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Using History to teach Mathematics

An International Perspective

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Abstract: The questions involved in the connection of the History of Mathematics with its Pedagogy were addressed at two recent conferences, one in Seville (Spain), the other in Braga (Portugal). This volume collects together all the relevant papers presented at these two venues. This review provides brief accounts of their contents.

1. Introduction

The question of the connection between the Teaching of Mathematics and the study of that discipline's History is a vexed one. Everyone assumes a link, but it is quite another matter to describe what that link is, and to proceed to discuss how that link might usefully be pressed into service in the classroom. This is the interface that provides the rationale for this book. Twenty-six papers are reproduced under five broad heads: General Ideas, Historical Influences on Pedagogy, Particular Subjects taught using History, History as Part of Teacher Training, and Chapters in the History of Mathematics itself. These will now be addressed in turn.

2. General Ideas

The first section begins with a brave attempt at an overview of the topic in question. Siu Man-Keung finds four uses for History in the teaching of Mathematics. These are A (for Anecdotes), B (for Broad Outline), C (for Content) and D (for Development of Mathematical Ideas). In summary, these may be grouped together under the heads of enrichment material, choice of syllabus and motivation of the student. A number of case studies illuminate the discussion.

Frank Swetz examines historical and intercultural material with a view to discovering and selecting effective teaching strategies, especially with regard to the organization of material, instructional discourse and motivation, use of visual aids, and finally the use of tactile aids. This last category seems a little surprising, but in the elaboration it refers to a wider class of visual aids (use of color, mnemonic diagrams, etc).

Anne Michel-Pajus gives first-hand accounts of several topics as taught with historical context in mind. The point is strongly made and reinforced by these examples that the current curriculum rewriting brought about by the availability of packages such as MAPLE and MATHEMATICA takes its place alongside other advances in Mathematics, and is not to be seen as a discontinuity of content.

3. Historical Influences on Pedagogy

The second head brings together four rather disparate papers all concerned in some way or another with historical aspects of pedagogy. Lucia Grugnetti finds (*inter alia*) that students presented with a historical context have the opportunity to evaluate their own problem-solving strategies, while the teacher may gain insight into the source of student difficulties.

Wann-Sheng Horng compares Euclid with the Chinese geometer Liu Hui and finds the former to be more concerned with overall structure, while the latter presents a more heuristic approach. In brief, Liu Hui's approach is seen as more suitable for beginners, and Euclid's for more advanced students when problems of rigor come more pressingly to the fore.

Fulvia Furinghetti uses an extended study on the area of a circle to illustrate her position that students have an inborn difficulty learning mathematics, can be assisted by "devices" such as puzzles and history, and that a knowledge of facts and their background is a necessary preliminary to full understanding.

Frank Swetz presents an intriguing array of problems, all set as such, and deriving from a variety of sources: Ancient Babylon, Egypt and China, Medieval Italy, Nineteenth Century America and England, the Greek Silver Age, and Edo Japan. Not only may we admire the diversity of the sources, but the problems themselves are intriguing.

4. Particular Subjects and how History may be used in teaching them

This section also groups together several papers with different focal points of interest. Luis Radford and Georges Guérette discuss quadratic equations and do so from a Babylonian geometric perspective, reaching in the end the familiar "formula". It is a wonderful *tour de force*, but if a reviewer may be allowed a moment of subjectivity, I would not myself use this approach.

Jean-Luc Dorier takes a historical approach to the teaching of Linear Algebra. The flavor of the paper is a reaction to the formalist "Bourbakiste" approach of an earlier generation, and emphasizes the more concrete provenance of highly abstract concepts.

Constantinos Tzanakis discusses the relation of Mathematics to Physics. The discussion is rather general, and individual connections are cited rather than explored in depth. It concentrates on the more "modern" aspects of Physics where the Mathematics involved is often at its most abstract.

The section also includes two papers on the broader issue of Mathematical Understanding and the obstacles to it. Both use a geometric context to develop their argument. Janet Heine Barnett considers the difficulties posed by incommensurables, by the rise of non-euclidean geometries and by the concept of infinity. Again the point is made that a knowledge of the historical development of a discipline can assist the teacher in understanding the sources of student difficulties.

Evelyne Barbin addresses the concept of "obviousness". The point is well made that the use of an algorithmic approach (co-ordinate geometry) often leads to complications avoided in the more pictorial techniques

of the more traditional school. Again to speak personally, I agree, but the availability of powerful Computer Algebra packages may be about to change this.

5. History as Part of Teacher Training

The training of teachers fits them for work in the classroom in a wide variety of situations and at a wide range of levels. Of the three papers presented in this section, one (the third) is of a general character, but with a middle school focus, while the others deal with the preparation of primary teachers. Ian Isaacs, Mohan Ram and Ann Richards describe their course in the Cultural Origins of Mathematics. The unit in question is a direct response to the culturally diverse, but sometimes ill-prepared, nature of their student intake.

Greisy Winicki, by contrast, presents a unified case study (of *Regula Falsi*), again offered to students without specialist background, but within a more uniform and very different cultural context.

Maxim Bruckheimer and Abraham Arcavi present an overview, stressing Active Participation (having the students actually "do" Mathematics), "Conceptual" History (making the mathematical *idea* the focus of the History), Relevance (finding material germane to the likely future classroom needs of their students) and Primary Sources (use of the actual texts from earlier times).

6. History of Mathematics Proper

The final section, and on all counts the longest, collects together eleven different papers dealing in specialist detail with some particular aspect of the History of Mathematics. Although the detail always does full justice to the complexities of the topic, the writing is in all cases "popular" in the sense of being lucid and accessible. In every case, too, the authors are world-renowned experts in the field they write about. This final section alone justifies the book although it has least *direct* relevance to the subject-matter advertised by the work's title.

This having been said, however, it should also be said in the space of the same breath that each of the distinguished authors *does* offer reflections on the relevance of the material presented to the classroom. The result is a valuable reference for teachers seeking enrichment material, and it will find a wider audience than this because it presents in accessible form historical research that in many cases is more difficult to track down. All the contributions to this section offer the reader valuable bibliographies.

Eleanor Robson describes the background against in which Mesopotamian Mathematics flourished. Siu Man-Keung gives us a glimpse into the riches of the Chinese tradition. George Heine tells us about Medieval Islam; Uwe Gellert discusses Renaissance Italy; Luis Moreno-Armella and Guillermina Waldegg look at the concept of number and do so from an epistemological perspective. Robin Wilson's account of Combinatorics revisits the Königsberg bridges, but includes much else besides. Torkil Heiede and Livia Giacardi both discuss non-euclidean geometry, the former in more broad sweep, the latter via Beltrami's letters to Hoüel, and illustrating her paper with some very nice drawings and photographs. Gavin Hitchcock's dramatic monologue enters a world of historical fiction, but in doing so offers us insights into the concerns of De Morgan and his contemporaries.

The last two papers concern more local and perhaps more contemporary topics. António Duarte, Jaime Carvalho and João Queiró present a short history of Mathematics in Portugal, and Ubiratan D'Ambrosio reviews that of South and Central America.

7. Summary

The book offers much to the practicing teacher, to the student of Mathematics Education, to the historian of Mathematics, and indeed to a wider readership than this. It would be too much to ask that it offer a definitive answer to the question posed at the outset of this review: what is the connection between the teaching of Mathematics and its History? Valuable insights are given, and there is much resource material for a teacher wishing to take the historical approach or to enrich the syllabus-content with historical material. Nonetheless, the caution expressed by one author (Tzanakis) is apt and bears quotation.

"[A] strictly historical approach [to mathematical instruction] is not didactically appropriate, since contrary to what is sometimes naïvely assumed, the historical evolution of a scientific domain is almost never straightforward and cumulative."

A similar caution is evident in Bruckheimer and Arcavi's emphasis on the history of the idea. But perhaps we can be a little less dogmatic. If, as several authors urged, a knowledge of the History allows us to see the reasons for student "hang-ups" and misconceptions, then we need to know a little of the messinesses of the historical course, as well. They even make nice juicy anecdotes that students enjoy.

The other area that remains open to question is the relation of Mathematics to Culture. This is what (in part) it is the task of History to illuminate. Opinions vary when it comes to detail, and we see in the papers presented here all shades of opinion. At one pole is Siu's approving quote from Hilbert: "Mathematics knows no races ... For mathematics the whole cultural world is a single country." At the other is Gellert's "Mathematics is not 'universal'."

This work will help us to formulate and consider such matters. It makes no pretense to answer them.

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