

ICMI Comparative Study

Mathematics education in different cultural traditions: A comparative study of East Asia and the West

Discussion Document

International Commission on Mathematical Instruction
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mathématique

Preamble

Education in any social environment is influenced in many ways by the traditions of these environments. As a consequence the results of such education will naturally differ with different traditions in different environments. Indeed, this is necessary since one of the intentions of education is to support the traditional continuity of structure and function of a special environment.

On the other hand, today we are observing a growing interdependence between environments like regions, states, countries, and different cultural areas of the world. In many respects they have to rely on corresponding or equivalent standards of education, and differences can cause irritations.

In mathematics education also, taking an international and intercultural point of view, we face this split phenomenon of difference and correspondence, linked with the perpetual challenge to improve the quality of mathematics education. A study attempting a comparison between mathematics education in different traditions will be helpful to understand this phenomenon in detail and to exploit it for the sake of mathematics education. From this, paths will be discovered leading to adequate and effective applications of differences, as well as correspondences, in national and international environments.

Due to the size of an ICMI Study, in manpower and in time, this enterprise must be limited to only a selection of cultural traditions. Those based in East Asia and the West seem particularly promising for a comparison, since similar interests in differences and correspondences have existed for a long time and experiences in equivalent research have been gathered.

A rich variety of aspects of mathematics education is to be considered in this comparative study, ranging from the host of social, economic and other contexts, curricula, teachers, students, goals, contents, methodology, media etc. to the nature of mathematics and the future of mathematics as well as mathematics education. Traditions of teaching and learning that are deeply embedded in history and culture will have to be compared, with a consideration of the rich experience growing out of them as well as their resistance to change.

At the same time, this comparative study must consider present developments in society, science and technology as well as ethics. Changing attitudes between generations are influencing the teacher-student relationship, as are the new information and communication technologies. In addition, these technologies define new roles for both the teachers and learners and the reaction is different in different traditions.

What kind of subjects will there be in schools of the future and how much planning is going on? In what ways will mathematics education of the future be comparable to that of today and how will it differ? What forces are competing in this field?

Exchanges of experiences and expectations will be an important part of the study and critical considerations will be inevitable.

Previous ICMI studies normally proceeded in three steps: Discussion Document, Study Seminar and Study Volume. In our case we will insert an Electronic Discussion Forum before and possibly even after the seminar.

First, the IPC offers a Discussion Document to the mathematics education community and people from interested contexts. We will welcome applications for a study seminar by invitation which we expect to take place in Hong Kong in October 2002. Contributions can come from individuals as well as jointly from colleagues who are already engaged in comparative activities about different traditions in mathematics education. This will allow an operationalising of the study by referring to case studies, for example.

Second, the Electronic Discussion Forum will allow statements about the theme of the study in general and corresponding comments and questions from any colleagues interested in the study.

Among other intentions the Forum should especially enable colleagues interested in the same or similar field of comparison to meet and to cooperate in preparing a contribution in general or a case study in particular.

Third, the Study Seminar will consist of presentations identifying and interpreting consequences from different traditions to a variety of aspects of mathematics education. Moreover, a great deal of work has to go into the comparing of observations and findings, for example in focus groups. In this way the seminar will arrive at recommendations for the applications mentioned above, serving to make differences and correspondences fruitful for national and international education.

Fourth, a Study Volume will be published for the mathematics education community and the interested public, containing the results from the communications and comparisons at the seminar.

I What is in this study?

1.1 Scope of the study

In this study, we confine ourselves to mathematics education of school age children (although we are not confining ourselves to mathematics education in regular schools) and related areas such as teacher education, and

will not specifically study issues such as vocational education or education at tertiary level. Problems in mathematics education to be investigated may include issues such as curriculum, assessment, policy, influences of information and communication technology (ICT) and multimedia, community and family background etc. These will be further elaborated in Section IV.

1.2 What do we mean by “cultural tradition” in this study?

“Tradition” is an equivocal word which can refer to many different things. As this study is on mathematics education, we obviously would like to study “traditions” which have a bearing on mathematics education. Two obvious choices are the *education* tradition and the *mathematics* tradition in different countries or regions. Indeed it is reported in the literature that mathematics education in a given country or system is very much influenced by the underlying education tradition and mathematics tradition. However, in this study we would argue that both the education tradition and the mathematics tradition are related to a deeper level of tradition, that of the culture. And it is this deeper level of *cultural tradition* with reference to which we would like to compare the mathematics education in different regions.

Culture is “one of the two or three most complicated words in the English language”. It may refer to “the fabric of ideas, ideals, beliefs, skills, tools, aesthetic objects, methods of thinking, customs and tradition”, or the “*configuration or generalization* of the ‘spirit’ which inform the ‘whole way of life’ of a distinct people”. For this study, culture refers essentially to values and beliefs, especially those values and beliefs which are related to education, mathematics or mathematics education. An example of a value that pertains to education is the importance attached to education in different cultures. An example concerning mathematics is the view of the nature of mathematics (e.g. whether or not it is essentially a pragmatic discipline). An example of a belief that affects mathematics education is the attribution of success and failure in mathematics (e.g. attribution to effort versus innate ability).

Although “culture” is a crucial concept in this study, it is not the intention of this paper to offer a comprehensive definition of the term. Rather, the brief discussion and examples above are meant to indicate the level of depth we are referring to when the term “tradition” is used and to stimulate discussion on what cultural values and beliefs are relevant to a discussion on mathematics education. Indeed, one important purpose of this study is exactly to identify the aspects in our cultural values which have impact, directly or indirectly, on mathematics education.

1.3 What are “East Asia” and “the West”?

This study is on the cultural traditions in East Asia and the West. In using the terms “East Asia” and “the West”, we do not merely refer to geographic areas. Our contention is that cultural divisions are much more meaningful than political or geographic divisions in explaining differences of educational practices in

mathematics. East Asia and the West in this study are therefore cultural demarcations rather than geographic divisions, roughly identified as the Chinese/Confucian tradition on one side, and the Greek/Latin/Christian tradition on the other. We acknowledge that neither of these “poles” is well defined, as with any label given to any culture. But we use the two terms to point to the scope that we want to confine ourselves to in this study.

In identifying these two “poles”, we are not claiming that the two cover all major cultural traditions in the world. Nor are we implying that these two are the most important human traditions. For example, it has been pointed out that there is a distinctive East European tradition in mathematics education which is definitely worth studying. Equally worth studying are traditions in South Asia (in particular that of India) and Africa. However, it would not be possible for a single ICMI study to cover all important traditions worldwide. What we hope to achieve, by choosing these two poles for study, is a balance between using pertinent examples to study the relationship between cultural traditions and mathematics education on the one hand, and choosing two major traditions that have attracted attention in the field of mathematics education on the other. More justifications on the choice of the two traditions will be discussed in section II.

1.4 What do we mean by a comparative study?

To compare means to identify similarities and differences, and to interpret and explain the similarities and differences identified. It may not be as easy as it is conceived. Given two things or concepts, there may exist infinitely many aspects of similarities and differences, and hence in a comparative study, we are always confining the comparison to a particular theme or some particular themes. For our study therefore, we are comparing practices in mathematics education (as defined in I.1 above) along the theme of *cultural traditions* (as described in I.2 above). Reminding ourselves of this obvious point is important. In studying mathematics education in different countries, we will definitely come across important aspects of mathematics education that are of interest to us. But in deciding which of those aspects should be included in our study, we have to ask ourselves whether or not those aspects are related to this theme of cultural traditions. Some aspects of mathematics education which we deem to be related to different cultural traditions will be discussed in Section IV.

After we have identified similarities and differences within a certain theme, the next question is what to do with them. A simple juxtaposition of similarities and differences does not in itself explain. There is a need for analysis based on certain theoretical frameworks, or in the absence of a suitable theoretical framework, a need for the establishment of one, based on the differences and similarities observed. More about how this comparative study is operationalised will be discussed in Section III.

II The rationale for this study

II.1 Why is this comparative study important?

Pressures and needs from outside mathematics education

Rapidly developing information and communication technologies have an enormous influence on mathematics, science, production, society, politics, education and even lifestyle. Increasing globalisation is encouraging the assumption of universalism in mathematics education. The increase in journals and books about mathematics teaching, the multitude of conferences in every part of the world, the availability of materials via the World Wide Web, and the activities of multinational computer companies all increase the pressure for adopting similar practices in mathematics teaching around the world.

At the same time, however, the globalisation processes are producing reactions from mathematics educators in many countries who are concerned that regional and local differences in educational approach are being eradicated. This is not just a mathematical ecology argument, about being concerned that the rich global environment of mathematical practices is becoming quickly impoverished. It is also an argument about education, which recognises the crucial significance of any society's cultural and religious values, socio-historical background and goals for the future, in determining the character of that society's mathematics education.

Policy makers also recognise the importance of adjusting to the changing world and mathematics education reform movements can be found in many countries at this moment. A number of international studies have also taken place in the last decade to provide policy makers with information on the relative standing and effectiveness of their education systems.

Pressures and needs from within mathematics education

Mathematics educators from all around the world are continuing to make efforts to improve the quality of mathematics education. One way to achieve this is through learning from different countries. For instance, many schools in the US are importing the mathematics text books of Singapore, while Asian countries are mimicking the approach taken by US textbooks that have been developed as a result of many specific educational projects. While these exchanges do not yet show clear evidence about which approach is the most appropriate for each country, they stress the need for more awareness of the cultural traditions in which the respective mathematics teaching is embedded. More in-depth studies on the relative merits of the different approaches within different education systems are needed.

II.2 Why is this study focused on a comparison between East Asia and the West?

A complete comparison of the background, perspectives and practice of mathematics education around the world is far beyond the scope of an ICMI study. Realistically we need to limit the range of a comparative study while still

ensuring that clearly different traditions are being examined. Recently, comparative studies such as SIMS and TIMSS have produced data indicating that there may be some systematic reasons for differences in achievement and practice between some regions. East Asian countries such as Japan, Korea, Taiwan, China, Singapore and Hong Kong consistently outperform western countries in North America, Europe and Australia in these international tests. These results have brought about a growing interest for policy makers and educators to find out the factors behind Asian students' high performance in mathematics.

Surprisingly, a superficial look at mathematics teaching in Asian countries indicated that teaching methods in these countries are not perceived as advanced as in Western countries. For instance, mathematics education in the East Asian countries mentioned above is typically characterised by the following: They are very often content oriented and examination driven. Large class sizes are the norm and classroom teaching is usually conducted in a whole class setting. Memorization of mathematical facts is stressed and students feel that their mathematics learning is mainly learning by rote. Teachers feel guilty for not teaching enough problem-solving during their classes. Students and teachers are subjected to excessive pressure from highly competitive examinations, and the students do not seem to enjoy their mathematics learning.

This parity between the high mathematics performance in Asian countries and a lack of modern teaching methods is puzzling. It prompts for a call for in-depth studies about mathematics learning that goes beyond classroom teaching. Cultural traditions, for example, could be highly pertinent to students' learning. In particular, there are marked differences in cultural traditions between the East and the West. These cultural differences provide us a unique opportunity to gain a deeper understanding about students' learning and achievement.

This study presupposes that the impact of cultural tradition is highly relevant to mathematics learning. Cultural traditions encompass a broad range of topics. It includes the perceived values of the individual and the society, as well as social structures such as the relationship between parents and children, or the relationship between teachers and students. There are clear differences in all these areas between Asian and Western traditions. For instance, some scholars have identified features of East Asian mathematics education, together with their underlying values, by contrasting them to the West.

Apart from scientific interests in comparing mathematics achievements in the East and the West, there is also a strong political impetus behind such studies. After World War II, Western countries have dominated world economic development for a long time, but in the last ten years, we saw the emergence of East Asia as strong competing economic powers. The United States government, for example, has given very strong support in international comparative studies of mathematics and science, with particular emphasis in the comparison between the U.S. and Japan. The Australian government, on the other hand, recognising the importance of its

geography proximity to Asian countries, has funded special studies aiming to get a better understanding of mathematics learning in Asian countries.

The culmination of all of the above issues has made a study between Asian and Western traditions in mathematics education not only interesting but also important. There is also some urgency for such a study to take place now, because with the current advances of technology and communication, the differences between cultural traditions are diminishing. Before long, we will not have such contrasting traditions to provide us with rich information to carry out such a study.

II.3 What can be achieved from this study?

The focus of this study is on differences in educational traditions which have cultural and social implications for the environment and practice of mathematics education. This focus does not mean a narrow interest on how to improve scores in international tests. The study is about gaining a deeper understanding of all aspects of mathematics learning and teaching, and about what each tradition can learn from the other.

To develop this point further, the study should not be a 'horse-race-type' of study resulting in a linear ranking list. Instead, in this study we want to start with a mutual understanding of the mathematics education systems and processes in different traditions that lead to more or less satisfactory success in the learning and application of mathematics. Given this degree of 'success', according to some agreed criteria, then we need to identify the conditions that contribute to this success (e.g. learning techniques, external influences, natural abilities of the children, language etc). Only then can any kind of transfer of methods take place, if desired, from one tradition into another. It is not a simple import-export business. As an old Japanese proverb puts it: *We cannot easily plant a good seed of another field into our own field.*

In summary, we are hopeful that this study can achieve the following:

- (1) By contrasting the different traditions, we gain a deeper understanding of various aspects of mathematics learning and teaching. For example, we may gain an insight into the cognitive processes of "doing mathematics", such as learning about place values, or grasping the concepts of abstract representations in algebra.
- (2) By contrasting different traditions, we develop a process of self-reflection on our traditional ways that we often take for granted. There is the opportunity to take a fresh look at our usual practices and beliefs, and, in the process, we gain a better understanding about our own traditions.
- (3) By contrasting the different traditions, we share between us the latest educational development and research. We learn from each other's successes and failures, and develop a common goal for improvement for the coming years.

III How will the study be operationalised?

This study is rather different from the earlier ICMI studies in that it is specifically concerned with comparing practices in different settings and with trying to interpret these different practices in terms of cultural traditions. It is also the intention of the IPC to ensure that the study will result in various products and outcomes including, of course, a book for the ICMI series, but equally important is the process by which the study will proceed. This means that there is a need to create certain kinds of activities for engaging the participants in operationalising the study, and through these activities to develop specific kinds of contributions to the study. The IPC has identified the following as being some of the most important activities and contributions for this study.

III.1 Identification and analysis of previous studies

As has been pointed out in the previous section, there is clearly a need to build on the several previous studies which have been carried out on East/West differences, and we wish to encourage contributors to the conference to be aware of the available background literature. In particular we note that there have been various achievement-based studies such as SIMS and TIMSS, and other studies such as PISA etc.

Contributions and proposals will therefore be useful which give some synopses and critical analyses of these studies. In particular, as well as focusing on the various ideas and results that have come from these studies, we will have an interest in the methodologies used in them. Most of the analyses performed on the data of these international comparison projects have been quantitative in nature, and we are more interested in a qualitative aspect which seems to have been ignored so far. The development of productive cooperative researching is one of the outcomes we seek and awareness of previous cooperations will be very helpful in this development.

III.2 Joint contributions

In the same spirit we wish to encourage joint contributions from colleagues who are already engaged in East/West cooperative or comparative activities, as well as from those who wish to use the study as an opportunity to begin to engage in such activities. Previous international conferences such as PME, HPM, ICME have already provided some contexts for cooperative international activities and it will be important to build on those activities. These conferences have already demonstrated the value of building trust between cooperating colleagues, as well as the importance of taking advantage of the different backgrounds and knowledge of the participants.

The nature of this ICMI comparative study is such that it will be highly dependent on trust and productive cooperation between the participants, and we realise that this kind of cooperation can take time to develop. There is much to learn from each cultural tradition before one can begin to seek meaningful contrasts and to develop productive explanations. In some senses the IPC sees this study as being an opportunity both to take stock of the progress made so far in this area, and also to sow the seeds for future cooperative research and development.

Developing joint contributions is one way in which study participants can begin to engage in the spirit of this ICMI study, and seeking these is one way in which the IPC and ICMI can help to foster future collaborative ventures.

III.3 Case studies

In a sense this ICMI comparative study is one large case study, as has already been mentioned, so the IPC sees the need for there to be several case studies, with a comparative flavour at the heart of the study, which focus on specific aspects of mathematics education (see Section IV). Thus it is important that participants are aware of the nature and value of case-study activity in order to develop the study to its fullest potential. It is only through such case-studies that the richness of educational and cultural interactions can be presented and interpreted.

The case studies hopefully will demonstrate the observation and understanding of a range and variety of typical phenomena in the different traditions, together with their importance in mathematics education. Cases should not only focus on demonstrating and analysing differences but will also explore aspects of similarity between the traditions. The depth of analysis made possible by such case studies will help to challenge the naive policy and practice of attempting to merely copy and transport specific practices from one tradition to another.

As well as welcoming contributions regarding previous case studies and the presentation of current comparative case studies, the IPC sees this study as creating the opportunity for the development of future comparative case study activity. It will welcome proposals for such work.

III.4 Variety of documentation

The IPC is aware of the value and importance of including a variety of documentation in this study. Mathematics education reveals itself through various media and materials. Government publications and documents demonstrate intentions, values and contextual features of a system's policies and practices, but in order to study the influences of the different traditions to the depth desired it is necessary to seek further documentation. The IPC will therefore also welcome contributions which are based on other kinds of documentation.

Textbooks give more information about intended practices as well as about the roles of the teacher and the students but other teaching materials are also revealing. Videotapes of classrooms are increasingly being used as research data in cross-cultural research studies. Aspects of student achievement are well used in comparative studies, but can give misleading and shallow information if not adequately interpreted through deeper cultural and social perspectives. Data on students' and teachers' attitudes, beliefs and values will be particularly interesting for this study as these often reveal more about the significant aspects of difference between cultural traditions.

III.5 Different perspectives on mathematics

Differences in mathematics education intentions and

practices are often stimulated by the different influences coming from various groups of professional working in and with the area of mathematics education. In addition different cultural traditions view mathematics itself in different ways, as has already been pointed out in an earlier section.

In this study therefore the IPC wishes to seek contributions from people who work in different parts of the mathematics education community. In a rich comparative study such as this, different perspectives are crucial, and colleagues who work in areas such as mathematical applications, informatics, and the history and philosophy of mathematics are encouraged to participate.

III.6 Different perspectives on the study of mathematics education

In the same sense as in the previous point, there are several different approaches to the study of mathematics education which need to be recognised in this study. Although the IPC sees case studies as being important kinds of contributions, it also does not seek to restrict the methodology of the studies presented. It recognises value for example in psychological, sociological, and anthropological approaches as well as in contributions from other areas of the educational sciences.

The IPC is also aware that as this comparative study develops, particular differences in methodology between cultural traditions may become revealed, for example concerning the position and role of researchers. It is conscious of the dangers of applying certain methodologies from one cultural tradition inappropriately in another cultural tradition. The IPC therefore hopes that one outcome of this study is an increased awareness in the international mathematics education community of the need for cultural sensitivity in carrying out future comparative studies.

IV Aspects of the study

IV.1 Context

Mathematics education does not take place in a vacuum, but there is always a host of different contexts within which the practice of mathematics education takes place. These contexts may be social, political, economic, philosophical or ethical, but they are of course all related one way or another to the underlying cultural values. What are the elements within these contexts which are relevant to mathematics education? What are the "givens" from which we organize our mathematics education, and what are the constraints within which we carry out the education?

IV.2 Determinants of the curriculum

In the literature on sociology of education, an important characterisation of an education system is to identify the significant figures or interested parties that determine education policy in general and the curriculum in particular. In this study, however, we are interested not in these general sociological issues, but in the determinants of the mathematics curriculum. Who are the major

players (e.g. mathematicians, mathematics educators, school teachers, bureaucrats etc.) in determining a particular mathematics curriculum? Are there significant cultural differences in the constellation of determinants of the mathematics curriculum? In addition to the issue of “who”, we are also interested in the mechanism through which the mathematics curriculum is determined. Are there cultural differences on how the mathematics curriculum is determined (e.g. centralized versus decentralized)?

IV.3 The role and place of mathematics in the overall curriculum

Mathematics occupies a central place in the curriculum of nearly all countries in the world, yet there may be subtle differences in the importance attached to mathematics as a school subject in different countries or cultures. As far as the education system is concerned, what role is mathematics playing in terms of sifting or filtering students through the education ladder in different countries? Is mathematics a highly aspired subject? What place does mathematics occupy within the overall curriculum (this may be reflected, for example, in the number of hours devoted to mathematics)? Is mathematics viewed as a service or instrumental subject, a subject for mental training, or a subject essential for the development of a cultured citizen? The different perceived roles of mathematics may affect the way mathematics education is conceived and organized in different countries and cultures. These are related to points (6) and (11) below.

IV.4 Teachers, teacher education, values and beliefs

The teacher is one of the most crucial elements in the implementation of the mathematics curriculum, and hence a study of mathematics education in different traditions should definitely focus on the teacher. Are there cultural differences on the image or role of teachers in the education process? Are there differences in the way mathematics teachers are educated or expected to be educated? What are the relative emphases on the subject matter (mathematics), pedagogy, and pedagogical content knowledge in the teacher education curriculum?

The literature shows that there is a high correlation between the attitudes and beliefs of teachers on the one hand, and their instructional practices on the other, and it will be both important and interesting to see whether there are cultural differences in teacher attitudes and beliefs. For example, how do teachers in different cultures perceive the nature of mathematical knowledge? The literature shows that mathematics is viewed by teachers as more or less a subject of absolute truth or as a fallible subject. Are there any cultural differences in these attitudes? Also, are there cultural differences in the patterns of attribution of success and failure in mathematics? Does the strength of the relationship between attitudes and practices differ in different cultures?

IV.5 Students, learning styles, attitudes, role of students in the teaching/learning process

Just as teachers in different cultures differ in their

attitudes towards mathematics and mathematics teaching and learning, students in different cultures are reported to differ in their attitudes towards mathematics and mathematics learning. Do these differences in attitudes lead to differences in learning styles? For example, are Asian students more passive in mathematics lessons? Does physical passivity necessarily imply mental passivity? Can we account for the different achievements of students by differences in attitudes and learning styles?

Also, does the pattern of gender differences in attitudes differ in different cultures? Are boys and girls treated differently in different cultures? How are students of different abilities (e.g. gifted children, disadvantaged students) treated in different cultures? Is the role of student in the teaching and learning process perceived differently in different cultures? These differences, if they exist, will definitely affect both the way teachers organize their teaching as well as the way students participate in the teaching and learning activities.

IV.6 Intentions and goals

Studies of different cultural traditions often reveal differences in the goals of education, and it is likely that there are also important differences in the goals and aims for mathematics education. This does not just concern the formal and published goals and aims in, for example, government publications, but concerns also the informal, understood but not stated intentions which people from a cultural tradition take as natural, assumed, and unimportant to discuss. So, are there formally described differences in the goals and aims of mathematics education between different cultural traditions, and what are they?

Intentions however are more general phenomena than goals and aims, and concern personal values as well as societal expectations. In this study it is hoped to reveal more of the informal intentions that the different cultural traditions hold for mathematics education as well as the more formal goals. Is it possible through this study to identify these informal intentions and to explore the systematic differences between them?

IV.7 Content

The differences between the content taught in various countries would appear to be important in this study, particularly in explaining variation in student's performance. School mathematics syllabuses might ostensibly be similar in different societies, but the arrangement of the content and the approach to particular topics could be very different. For example, the content of geometry taught in school might have different emphases on coordinate geometry, vector geometry, Euclidean geometry, transformation geometry or indeed on the integration of these geometries in different countries. Whether algebra is or is not considered as a means for mathematical argument can result in very different approaches to algebra.

So, the question is, how do the different aspects of content chosen and the approaches taken interact with, and how are they influenced by, particular cultural values?

School mathematics is developing. For example, the

modeling of situations through programming and computer algorithms has developed a certain appropriate content for school mathematics, such as testing the correctness of algorithms. How do different cultural traditions react to such new content?

Comparison of content can also focus on some specific issues. For example, how are ideas of proving and testing in mathematics introduced and developed? How is discrete mathematics implemented in school? What has been the legacy of the New Math that was exported from the West to many countries in the 50s? Such issues may also have significant cultural roots.

IV.8 Methodology and media

Methodology of teaching and learning is one of the master keys opening up differences in mathematics education, both in theory and practice. Researchers have shown rising interest in comparisons in different countries for the last two decades and they generally agree that differences are both substantial and striking.

Major and coherent questions are:

- 1) What are the real differences of teaching and learning in the classroom?
- 2) Why are there differences?
- 3) How can studies of differences help us to improve teaching and learning?

Section II.3 has already been dealing with question 3) and we will not go into any more details here.

As to question 1) differences have been observed and studied in many aspects like classroom organisation and routines, teaching sequences, instructional expectations from students, teacher-student interactions, representations for mathematical concepts and procedures. Are these the most influential aspects of differences and which others should be considered? For example, what about the process of planning, analysing and evaluating lessons? Or a deeper insight into the changing teacher-student interaction influenced by general changes in the attitudes of different generations towards each other and the growing interaction of students with each other about learning and with informal teaching and learning resources, such as the internet?

There have been attempts by researchers to characterise differences in mathematics lessons in Japan (e.g. as built around a consideration of multiple approaches to carefully chosen practical examples or activities). In lessons in American schools some researchers see teachers presenting information and directing student activities and exercises with a unique feature of the multiplicity and diversity of both topics and activities.

How can these issues be investigated further and understood more deeply?

As to question 2) reasons for differences observed have been found in different theories about teaching and learning. Education in general and teaching methods in particular do rely on very different fundamentals e.g. philosophy-oriented, science-oriented or application-oriented (or dialectical, hermeneutical, empirical-analytical). Schemes for the structuring of a lesson also can follow a dialectically oriented model of the process of education (*Bildung*), a process of development as in

nature or technology, or a model oriented at the problems to be handled.

Theories about learning are often related to psychological considerations, resulting in a behaviourist or action-oriented view, e.g. Which concepts are dominating in different traditions and what are the consequences, good and bad?

In practice, teaching and learning is strongly dependent on the formal organisation of lessons and strings of lessons, following principles like education (*Bildung*) oriented, subject matter oriented or student oriented for the structuring of lessons. So to answer the question why there are differences we have to find out in detail about a mixture of philosophical, social, psychological, cultural, political, economical and even ecological intentions and principles for teaching and learning.

Strong changes in teachers' and students' activities in class are reported from many countries. Control of teachers on the activities of students is diminishing. Students rely on outside sources or on information from their peers. They become more active in class and they want to have influence on contents and goals. Models like "students teach students" or "learning by teaching" are practised. How do mathematics educators in different traditions judge these activities? How do they try to exploit them for mathematics education purposes or how do they try to avoid them happening?

Some of the observations just mentioned originate from the new role of the didactic component "media". Compared to classical media like blackboard and chalk or the OHP, modern information technology based media like multimedia computers have a stronger touch of educational intelligence. They can furnish students with information as well as with some advice for learning and about correctness of findings. Communication technology can open up the classroom for geographically and - this is more important - culturally distant information, giving rise to activities like "distance learning". These experiences have a very strong impact not only on cognitive but also on emotional intentions in class (motivation, creativity, ambition). Our study will not only have to evaluate the quality of these attempts but also compare reactions to this kind of challenge in different traditions.

Do we see modern media as a new component in the conservative teacher-student structure of the classroom? What chances and what risks do we see in the new media and what are we doing about it? How can we explain the unchanged importance of teachers for the learning process to society, and how the new role? Even if we see the risks of the new media, can we avoid them? Many societies have seen a "metacognitive shift" going on at all levels of information and communication processes. There is less interest in the contents of these processes and growing interest in the medium carrying the contents. How do we uphold the intentions of mathematics education as far as contents and methods are concerned?

IV.9 Assessment of students' achievement

There have been different views and approaches to the assessment of students' achievement in mathematics. For example, in East Asian societies, students, teachers, and

parents view written tests and examinations as one most important things in a students' school life and as a key to the success of their future life. There are also high-stakes college entrance examinations. This does not seem to be the case in many Western societies. Moreover, there have been great differences in the approaches to assessment of students' achievement in different societies with different traditions. For example, in Western countries (e.g. the United States), standardized tests and multiple choice questions have been used for a long time in the assessment of students' achievement. Recently, alternative assessment methods have been increasingly used in Western schools. Again, this is not the case in Asian countries, though we can see some influence of the standardized test and alternative assessment in some Asian educational systems. The question is, what are the exact differences in assessment of students' achievement in different societies with different traditions? Why are there such differences? Will there eventually be a universal view and approach to the assessment of students' achievement because of the growth of globalization and information technology?

IV.10 Different aspects of achievement

Many available comparative studies, especially large-scale achievement assessments, have shown that East Asian students overall outperform their Western counterparts in mathematics. However, some recent researches have suggested that the superior performance of East Asian students is more evident in certain aspects of mathematics achievement, such as using basic skills of computation and algebraic manipulation, solving routine problems and school mathematics problems by applying algorithms, and it is less so in some other aspects. In some studies, researchers found that Western students performed as well as, or sometimes even better than, their Asian counterparts in aspects such as using visual and graphical representation and solving open-ended problems.

Some people have also argued that East Asian students might be better at abstract thinking in mathematics, but Western students might be better at intuitive thinking. Is this really true? In other words, do Asian students perform better in some aspects of mathematics achievement, and worse in some other aspects than Western students? If so, do the differences in these different aspects of achievement reveal certain differences in cultural traditions?

Furthermore, it has been shown in some international studies that students in East Asia, such as in Japan, do not like mathematics as much as, but they performed academically better than, students in Western countries. The question is, to what extent are students' attitudes towards mathematics and mathematics learning affected by their cultural traditions, by their social environment, or by their school learning experiences?

IV.11 Views on the nature of mathematics

Researchers have documented the fact that different kinds of mathematics have developed through different cultural traditions. For example, there are significant differences between ancient Chinese mathematics and ancient Greek

mathematics. The former is more algorithm-oriented and application-oriented, and the latter is more oriented towards deductive reasoning and not towards the application of mathematics.

The question then is, how do different cultural perspectives on the nature of mathematics influence mathematics education in different traditions? Moreover, what is the relationship between the cultural traditions and the societal views on mathematics and mathematics education in different traditions. In other words, how do they influence or interact with each other?

IV.12 Uses, misuses, and abuses of mathematics

There are different views on the relationship between mathematics and society. It has been claimed that mathematics is essential to a modern society, and mathematics can help people become informed citizens and make wise decisions in their individual and social life. This seems particularly true in the modern information age. However, it is not uncommon to see that mathematics, including statistics, is misused or even abused intentionally or unintentionally on different occasions and for different purposes. The question in this connection is, how is mathematics used, misused, and abused in different societies with different traditions? Why is mathematics used, misused, and abused in this way or that way in different societies? What can we do in mathematics education to help students as well as the general public to understand mathematics and its usefulness appropriately so they will not mathematically mislead others or be misled by others with numbers, data, graphs, statistics, or other forms of mathematical information.

IV.13 Non-formal mathematics education

In addition to formal school mathematics education, there are other forms of mathematics education, e.g., Juku schools in Japan, Bu-Xi-Ban in Hong Kong and Taiwan, private tuition in Singapore, mathematics clubs within schools, internet-based and home-based learning, to name a few. In this regard, we are interested in the following questions: what are the other forms of mathematics education existing in different societies with different traditions? Why do they exist in different societies? What role do they play in the whole picture of mathematics education? How do they interact with formal mathematics education?

IV.14 Evolution of mathematics education

Mathematics education has a long history in human civilization. The question of interest is, how has mathematics education been developing and changing in different countries or societies in different traditions? As the world is increasingly globalised and information technology helps people in different places communicate and share their questions, ideas, and information more freely and conveniently, will there be a universal approach to mathematics education? What can we learn about mathematics education from the past in order to ensure a better future for mathematics education?

V..Contributions to the study

V.1 Call for contributions

The ICMI Study *Mathematics education in different cultural traditions: A comparative study of East Asia and the West* consists of

- an Electronic Discussion Forum,
- an invited *Study Conference*, and
- a *Publication* to appear in the ICMI Study series.

A discussion web-site is being set up, and members of the mathematics education community are invited to participate in discussion on the major topics and problems identified in this discussion document or related issues. Please refer to the official web-site at the end of this document.

The invited Study Conference, with a size of about 80 to 100 participants and a duration of 5 days, is scheduled for October 2002, in Hong Kong.

The IPC, as well as ICMI, is interested to have approximately equal number of participants from East Asia and the West, like the composition of the IPC. English, however, will be the language of the conference. We are well aware that this may mean a handicap for many individuals whose first language is not English, but we would nevertheless like to encourage such people to participate. We would also like to encourage the native English speakers to take special care of this situation. We will have little chance to succeed in a real comparative study if we do not succeed in managing the language problem in the Study Conference.

It is expected that every participant be active in discussion and other modes of activity during the conference.

Participants should finance their own attendance at the conference, and invitation to attend the conference does not imply that financial support will be provided by the organisers.

Individuals and groups are invited to send in abstracts of their anticipated contributions on specific questions, problems or issues raised in this document to **both** co-chairs as soon as possible. Submissions should be up to two pages in length and may be e-mailed, faxed or posted as hard copies. Based on the abstracts, some contributors will be asked to produce longer versions of their submissions for the consideration of the International Programme Committee (IPC). Invitees to attend the Study Conference will receive comments on their submissions from the IPC and will be asked to produce camera-ready revised versions of their submissions for publication in the pre-conference proceedings.

The major outcomes of the Study, based on the contributions to, and the outcomes of the Conference, as well as results from the electronic discussion, will be published as part of the ICMI Study series by the end of 2003, and will be presented at ICME-10 in 2004.

V.2 Time-line

The planned time-line for the Study is as follows:

From March 2001

Individuals and groups start sending in to both co-chairs abstracts of their anticipated contributions in reaction to

this Discussion Document

From May 2001

Contributors invited to produce longer versions of their submission for the consideration of the IPC

30 September 2001

Deadline for submission of longer versions of contributions

October 2001

IPC meeting

November 2001

Invitations to attend the Study Conference sent to selected individuals

15 June 2002

Camera ready papers for the conference to be submitted to the co-chairs

August 2002

Publication of pre-conference proceedings

October 2002

Study Conference in Hong Kong

December 2003

Publication of Study volume

July 2004

Presentation of Study results at ICME-10.

V.3..International Programme Committee and contacts

The members of the International Programme Committee (IPC) are:

Annie BESSOT (France)
 Alan BISHOP (Australia)
 Lianghuo FAN (Singapore)
 Walther FISCHER (Germany)
 Klaus-Dieter GRAF (Germany, Co-chair)
 Bernard HODGSON (Canada, ICMI Secretary)
 Frederick LEUNG (Hong Kong, Co-chair)
 Fou Lai LIN (Taiwan)
 Francis LOPEZ-REAL (Hong Kong, Chair of the Local Organising Committee)
 Kyungmee PARK (Korea)
 Katsuhiko SHIMIZU (Japan)
 Jim STIGLER (USA)
 Margaret WU (Australia)
 Dianzhou ZHANG (China)

Enquiries on all aspects of the Study, as well as suggestions concerning the content of the study conference programme should be sent to **both** co-chairs:

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The official web-site for the Study is

<http://www.inf.fu-berlin.de/icmics>