

Bruno D'Amore:

## Elementi di Didattica della Matematica

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As Colette Laborde says in her foreword, Bruno D'Amore has accepted the challenge to write a big and general work of 454 pages on didactics of mathematics respectively mathematics education at the end of the century, having well realised the risk of a venture that demands to comprise a turbulent and by no way consistent development of a still young and in some aspects immature discipline.

In the first two chapters, dealing with the question what didactics of mathematics/mathematics education really are alike, the author opens his view (and his studies of substantial literature) for different approaches to an adequate answer. After terminological reflections and quoting definitions and descriptions of the content of the discipline given by different authors, he arrives in **chapter 1** at the distinction of two intentional mainstreams. One of them, „didactics A“, mainly engaged in the production of methodological ideas and materials for mathematics classroom, handles didactics as an art. The other, „didactics B“, emphasising empirical research into pupils' learning processes, allows to describe the discipline as an epistemology of learning.

Didactics A seems to be of fundamental importance for the author. The critical analysis and development of ideas constitutes the field of interest for pupils and teachers. The historical aspect reflects about the origin of ideas, problems and theories. And also the anecdotal aspect has by no means a banal function. Didactics A is seen prestigiously represented mainly by the works of Zoltan Paul Dienes, Emma Castelnuovo and Maria Montessori, and by materials like the “numbers in colour” (Gattegno), “mathematical laboratories”, the “minicomputer” (Papy), the geo-board and the multi-base abacus. It has its limits as well, mainly in the problem of cognitive transfer. Therefore, empirical research into the pupils learning processes, as intended by didactics B, seems D'Amore indispensable. But how difficult it can be to draw consequences from this kind of research shows the example of New Mathematics. The idea to introduce pupils into numbers by previously engaging them into work with sets and one-to-one-correspondence of sets, referring to investigations of Piaget into the development of the number concept in the children's mind had soon been seriously questioned, mainly on base of further and more intensive research into the same topic.

**Chapters 2** enters into didactics B as epistemology of learning mathematics. The author discusses in detail the problem of theory import from other disciplines like pedagogy, mathematics, philosophy, psychology and sociology. In order to find a way towards a proper theory of didactics of mathematics he refers to Kuhn's concept

of paradigm, to Lakatos' research program, to Bunge's idea of science as a growth of knowledge, to Vergnoux's typology of pedagogical statements, to Durckheim's concept of practical theory, and to Romberg's particular characteristics of a consolidated and stabilised science. He mentions papers of the group TME (Theory of mathematics education) founded on ICME V (1984), and texts of Artigue and Ernest.

The profound links to psychology raise from three theoretical files and models on instruction, (namely the cognitive, the social and the content related interaction in the classroom); and it becomes visible in the regular international conferences of PME (Psychology of mathematics education) and by the works of researchers like Fischbein and Vergnaud, who also try to respect the particularity of mathematical knowledge and learning mathematics. Constructivistic epistemology is reported as another fundament for a theory of mathematics learning (Glaserfeld, Duval), and it is confronted with Godino's (1991) way of modelling mathematics learning in terms of the functioning of a calculator. D'Amore quotes attempts of defining didactics of mathematics by Laborde (1989), Brousseau (1989), Chevillard (1985). He highlights Brousseau's idea of the development of a didactical and didactical situations, and its extension by Perrin-Glorian (1997), adding situations of action, of formulation, of validation and institutionalisation. The author refers to Brousseau's concept of cognitive obstacles, Chevillard's didactical transposition, and other concepts developed in the French school of didactics of mathematics. He devotes nearly a whole sub-chapter to Van-Hiele's phases of mathematics learning, and finally arrives at recently published articles about didactics of mathematics as an academic discipline, emphasising Godino & Batanero's decomposition of the complex field of the discipline into practical action, which mainly takes care of the teacher and his/her needs, didactical technology, which is engaged in the development of curricula and mathematical text books, and research, which happens on the field of university. A graphical representation shows mathematics education as a solid in which knowledge from different disciplines is “filled in”: from didactics of mathematics, semiotics, epistemology, psychology, mathematics, sociology, and from pedagogical sciences (pedagogic, general didactics, etc.). D'Amore himself wants to stay with his distinction of didactics A and B and sums up the discussion as follows: “The performance of research which informs and constructs didactics of mathematics seems to have as main instruments the description, reflection and production of didactical systems, while the instruments of mathematics education seem to be more linked to solving problems in given situations and contexts.” (p 96).

The subsequent chapters deal with particular topics of research on didactics of mathematics, **chapter 3** particularly with the *didactical contract*, invented as a theoretical category by Brousseau (1978). D'Amore tries by means of examples of different kind to relate it to classroom reality, e. g. the pupils' expressing their mathematical ideas in a context which appears them completely controlled and evaluative, the pupils' expectation that in mathematics all is about calculation,

and their restriction in problem solving on the facts presented in the problem text. As a typical phenomenon of the didactical contract he discusses the “captain problems”, described by Baruk; quoting Perret-Clermont, Schubauer-Leoni and Trognon and literature about problem solving in general, e. g. Zan. After presenting further examples like the famous Schoenfeld problem (1978) about the transport of 1128 soldiers in buses of 36 seats each, and about solving incomplete problems from own research work D’Amore arrives at the conclusion that there are diverse approaches to the concept of didactical contract, defined as the whole of – in most cases not explicit – rules, real and appropriate clauses, which organise the relationship between the educated content, the pupils, the teacher and the expectations in mathematics classroom. The meaning of the term „didactical contract“ has changed; it has been modified by different authors, and extended by Chevillard (1988) to a meta-contract. In case of experiments in the classroom for reasons of research, D’Amore sees an “experimental contract” in effect, which is quite different from the contract in every day classroom.

**Chapter 4** is about *conflicts, misconceptions, and intuitive or parasite models*. Conflicts arise from the tension or difference between a previous imagination a pupil had constructed, found valid and reinforced in course of his/her school career on the one hand, and a new concept which comes up in the classroom on the other. Examples presented: concepts of geometrical figures appearing in uncommon position or form, problems with calculation in the transition from integers to fractions and decimals, and particular problems with addition and subtraction of natural numbers as investigated by Fischbein. D’Amore distinguishes between “internal” conflicts, caused by lack of coincidence of concepts or of intuitive and mathematical models, and “socio-cognitive” conflicts due to the teacher’s creating new situations. A pupil may realise now that his/her concept is no longer shared by the rest of the class. At the basis of conflicts are often misconceptions, i. e. concepts possibly not correct in the frame of a more elaborated and critical cognitive system. The author thinks of mathematics learning in the individual pupil’s school career as a passage from (mis-)conceptions via more and more elaborated and comprehensive towards correct models of concepts, expected and wanted as an effect of didactical activities. In this context he refers to Piaget’s theory of assimilation, accommodation and equilibration.

**Chapter 5** headlined by “*models and schemata*” starts with big attempts to clarify these terms. The text quotes definitions of mental images given by Vecchio (1992) and di Holt (1964), the concept of image developed by Paivio in “Imagery and Verbal Processes” (1971), and Kosslyn’s talk about analogical and propositional representation (1980). Intentions are the seen as the basis of all kind of spontaneous imagination of cognitive competencies by Katz (1983, 1987). D’Amore proposes a terminology which describes the formation of a mathematical concept as a kind of spiral process, repeating steps as follow: a first solicitation, a mental imagination, a second solicitation based on realising that the mental imagination

is not adequate for the solution of a new problem, leading to a conflict until the mental image seems adequate to the mathematical concept. In his psychological research into mental models Bruner found three external models: the active, the iconic and the symbolic representation of knowledge. According to Ackermann-Valadalao (1983) models intervene into the process of problem solving in two ways: in a descending phase, when the problem solver has to use it in order to arrive at the solution, and in a ascending phase, when the real problem situation guides him/her to a model. The model may already be formed, or it is on the way of being constructed, and it can be adequate to the problem situation or not. This theory is related to an example of Gallo: Given is a straight line with a point A on it and a point C apart of this line; the pupils are asked to construct the quadrangle ABCD with AB on the given straight line.

Another theory distinguishes between normative and descriptive models. In the first case the teacher makes suggestions about how to decide and how to form an adequate model; in the second case he/she restricts him/herself to indications about the procedure to be followed or about a way of resolution. A cognitivistic interpretation regards mental models as an analogical and conscious representation of knowledge. According to (1983) a mental model must be computable (i. e. it can be simulated by a computer) and finite. At the basis of mental models are three elements, namely primitives, simple concepts and complex concepts. In addition the mental model has to be isomorphic to the represented situation, economically convenient and compatible among each other. In addition Johnson-Laird distinguishes physical and conceptual or abstract models. D’Amore describes models as monadic, relational, meta-linguistic or holistic.

Other researchers see differences between a model, a mental representation, and an imagination of a particular situation. Duval (1996, 1997) has developed an elaborated theory about this topic. He thinks that representations can be intentional or automatic productions. Intentional productions can have semiotic representations, classified into non-analogical ones (working with or without a internal context) and analogical ones (with a discernible interpretative meaning or an autonomous visual meaning). Automatic productions own reproductions that are immediate (not motivated), or they are motivated by the attempt of imitating a model; in the first case they may be external or internal, in the second case they can consist of different types of images, depending on the ability of imitating valuable models.

Close to the previous categories are the concepts of frame (Bateson 1972, Minsky 1975, Winograd 1975) and script (Schank & Abelson 1977). Both refer to a cognitive structure, frame more in terms of a general cadre, script more in terms of a specific procedure with some meta-linguistic components derived from previous experience in analogical situations. Quite close is also the concept of scheme; schemata have, according to Rumelhart & Otoni (1977) four characteristics: they must remain variable, they can include sub-schemes, they organise and modules knowledge, they represent knowledge, but do not

constitute definitions. Later on, Rumelhart (1980) characterised schemata by four analogies, saying they are like scripts of a comedy, like informal theories, like programs of a computer and like linguistic analysers. D'Amore points out that the word "scheme" is also used to label a graphic representation of any kind of procedure. The final sub-chapter deals with figural concepts in terms of Fishbein (1993). Geometrical shapes should not be identified with figures drawn on a sheet of paper. On the other hand they are not pure concepts; they are rather a visual imagination. The objects of study and manipulation in geometrical reflection are, therefore, mental entities, which can be called "figural concepts".

**Chapter 6** is about *concepts and obstacles*. The author firstly quotes a lot of general definitions and classifications of "concept", philosophical ones (e. g. Abbagnano) and psychological ones (Vygotskij, Piaget & Inhelder & Szeminska, Gal'perin). Descriptions with more reference to instruction are given by Bruner, Dewey, Gagnè, Klausmeier & Gathala & Frayer. Piaget also talks about the role of language in learning and forming of concepts, saying that a concept is something signified by a word as the verbal representation designating the concept. This position is different to Vygotskij's, who regards language as a mediator between individual and culture. Vergnaud (1992) sees at the basis of concepts three logical types of invariants, namely propositional, functional and argumentative ones. For him a concept is then a triple (S, I, S), where S is a set of situations that give meaning to a concept, I a set of invariants on which operativity of schemata is based, and S a set of linguistic and non-linguistic forms allowing to represent the concept symbolically.

Obstacles which hinder the learning of mathematics have been described by Brousseau (1976), afterwards in more systematic way by Perrin-Glorian (1994). An obstacle can be seen as an idea which, in the moment of concept formation, proved effective for the solution of a previous problems, but not appropriate in the attempt of solving a new one. In need of overcoming, obstacles didactical situations have to be studied that can convince a pupil of the necessity to modify his/her concept. Brousseau points out that an obstacle is not a lack of knowledge, but a knowledge itself; the pupil makes use of this knowledge in order to give adequate answers in a certain context, and afterwards tries to apply it in an other context as well. The obstacle produces contradictions but the pupil resist to recognise them as such. It often reappears, even after having been overcome once. D'Amore distinguishes and discusses three types of obstacles: ontogenetical, didactical and epistemological ones. Dealing with epistemological obstacles Bachelard arrives at the concept of error. Enriques and Douroux also talk about errors in mathematics (history). And Artigue (1989) is quotes with the statement, that epistemology can help didactics to control the relationship between the objects in which it is engaged and mathematical knowledge.

**Chapter 7** talks in the first part about the triangle *teacher – pupil – knowledge* which has been taken into consideration in different way, e. g. by Cornu & Vergnioux (1992). It guided Chevillard (1985) to the

concept of *didactical transposition*, describing the way from mathematical knowledge across knowledge to be taught to (really) taught knowledge. Mainly the last two must be neither too close nor too close to each other. Didactical transposition produces a certain number of effects: simplification and de-dogmatisation, creation of artefacts, or production of completely new objects. It participates in the transformation which allows the school subjects and programs to ascend to knowledge, and it becomes effective by the interpretations and examples the teachers use in their everyday practice as well. D'Amore points out that specific studies of didactical transposition have unveiled numerous quite important examples of differences between original and didactical contextualisation. They give insight into teachers "didactical engineering" (Artigue 1989) in the creation of their curriculum about a particular piece of mathematical knowledge. They have to consult knowledge about mathematical and didactical models, about relevant epistemological obstacles, about general instruments of curriculum construction, in order to arrive at a curriculum adequate and specific from the mathematical and didactical viewpoint. Coming back to the didactical triangle, D'Amore gives meaning to the flashes between its vertices, saying, that the relation between teacher and pupil is "a pedagogical (asymmetric)" one, the relation between teacher and knowledge "refers to the epistemology of the teacher", and the relation between pupils and knowledge "conceptualises culture, school and knowledge". The final sub-chapter comes back to the theory of didactical situations, Brousseau's distinction between a-didactical, didactical and non-didactical situations and other classifications already presented in chapter 2.

**Chapter 8** relates mathematics and didactics of mathematics to *language problems*. Some authors even designate mathematics as a language, since it has its own syntax, semantics and pragmatics. According to Duval the word language can mean a semiotic system with its own function, different forms of discourse produced by making use of a language, a general function of communication among individuals of the same species, and the use of specific codes more or less socially shared. Psychologists try to answer the question, whether the semiotic system of a language is necessary to make logical thinking work, and the development of scientific knowledge as well. D'Amore reports about the different positions of Piaget and Vygotskij on spontaneous oral language and the phenomenon of ego-centrism. In classroom communication the complexity of learning a "scientific discourse" by pupils has to be seen. The technical language of mathematics is influenced by every day language, what results in a paradox: On the one hand teaching and communication has to favour learning, and the language must not become a source of obstacles for understanding. For that reason the teacher should avoid to use a specific language and rather stay with normal language. On the other hand, mathematics is characterised by a specific language; therefore, it should be one of the principal objectives to teach it and to make sure that the pupils get the chance to learn it. In fact, language in mathematics classroom is neither the mathematicians' nor just a common language; it realises a

particular synthesis of both.

In any case, for technical symbols has to be guaranteed that the pupils do not apply them in a meaningless way and just carry out algorithms mechanically. The symbols and the algorithms should be linked to the understanding of related concepts. For a meaningless use of technical language elements by pupils D'Amore created the term of "mathematical jargon". He and other researchers have studied the relationship between every day and technical language in the mathematics classroom by characterising their differences and the difficulties which can arise for the pupils from that. Maier, e. g. describes interference of meaning which often happens in the pupils mind, causing difficulties for the understanding of problem texts and for an adequate formulation of mathematical facts. Duval worked intensively on the problem of semiotic representation and its relationship with cognitive processes. He was interested in pupils' being confronted with different symbolic representations of the same concepts, e. g. with the signs 0,5,  $\frac{1}{2}$  or  $5 \cdot 10^{-1}$  for the same rational number. And this is a topic to which D'Amore contributed in a research project reported in detail. The pupils were presented four different representations of the relation "... is situated in ..." between a set of city names and country names (Caroll diagram, Cartesian diagram, Venn-diagram and verbal proposition) and they were asked what they think about them. In case they did not recognise or talk about same meaning they were put a second question: Are there two representations saying the same thing? Afterwards the test persons was requested to decide which representation appears them to be the clearest, and which one they would use as a teacher of elementary school for younger pupils. The interesting results raise some important consequences for mathematics classroom.

**Chapter 9** is about *tasks, problems and problem situations*. These terms are discussed, quoting different definitions from the literature. Problem solving has been a topic for mathematicians (Polya, Hilbert), psychologists (Duncker, Gagné, Lester), and didacticians (Schoenfeld, Kilpatrick, Chevillard, and the Grenoble crew). It seemed interesting to ask pupils as well, what it means for them to solve a problem. The author is highly engaged in insoluble problems that, according to him, appear in three forms: the text looks like a standard problem, there is a "logical rupture" between dates and question, and there is a lack of dates not available, but necessary for solution. These problems are able to unveil much about the pupils activities in problem solving. This process depends, as Lester (1983) say, of parameters as follow: knowledge about the subject available to the problem solver, control, affective factors (imagination and convictions about mathematics, the school, and others), and socio-cultural conditions. Kilpatrick (1987) restricts himself to three independent variables that characterise the process, namely the subject (the solver), the problem content, and conditions (customs, so called "externals"). D'Amore & Zan (1996) classify the important factors into process and product variables. They see an interrelation among the factors problem content, environment, subject (problem solver), processes and products, and they complain that so far in research only

pairs of these factor have been related to each other.

An appendix to this chapter reproduces a research report of Cassani, D'Amore, Delconardi, and Girotti about "Routine problems and "unusual situations" (1996). The routine problem demanded to calculate the volume of a pyramid, knowing the side lengths of the square at its basis and the length of an edge linking a basis vertex with the top. As "unusual situation" the testees were given a "real" pyramid model, and they were ask to calculate its volume, too. This problem has been presented either before or after the routine problem, pupils solutions have been observed and compared. Observations and results guided the researchers to interesting consequences for problem solving in the classroom.

**Chapter 10** deals with *cognitive styles and pedagogical profiles*. After referring to the ideas of Neisser (1967) about "Human information precessing" it provides a detailed presentation and discussion of components which are, according to Bransford (1979), effective in the process of knowledge construction. Factor A concerns the nature and presentation of concepts and allows distinction between components characterising the content and modes of organisation and presentation. Factor B is about the character of the learning subject, its abilities, cognitive styles, personality, knowledge and attitude. Important concepts are, with respect to abilities, the processes of assimilation and accommodation at a scheme, and with respect to coginitive styles, according to Witkin's (1962), gestalt specific and analytical proceeding. Factor C, containing activities, strategies, and processes requested from or activated by the pupil (attention, repetition, production of imaginations, etc). Factor D refers to intellectual activities like cognition, memory, problem solving and transfer. The process of problem solving is modelled in quite different way by associational, cognitivistic and constructivistic epistemologies and theories of learning. Pedagogical profiles and cognitive styles have been described by De La Garanderie; he points out that the cognitive styles do not depend of natural facts only but can also be of momentary manner like interest, motivation, will, etc. The construction of meaning must not be seen as a linear process, it shows fractions, phases of slow progress and of quick moving forward, too. This allows to describe pupils' learning as a permanent cognitive reorganisation, i. e. the meaning built up are always in discussion and underlie permanent modification.

**Chapter 11** has the title *intuition and proofing*. For "intuition" there is plenty of philosophical definitions, beginning with Plato and Aristotle across philosophers of medieval times (Thomas of Aquinas and Occam) and the times of enlightenment (Decartes, Locke, Leibniz) up to modern phenomenologists (Husserl, Croze). Gestalt psychologists have also used this concept as well, and in didactics of mathematics Fishbein distinguishes three levels of mathematical understanding, the formal, the algorithmic and the intuitive one. He classifies intuition into anticipation and agreement. Anticipation comes near to what Poincaré and Hadamard talked about with reference to problem solving and what Bruner meant by intuitive thinking. Agreement can be attributed different

levels of credit; a conviction may be sure already before, or not earlier than in course of or after a proof. We can also think of a more accessible and immediate representation of abstract mathematical concepts, ideas, relations, operations, etc. by help of an intuitive model. In this case a lack of intuition could be seen as reason for several kind of unsuccessful understanding on the cognitive area.

For mathematicians a proof seems to be a fact, a natural attitude. But for pupils things seem to be more difficult, mainly if proofing ought to be seen as equal to convincing. All didactical discussion about the learning of proofing seems to mention at least two main points, the act of proofing itself, and the insight into how it works. Proofing is not completely different from carrying out an exercise or solving a problem, since there are many proofs which include calculation, exercise and problem solving. Antibì (1993) prefers the idea that in mathematics solving a problem always demands proofing as well, how simple the problem ever may be.

D'Amore unveils a panorama about some research into proofing in mathematics education. According to Bell (1976) proofing can mean a verification or justification, an illumination, and a systematisation, the last referring to the formal concept of proof in mathematics; he distinguishes between empirical and deductive proofs. Balacheff (1988) contributed much to clarify the categories explanation, experiment, proof, reflection and process of validation, which are seen as a sequence of activities ordered in time. Proof is characterised by de-contextualisation, de-personalisation and de-temporalisation. Perelman (1977) highlights the distinction between argumentation, defined as a discourse in order to convince others, and proofing, which is to convince the proofing person itself, answering the questions "Is it true?" and "But why is it true?". D'Amore relates argumentation as part of a convincing discourse to theories of rhetoric, quoting relevant literature. But with most extension he reports Duval's reflection about the difference between argumentation and proofing, extending the categories by experimenting, and arriving at a table which describes the differences with reference to six questions, namely "How comes the starting point to the focus of discourse?", "Which result is laid out by the discourse production?", "Which aspects do propositions in the discourse consider?", "Are contributions contradicting propositions mentioned?", "Are contributions indicated by propositions?" and "Is there continuity in the discourse?"

**Chapter 12** presents three concepts which refer to the word "domain". Vergnaud (1990) talks about *conceptual domains*, defined as a set of situations, concepts and symbolic representations which are so much linked to each other that it would be illusionary to analyse them separately. An example is the multiplication structure in arithmetic. It would be misleading to study multiplication in an isolated way, since a large domain of knowledge is requested to describe the cognitive evolution of this concept, and also the procedures the pupils' have to remind, the concepts they have to make use of and the symbolic representations they have to refer to are diverse and by no way singular. The terms "*domain of*

*experience*" and "*semantic domain*" have been applied by Boero (1989) to issues different from mathematics. But D'Amore claims that they can be of big importance also in didactics of mathematics, the last one mainly with respect to linguistic problems on which he himself has worked a lot.

The final **Chapter 13** comes back to the topic of chapters 1 and 2, discussing particularly the relationship of general didactics and didactics of mathematics. With respect to problems of existence and legitimation two contrary positions can be found: General didactics do not exist at all, what exists is nothing but didactics of something versus specific didactics for every particular subject do not exist but as segments of general didactics, really is the only thing existing. The author points out that discussing these positions presupposes a clarification of what is meant by "didactics". An epistemological perspective arrives at the alternatives that only specific didactics or only general didactics have a significant epistemological status. Again it has to be realised that the term "epistemology" has some ambiguity in it, and the different positions may be due to different imaginations about this concept. Things take a more pragmatic turn in case of teacher education is taken into consideration. The propositions that general didactic contain nothing but (banal) the description of psychological attitudes that can completely be replaced by common sense versus specific didactics coincides strongly with the subject discipline, in case of didactics of mathematics with mathematics, must not be seen as contradictory positions. The author decides that for good teaching it seems necessary but not sufficient to have a profound knowledge of the subject to be taught, and for professional teacher education a high training in mathematics has to be completed by an education on the pedagogical domain. Thus, the best way to educate teachers of mathematics (for every level of schooling) are integrated studies of mathematics, didactics of mathematics and general didactics.

The volume of D'Amore initiates and allows the researcher in didactics of mathematics (mathematics education) a serious reflection on the character and the main topics and objectives of his/her discipline. It gives him/her, but a teacher student and an interested teacher as well, a broad overview on the most important fields of theory and empirical research. The author tries all, mainly by reference to much literature, to achieve a definition or clarification of basic concepts and the main terminology open for different aspects and contributions. The attempts are so profound that the book in some passages takes the character of an encyclopaedia on didactics of mathematics. The theories D'Amore reports about emphasise French didactics, but there are also some excursions into Anglo-Saxon, German and Spanish literature. Interesting reports about empirical research are, if not taken from French didactics outcomes of his own investigations, which all are designed creatively and near to school reality, and often realised in co-operation with teachers and other researchers.

Concept clarification, presentation of theory and reports on empirical research coincide in the volume to a composition which really reads exciting. This is also due to a language quite clear and expressive but vivid at the

same time. In summary a publication which seem worth to be read by everybody interested in didactics of mathematics, and which should be used extensively in European teacher education.

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