; Küchemann, 1981). According to Grenier&Laborde (1988), transformation can be understood at different levels. Among others, these include:

Level 1 – relationships between two figures or two parts of the same figure. The concept at this level seems to be connected to the context figures and hence involves the transformation of figures.

Level 2 – applications that map points in the plane onto the plane itself.

In the French lower secondary school, geometric transformations are introduced at level 1 (*Collège*, 11-15 years), but from *Lycée* (15-18 years) they should be studied at the second level. This is a change from a transformation that operates globally on a figure (a movement) to an application that operates on points and over figures as parts of the plane composed by points. We analysed the students possibilities to establish relations between these two aspects which we name global (in synthetic geometry) and point-wise (in a functional interpretation). It is in this context that we highlight the intervention of the concept of locus and the use of an dynamic geometry environment.

1 Sets of points, trajectories and loci in the functional approach

The notion of Locus is introduced naturally in the study of transformations, once they are defined while application that map points on the plane are not as easily understood. Indeed, studying the French textbooks, we perceive that this notion appears, generally, in the transformations context (case 1), showing that these transformations are very efficient tools to solve loci problems. On the other hand, the term "locus" may also be used in the application context (case 2) whose meaning is not the same. Let us highlight the differences.

In a last case (2), locus is a set of points having a certain property: a characteristic condition determines whether a point belongs to the set. This is the case, for instance, of the "classic" point-wise characterisation of objects, such as perpendicular bisector, angle bisector, conic, and more particularly circle.

In a first case (1), we refer to a set of points that are images of a set of points defined as the image of an object under an application or transformation. "If we call f the function $M \rightarrow N = f(M)$, searching the locus of N is searching the set of all points f(M), that is searching the image of line L under f" (translation RS[2]); see Antibi&Barra, 1996: Transmath 1ère S, p. 376.

So, locus is defined in a functional form, theoretically we have to consider that a variable point P which belongs to a figure F considered as a set of points (a straight line, a circle,...) corresponds to a point P', image of P under an application f. Locus has a double meaning: it legitimates the change from a synthetic figure (a global point of view) to a figure as a set of points, and it allows to recompose the figure.

When analysing textbooks we also notice that the notion of locus is often presented - on the high school

[1] In this paper we do not distinguish between the terms transformation and application. It is interesting to observe however that this is not always the case in secondary school teaching, where transformation is reserved to designate bijections of $\mathbf{R}^2$ (or $\mathbf{R}^3$).

[2] "Si on appelle f la fonction: $M \rightarrow N = f(M)$, chercher le lieu géométrique de N c'est chercher l'ensemble de tous les points f(M), c'est donc chercher l'image par f de la ligne (L)."

level - in a dynamic way: “Supposed a point M moves on a fixed line (L), another point N which is linked to M, will move too. The problem consists of finding out on which fixed line (L1) point N will move. This line (L1) is called 'geometrical locus' of N if point M moves on line (L)” (translation RS[3]; see Antibi&Barra, 1996: Transmath 1ère S, p. 376).

The locus (or the transformed figure) is then identified with the trajectory of a point N constructed as the image of a mobile point M in the figure (L). This dynamic point of view avoids the use of quantifiers and the language of set theory. Even though this didactic choice doesn’t correspond to the notion of movement presented in Physics, inspired by a historic study, we adopted the dynamic interpretation “*mobile point on a curve*” as an intermediate phase with the aim that the students may grasp a point oriented conception of a transformation.

The didactic necessities that determine the distinction between trajectory and locus are equally present in the creation of the tools “Trace” and “Locus” of Cabri II, which are described in the following section.

2 "Locus" and "Trace" in Cabri-géomètre

The distinction between trajectory and locus as described above is reflected in the form of two distinct tools present in version II of Cabri-géomètre, “Locus” and “Trace”. A locus as produced by the “Locus” tool behaves in many ways like other Cabri objects: for example, it remains visible on screen, moving accordingly as the elements upon which it depends are manipulated (no longer disappearing as in the preceding version). A point can be constructed on it and moved in the same way as a point on other Cabri objects and it can be defined as a final object in a macro-construction. In addition, it is possible to obtain a locus of objects such as lines, rays, segments and circles and hence generate their envelopes (see Figure 1). However, a Cabri-locus differs from other Cabri objects in that it cannot be used in the construction of intersection points.

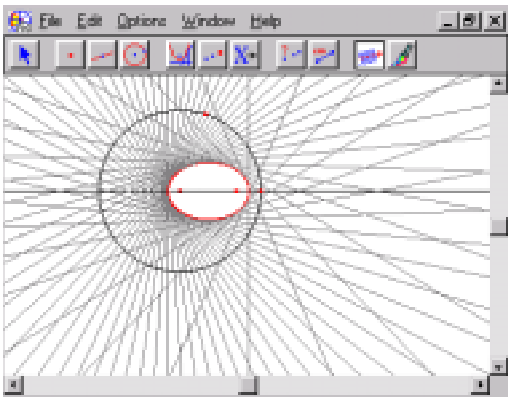

Figure 1 – Ellipse as envelope of a line

In the case of the loci of points, the subject of the work reported in this article, the "Locus" tool of Cabri II produces a set of points, L, such that each element is defined in function of an element from the set E: $L = \{f(P), P \in E\}$, and will appear on the screen as a sketch (a representation) of the set L for a finite number of f(P)[4]. To define such a locus it is necessary to select a point P' for which the locus is desired and then the point P on which P' depends (where a functional relationship exists between P and P'). The point P is a “variable” point that belongs to particular set of points of the plane (a line, a circle, a line segment...) and the point P' is related to P by a geometric construction. The points P' of the locus are calculated by the software and obtained directly and it is not necessary to drag the point P. The locus is immediately represented in its entirety – which was not the case in Cabri I where the “Locus” tool is related to dragging (that is, has a dynamic aspect). In fact, in the first version of the software, using a “point on object” M (with one degree of freedom) and a point M' (with zero degrees of freedom) that depends on M, the locus of the point M' is produced as the trace of its successive positions when M is dragged. A (manual) locus can also be produced by selecting any point on the screen – including free points (points with two degrees of freedom) – and observing the trajectory generated when this point is moved. This second use of the “Locus” tool of Cabri I corresponds to the “Trace” tool in Cabri II. “Trace” allows the user to instruct certain objects on screen to leave a trace when they are moved, either manually using the mouse or through the use of the “Animation” tool. The trace does not exist as an object of Cabri, only as a set of pixels highlighted on the screen. Roughly speaking then, in Cabri II, “Trace” emphasises a dynamic interpretation of the representation of a trajectory of a point, while “Locus” is characterised in a functional manner by a one-to-one correspondence between two points P and P', representing, at least implicitly, the image of a set of points for a certain application.

Under the conditions described above, not all loci can be obtained through the use of the “Locus” tool of Cabri II. The restrictions relate to the type of transformations of geometrical configurations that are possible using the drag-mode. For example, in Figure 2, since point M is a free point, it is only possible to sketch the required locus (a circle) using the “Trace” tool, whereas Figure 3 below

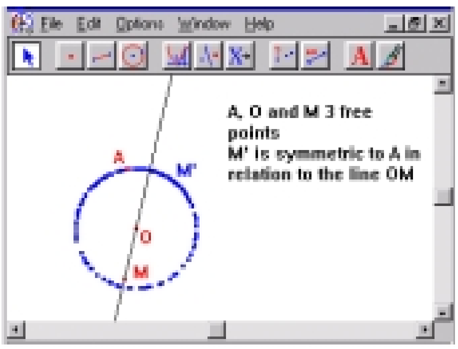

Figure 2 – Visualisation of the locus of M’ using “Trace”

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[3] “on suppose qu'un point M se déplace sur une ligne fixe (L). Alors, un autre point N, associé à M, bouge aussi, et le problème consiste à chercher quelle est la ligne fixe (L1) décrite par le point N. Cette ligne (L1) est appelée lieu geométrique du point N lorsque le point M décrit (L).”

[4] n: number of points of the locus, with $5 \le n \le 5000$.

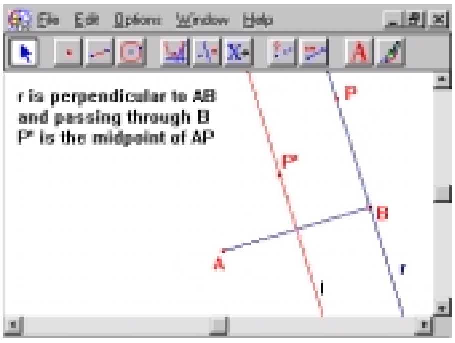

Figure 3 – Perpendicular bisector of AB using “Locus”

presents a case in which, although the ‘Locus’ tool can be used, an auxiliary construction is needed in order that the desired locus (the perpendicular bisector of AB) may be generated.

Schumann and Green (1997), mentioning Cabri I, they show some uses of the interactive generation of loci. In our work, we are interested in a specific kind of problem described by them as “in investigations of the position and shape of the image of a transformed original shape” (ibid., p. 80). From the characterisation of the tools "Locus" and "Trace" of Cabri II, we formulate the hypothesis that they offer new possibilities of interpretation of functional dependence, being able to follow the didactical introduction of the concept of function in Geometry.

With this in mind, a study was designed to investigate how the Cabri-géomètre environment, and especially the distinctive and comparative use of the “Locus” and “Trace” tools, accommodates (or perhaps favours) an approach to the notion of geometric transformations that brings into evidence both their functional character and the importance of the preservation of properties. The remainder of the paper presents some of the situations from the didactic sequence designed during the study. The overall aim of the sequence was to “problematise” the construction of an image under a transformation and to relate this to the notion of locus.

3 An experimental study in the context of transformations

The experimental sequence was composed of four situations to be worked upon during seven one hour sessions by a class of 33 students (aged 15-16 years) from a public school in the south-east of France. It is important to observe that the students who participated already had had around six months experience with Cabri in their mathematics lessons as part of a project[5] aimed at the design of Cabri-integrated learning scenarios (of with the transformation study formed a part). This meant that the students were familiar with most of the Cabri tools and that the principle of robusticity of constructions in Cabri had been previously negotiated with, and accepted by, the majority of students.

During six of the research sessions, the class was divided into two groups, while the seventh session was conducted with the whole class. During their interactions with the proposed activities, the students worked in pairs in the computer laboratory. Five pairs were selected for case-study. The analyses[6] presented in this paper mainly refer to two of the four situations of the sequence entitled *“Affinity”* and *“Oblique symmetry”* respectively and focus upon aspects related to the understanding and use of the “Trace” and “Locus” tools of the case-study pairs (for a complete description of the sequence, see Jahn 1998).

3.1 Generating a conic

This situation was designed with the aim of characterising a locus as the image-set of another set by an application applied to points in a geometric setting. The problem proposed to the students was to construct, point by point, an ellipse as the image of a circle (C) whose centre lay on a line (d) under an orthogonal affinity with the line d as axis and a ratio of 1/2. This affinity was introduced as a simple geometric construction (see, figure 4a), whose steps, along with the associated Cabri-tools, were presented on a worksheet given to the students (a *“guided”* construction). Starting from this construction, the idea was that students would examine the correspondence between a point M on the circle C and its image-point M' (without speaking of transformations) and, as a consequence, would consider the set of points M' as point M describes the circle; that is, point M was to be treated as a variable point. Figure 4b reproduces part of the worksheet given to students.

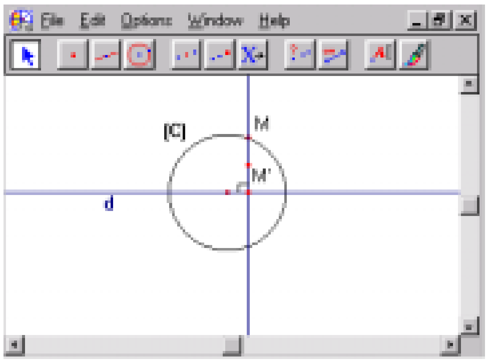

Figure 4a – Affinity of circle (C)

Drag M and observe the movement of M'
✐ *Description of observations:*
➢ You have just built a construction that allows the association of each point M on the circle C with a point M'.
💻 Determine the set of points M' as M varies on the circle.
✐ Explain how you obtained this set.

Figure 4b – Students’ worksheet

Students at this level of schooling were not expected to recognise the properties of an ellipse – a topic they had

[5]“*Conception et évaluation de scénarios d'enseignement avec Cabri-géomètre*”, a project of the team EIAH of the Leibniz-IMAG laboratory, IUFM of Grenoble, with a grant from the Région Rhône-Alpes and the INRP (1999-2000).

[6]The analysis was based on data collected from each section. It contained experimental protocols (transcription from audio), observers’ notes, written work sheets from pairs of students and Cabri files (figures and macros).

not yet studied – instead the activity enabled a response in the form of a representation of the image-curve on the computer screen, produced by means of the Cabri tools.[7] In practice, as the set they had asked to identify did not represent a figure known to the students (such as a line or a circle, for example), they needed to build it point by point; that is, producing a visual representation of the complete set necessitated the reproduction of the initial construction on various positions in the circle C or, the use of the tools "Trace" and "Locus".

It was effectively the last strategy that the students employed. Although they at first paid more attention to the geometric properties of M' than to the nature of the image set described by this point, two pairs (B1 and B4) concluded that M' described a curve. It was by dragging point M that they modified their original conjecture that the image of the circle C would also be a circle, with both pairs going on to suggest the image consisted of two arcs before the students in B1 settled on the term "oval" while those in B4 identified the set as an ellipse. Because they wanted to visualise the trajectory of M', already experienced dynamically through the dragging of M, these pairs privileged the use of the "Trace" tool and did not use "Locus" as their first option.

S2 – B1
Lud: *Determine the set of points M' as M varies* [reading from the worksheet]
Lau: Isn't it this... it's a curve, isn't it?
Lud: We have to have a "Trace". Does it still have "Trace"? [to the observer]
Obs: Yes, I think so... It looks like you have the complete.
Lud: Let's see what happens. [activates "Trace" of the point M' and drags M]

These students attributed to the "Trace" tool a function of representing the curve of the trajectory of a point, allowing them to better visualise or understand the object in question. On top of this, the output of the "Trace" tools seemed to completely satisfy the students, to the extent that the "Locus" tool had only a contractual role (generating the robust construction emphasised by the teacher) or was used in the second part of the task as a means of constructing an object on which various points M' could be placed – more precisely the five points that were necessary in order to define a conic using the "Conic" tool of Cabri.

Three pairs did make use of the "Locus" tool – although only after first obtaining an image of the locus using "Trace". In fact, for the students the functions of the two tools are very similar (almost equivalent), except that they saw that the output of the "Locus" tool could be recognised by the software – at least in terms of its points.

S2 – B2
Hor: Where is "Refresh drawing"?
Lil: In "Edit"
Hor: Really, "Trace" and "Locus" are kind of the same, aren't they?
Lil: I don't know!
[...]
Hor: OK, I put that we used "Locus" and...
Lil: Of M'! And "Trace" as well.
Hor: Yes, but there we have the locus, the actual curve.

As previously described, the use of the "Locus" tool assumes some understanding of a functional relationship between two points and its application in practice reflects this relationship. Some hesitation on the part of all five of the case-study pairs was observed during considerations of the arguments of this tool. The excerpt below, for example, illustrates how the students in B5 were confused about the respective roles of the two points they were selecting:

S2 – B5
[Bea had selected M then M', as they tried to apply "Locus"]
Aman: With "Locus", it didn't redo it!
Bea: How do we use it? Wait, can you help me do it?
Aman: Get "Locus".
Bea: Of M or of M'?
Aman: Of M and afterwards click M'. Or the other way round... I don't know anything!
Bea: This and this [choosing M' then M]... Magnificent!

Moreover, the situation "Affinity" allowed the students to experience the differences in the ways of using and in the products of the Cabri-tools "Trace" and "Locus" of Cabri. At the end of this activity, related to Cabri-dragging, a first difference was established: this led the pairs to attribute to "Trace" the function of providing a provisional sketch and to "Locus" the function of determining the geometric object introduced by the initial construction – "the curve described by M' " – that could be used in attempts to validate their results.

3.2 Transformations which transform: the case of Oblique symmetry

The situation "Oblique symmetry"[8] was directly located in the context of geometric transformations. It consisted of an investigation of the images of various objects – points, lines, polygons and circles – by a symmetry in a given axis and parallel to a given direction. Using the tools of Cabri, it was possible for students to have access to this unknown transformation without explicitly defining it. In effect the transformation represented a "black-box" and the first task of the students was to characterise it by studying the behaviour and properties

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[7] The second part of this activity, not discussed in this paper, involved the identification of the locus of M' as a conic (and the relevant Cabri-tool was hence introduced), with the students asked to produce a mathematical justification in an analytic setting, via equations.

[8] A non isometric transformation was chosen to allow for a possibility of a point-wise approach to transformation, avoiding the immediate recurrence of conservation theorems (see figure 5 next page).

of a pair of points (P, P')[9].

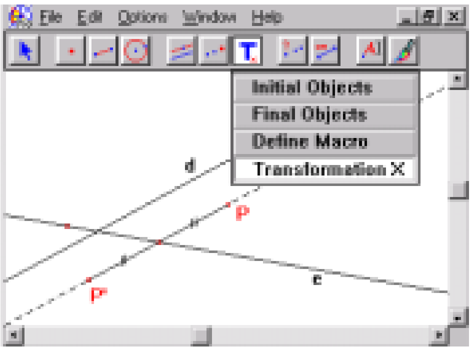


Figure 5 – The black-box “Transformation X”

In the second part of the activity, the students were encouraged to consider the image of a figure as a set of points, or rather as a locus, in a functional setting. The task consisted of constructing the image of a circle under “Transformation X”. In terms of the passage *from points to figures*, the question was: how could the image of a circle be constructed on the basis of the image of a point, if the properties of the transformation where unknown.

In order to obtain an initial idea, it was proposed that the students consider four distinct points of the original circle along with their respective images and then modify the position of the circle-points whilst observing the behaviour of the images. The aim of the activity was to put in doubt the theorem of the isometries *“the image of a circle is a circle of the same radius”*, that was very familiar to the students.

This activity was very important when students considered only global aspects. It allowed students to visualise and understand that, in this case, the image of a point on the object-circle (C) is a point of the image of (C).

A prediction in the form of a conjecture formulated by the students in response to the subsequent question (obtaining the set of point-images of the complete point-set of the circle under the transformation X) indicated whether this role was achieved. The final task consisted of combining the macro-construction “Transformation X” and the “Locus” tool to obtain the image of the given circle.

In terms of the behaviour and strategies of the students, the “Locus” tool tended not to be the immediate choice. It was students’ analysis of the images of the four points that created the first favourable rupture: four pairs began to doubt their initial idea that the image of a circle is always a circle of the same radius – a first “deformation” of the circle was observed by these students. The fifth pair, insisting on the idea that the image-figure has to have the same form as the original, tried to construct a circle passing through the four image points, however this strategy was invalidated as the construction failed. The other students embarked on a search for a tool that could eventually sketch the required image. Always with the help of dragging the initial points, various conjectures about the image of the circle emerged during this phase: parabola, oval, semicircle, ellipse…, and the attempts to identify the form of this figure-image induced the students to use either the “Trace” or “Locus” tools. More precisely, among the five case-study pairs, only one utilised directly the “Locus” tool, the other all chose first to use “Trace”.

S3i – B1
Lau: Wow, its an oval!
Lud: Yes! Ah yes, it makes an oval!
Lau: Look, did you see? Actually its as if we can see the circle inclined!
Lud: Can’t we use "Trace"?
Lau: Ah yes! Let’s try! "Trace", where it is? No by the side! Select A' and... Yes, so really we can predict the shape.

Once again, the passage to the “Locus” tool seemed for the majority of students to be contractual on the influence of the teacher and its use was motivated by the necessity of saving the figure (the last question). For the students, the “real” trace of the curve was only possible through the use of “Locus”.

S3i – B4
[after using "Trace"]
Nad: How do we do "Locus"?
Gér: Why do you want to do "Locus"?
Nad: To trace.

As a result of the analysis of the “Oblique symmetry” situation two suggestions related to the students interpretations of the two Cabri-tools can be made:

* “Trace”, except for B1, is always privileged. The idea of trajectory is strongly present for the students who can control perfectly the use of this tool. There were numerous comments that the “Trace” tool allowed an examination of the figure described by a point when an associated point is dragged.

* “Locus” was not easy to use: the students had difficulties in understanding the order of its arguments (arguments which represented its underlying functional relationship). Its use was generally motivated by the limitations of the output produced by the “Trace” tool: this output could not be saved and had to be deleted and remade each time the elements on which it depended were moved.

S3i – B1
Lud: Doesn’t it have "Locus"? Let’s use "Locus".
Lau: "Locus"? Must be round about here...
Lud: No by the side! How do you use "Locus"?
Lau: Uh...
Lud: You have to get 5 points [probably he is thinking of the "Conic" tool]
[They read the help message]
Lud: Did you understand? Locus of an object...
Lau: Circle !
Lud: A point, isn’t it? In relation to another that moves on an object?
Lau: So we do C and C'!
Lud: Or C' and C...

[9] A macro-construction simulating an oblique symmetry, which applied only to points, was available to the students.

The fourth and final situation (entitled "*Conchoid*"), the study of an exotic transformation in which alignment was not preserved, reinforced the considerations already presented: out of five pairs only two pairs represented the images of figures using the "Locus" tool without first utilising "Trace" during an intermediary step.

When the comments and interpretations present in the protocols of the rest of the students on the class were also considered, it was found that the "Locus" tool seemed to gain a particular role in the transformation problems: it is the form by which the *global* image of the figure can be obtained, starting from a figure and one of its points. In this way, "Locus" appears to be characterised globally.

S3 – B2
Prof: What do you want to do?
Hor: "Locus".
Prof: And what does "Locus" do?
Lil: It draws a curve... "Locus" gives the... uh... the figure!

An evolution could be observed during the experimentation, as, by analogy to "Trace", the "Locus" tool comes to be interpreted dynamically. Using "Locus" by the end of the third and also in the fourth situation, the idea of trajectory arose again.

S3 – B5
Teach: And then you go to "Locus". What are you saying? Do you want the locus of what?
Bea: The locus of A.
Aman: No, of a circle!
Bea: I don't know!
Teach: What do want to see?
Bea: I wanna know what happens when point A moves on the circle.
Teach: OK. That means that you want to see, what?
Bea: The locus.
Teach: Of what?
Bea: Of A?
Teach: No, A is a point of the circle.
Aman: Of A'.
Bea: Yes, of A'.
Teach: So, make this one [point A'] and then you assign the starting point.
Aman: If I use "Trace"?
Teach: It is the same thing. You trace that one [A'] while...
Bea: I see A' when A moves.

S4 – B5
Béa: You have to use "Locus". First make the point [with the macro-construction] the locus of this point when the other moves.

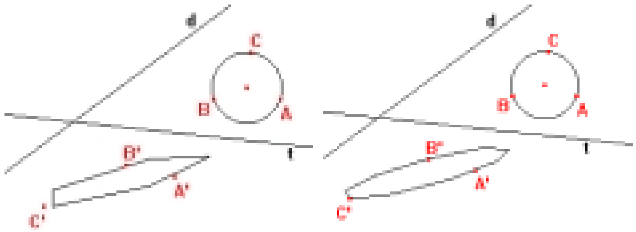
In this way, students begin to make use of functional perspectives, referring to the possible positions of the initial points and the corresponding positions of the associated image-points. It contributed to return to a point-wise conception for "Locus".

S3 – B4
Jér: We constructed the image of a point and Cabri… uh… "Locus" I mean, gave the other automatically!
The image of all the points of the figure when I move that there…

"Locus" also constituted a form by which the students could approach continuity. Trying to understand the type of representation made by the computer and the preferences relative to the tool (the number of points of the locus could be increased or decreased using the keys "+" and "-"), some students commented:

S3ii – E1
Jos: There is a load of them... you have to have loads to show a curve... there [pointing to the screen] we put 100! But you can have more...
Prof: What is it doing?
Jos: The computer? It doesn't do it of all of them! But it gets lot and draws little segments, I think…

S3i – B4
Jér: Look! You can have various shapes! [increasing the number of points of the locus which began with 5]
Thi: What?
Jér: So there can be various shapes... Pentagon, with 7... 10, look! (see Figure 6)
Thi: It's not just a few. It joins the points... you have to put lots.
Jér: I'm increasing it... [using the key "+"]
Thi: To approximate the curve.



(a) (b)

Figure 6 – Loci with reduced number of points

At the activity 2, demonstration was done by using Analytic Geometry, that allowed to characterise algebraic conditions for a point of the locus and to compare it with a conic defined by 5 points of the conic. The conic equation was done by Cabri. In the activities 3 and 4 however, mathematical validation was not required. There is evidence that the students did not stop on the levels of actions and perceptions. For instance, in activity 3 the task to prove that distances were not invariant was motivated by students' will to explain (or to understand) why the image of a circle was not a circle. As stated by Laborde (1998, p. 90) "By coming up against the impossibility to use an invariant, one realises its remarkable character. Here we are at the heart of Mathematics: it is one of cases when a property is verified by those cases where it does not hold" (translation RS[10]). In our case study, it was facilitated by the dynamic geometry environment, particularly by the

[10] "C'est en se heurtant à l'impossibilité d'utiliser un invariant, que l'on prend conscience de son caractère remarquable. Il s'agit là d'une de ces mises en relation cruciale pour les mathématiques: celle des cas où une propriété est verifiée avec les cas où elle ne l'est pas"

tools "Trace" and "Locus" that allowed to diverse experimentation, to study exotic cases and to establish mappings. Many questions are opened on the issue of proof with Cabri. Although it was not the scope of this study it was worth noting what happened.

4 Final remarks

The idea that a transformation can deform objects was very strange for the students. They discovered unfamiliar figures when transforming lines or circles. There was a need for a pointwise investigation to obtain the image of a figure by a given geometrical transformation. The tools "Trace" (for a dynamic approach) and "Locus" (for a function approach), as we showed in this article, were very efficient. Moreover, applied situations allowed the students to be aware of the range of validity of conservation theorems – one property that is not validated allows to differentiate one transformation from another rather than to be considered trivial.

The dynamic interpretation (easily represented by the tool "Trace") was not avoided during this investigation. Furthermore, the proposed problems of image representation do turn the intervention of "Locus" dispensable. In fact, "Trace" guide students to the solution of the problem; "Locus" is used when one needs the technical characteristics of the object created by the software or the criteria of constructions validation, e.g., resistance to dragging. It is still important to explore different types of problems and other applications with "Locus" to make sure that "Trace" is not substituted only because of the didactical contract. According to Schumann&Green (1997, p. 87) "... to draw loci using Cabri-géomètre and to do so using conventional tools on paper involve quite different experiences and skills. It is wrong to discredit the traditional approach – both it and computer-based methods are valuable". We advocate the use of pencil-and-paper after the computer exploration and we suggest to deal with characteristic solutions and the validity of problems in inverse order.

Concluding, we emphasise that representations and knowledge are intimately related to the respective Cabri-functionalities, they are closely linked to this context. This is true because the situations have been conceived to be developed using the computational environment. Cabri was particularly convenient for our purpose. In addition to that, the dynamic approach of the software allowed the students to acquire the notion of transformation by the point-wise approach - which is crucial in their understanding. Considering this condition and according to students interpretations, it would be interesting to have the tool "Locus" generated by the user manually, reintegrating a dynamic characteristic and, why not, becoming a real object compound by the system.

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