Baumslag, Benjamin:

Fundamentals of Teaching Mathematics at University Level

London: Imperial College Press, 2000. – 240p.
ISBN 1-86094-214-8

Ed Dubinsky, New York (USA)

This is a book of opinions. The author has taught mathematics at the university level for many in years, in several different universities in a number of countries. He has done this in a conscientious manner, attempting to be aware of everything that has been going on and trying to do the best job of teaching that he could. As a result, he has developed points of view on a vast number of topics, issues, ideas, etc. related to university level mathematics teaching and the purpose of this book is to share those opinions with readers. The material is quite comprehensive and just about every aspect, from how a teacher should stand in relation to the blackboard to research in learning mathematics is treated. Although the book might be useful to someone who had not the slightest idea about the enterprise of teaching university mathematics, I did not find any of the opinions to be remarkable or surprising. The most superficial topics (such as checklists for how to discuss the statement of a theorem, or what kinds of breaks to take in a lecture) are given detailed treatment whereas major issues of teaching and learning (such as those related to the mathematical concepts of function or limit) are handled in analmost offhand manner.

For example, the author spends two pages giving three methods for teaching limits. The first is for the teacher to introduce the idea of successive approximations, the second is for the teacher to describe the limit informally as the expected value based on neighboring information, and the third is for the teacher to describe the epsilon-delta idea in terms of controlling error. It is certainly useful for beginning faculty to know that such methods are used by experienced teachers to help students understand deep and difficult concepts. But it would be perhaps even more useful for the neophyte to be aware that, in spite of such methods, very few undergraduate mathematics students develop very much understanding of such concepts.

We see in this example, two major characteristics of Baumslag's point of view. One is the emphasis on what he (and others) refer to as "teacher centred" teaching. Although many other approaches are mentioned, it is clear throughout the book that Baumslag's emphasis is on what the teacher does: present, describe, introduce, and so on. There is very little about what may or may not be happening with the students – and this relates to the second major characteristic of this work. Again and again an opinion is stated about what is a good or a bad thing for the teacher to do. This is, however, almost never related to student learning. That is, Baumslag makes not the slightest attempt to justify any of his many opinions on the basis of student learning. He does of course say that this or the other suggestion for what a teacher might say or do will lead to student learning, but he gives us no way of making anything like an independent judgement. He tells us about his many years of experience (but not of any exceptional successes) and asks us to more or less take his word for it.

Indeed, Baumslag seems almost defensive about suggestions that alternative approaches to teaching can lead to results that are better than what happens in traditional teaching. Consider, for example, the following curious excerpt.

> "One of my colleagues explained with great pride that the new method of teaching he had introducedhad meant that the students had been forced to work much harder. As a consequence, they had learneda lot more. He therefore felt that his new method of teaching was a success. I cannot agree. I find itquite natural that if somebody works harder in an effective manner he will learn much more." (p. 49.)

In my view, the key here is the term "effective". There are many university situations in which it is not difficult to get students to work very hard. I have conducted courses in mathematics to engineers at several universities, some of them amongst the highest ranked. In these experiences, I observed that students worked extremely hard, but they did not learn very much mathematics (beyond the ability to repeat precisely any calculations that had been carefully, explicitly, and fully laid out for them). It was only when teaching methods could be found that got them to think for themselves about mathematical problem situations, and still spend many hours working at mathematics, that exceptional learning took place. So if we can take at face value (since Baumslag does not deny it) that his colleague's students worked harder than normal and learned a lot more, then I think it is fair to agree with the conclusion that the method was a success. Indeed, I and most of the mathematics faculty I know are overjoyed when they have such results – no matter what the method of teaching!

Although we may well concede that Baumslag is entitled to his opinions, I must say that I am rather disturbed by the statement he makes in the very next sentence after the above quote.

> "The whole point of providing instruction is to reduce the work required for learning."

I taught college level mathematics for 44 years and I find one conclusion unavoidable. Learning mathematics is hard work and learning more mathematics is more hard work. Baumslag goes on to say that he would have been impressed if his colleague had claimed that the students had "learnt more with less work." I worry about teachers whose goal is to get students to do less work. It is easy to create a false impression of successful learning, for example by giving exams with questions that appear hard but are relatively easy for the students because they have done little else but drill on prototypes of precisely those questions. Your students will love you, the administration will be happy, and you may win teaching awards. But your students will not learn.

These are not the only opinions Baumslag puts forth that I find unfortunate. Here is a sampling of some others, followed in some cases by opinions of my own..

"When I was at school, I studied Euclidean Geometry, from the age of 12. It was a matter of proving theorems from axioms and definitions. I and my contemporaries had a very good idea of what a proof entailed." (p. 19.)

This high ability in mathematics on the part of students in the past is a belief held by so many people that no one seems to think it needs to be supported by any facts. The only data I know of suggests the opposite: students in mathematics throughout the 20th Century were, on average, not substantially better at mathematics than those of today.

"The event of the computer means that people can now solve extremely difficult problems with less knowledge than before." (p. 25.)

"Explaining and giving drill on using the rule is training, explaining why the rule works is education." (p. 31.)

At the risk of repeating myself, I must counter this opinion with the view that explanations are never education. Getting the students to understand why the rule works is education.

"Not everybody can learn mathematics." (p. 33.)

In a general consideration of new methods of teaching, Baumslag writes:

"The problem is reminiscent of the alchemists search for converting lead into gold; nobody knew whether it was possible or economic, and much time and energy were frittered away." (p. 49.)

It is particularly disturbing to think of so many years of teaching to so many students has been conducted by one who has such a view about efforts to explore the unknown in relation to teaching and learning. I have recently read that Isaac Newton was very interested, perhaps in an active way, in alchemy. If he were living in Newton's time, would Baumslag have said that Newton was frittering his time away when he was trying to understand the relation between lead and gold but not when he was trying to understand the influence, at a distance, each could have on the other's motion?

"Teacher-centred teaching has always been effective and is one of the most natural ways of learning." (p. 49.)

"Most teachers are satisfactory." (p. 50.)

In reference to students who study mathematics in order to "help them understand some other topic, for instance, engineering:

"In such a case, the need is more for training in procedures and algorithms rather than in the proper understanding of mathematics itself." (p. 51.)

When I joined the faculty of Purdue University in 1987 and began the development of new ways of teaching Calculus (in courses largely populated by engineering students) I found the above opinion to be the prevailing view in the mathematics department. I was told that this was the position of the Engineering and Science Faculty and therefore the department's calculus courses emphasized training in procedures and algorithms. I decided to check this before developing alternative strategies. Together with my colleague, Keith Schwingendorf, we contacted the leadership of every Engineering and Science department (and all of them can be found at Purdue) and asked for the names of faculty in their department who could give us definitive information about what their department wanted students to bring them from their calculus courses. In some cases the chair responded, in others an individual faculty member was designated and in others, we met with a committee. To all we said, "You require your students to take courses in calculus. What would you like them to learn in these courses?"

The responses, which did not vary very much were astounding. The immediate response was in every case something like: "Well certainly don't emphasize procedures and algorithms. We can quickly teach them to do that with Maple or Mathematica. And don't bother much with applications. We prefer to teach the X (here read one or another branch of Engineering or Science) material ourselves because we are the experts. We want you to teach them the mathematics in which you are the experts." The implication, occasionally explicit, was a concern that in general, mathematicians might mess things up in a field in which they might have had little training.

The conversation then went on to consider understanding mathematics and most of the engineering and science faculty expressed the view that the proper understanding of mathematics itself was what they hoped their students would get from their mathematics courses. One individual, who taught Physical Chemistry put it rather succinctly. He said: "All I want is that if I am teaching students who have studied calculus and want to explain some physical concept, like work, in terms of adding up a large number of small approximations by constants and passing to a limit, then I would like my students to act as if this general procedure was something they had seen before."

At the time my conclusion was, and it has remained so, that this emphasis on procedures for engineering and science students, advocated in the above statement by Baumslag, is an idea that comes not from Engineering and Science faculty, but from those who teach mathematics! This is not the place to discuss why this had ocurred, but I do wish that Baumslag had.

I will not suggest it represents an opinion, but I wish someone had pointed out to the author that if the audience for this book includes the United States, then a number of potential readers will be offended by the use of the term, "denigrating" (p. 52.)

I do not disagree with all of Baumslag's viewpoints. For example, I applaud his preference for texts that show how results are obtained as opposed to those that "…use a very terse style which require one to begin at the beginning and master each step perfectly before proceeding…" that "…assemble the precise material needed, and faultlessly and without explaining any connections, derive the major results in the most general form." (p. 73). But I find far more questionable opinions than those I can support.

The strength of this book is that it touches on so much of what one should be thinking about with respect to undergraduate mathematics education. If this material were given a better treatment, I would recommend that beginning college teachers read it. But the overemphasis of trivia, banal suggestions, and unfortunate opinions presented with no regard to supporting evidence leads me to conclude that the experienced teacher will learn little from it and the novice can probably find better introductions to teaching mathematics in an essentially traditional manner, such as the recent book by Krantz on the same topic.

Author

Dubinsky, Ed, Prof. Dr., 265 North Woods Rd., Hermon, NY 13652, USA (May – Oct.); 5638 Springdale Rd., Apt. 3, Cincinnati, OH 45251, USA (Oct. – May).
E-mail: edd@mcs.kent.edu
Internet: http://trident.mcs.kent.edu/~edd/