The Creation of Actuarial Science

Chris Lewin, London (Great Britain)

Abstract: Actuarial science was created more than 300 years ago. The article traces the influences which led to this development, which has been of such fundamental significance for calculations involving risk and finance and has enabled life assurance companies and pension funds to be financed on scientific principles. Actuarial techniques are nowadays starting to be applied in wider fields and it is suggested that the history of actuarial science could be taught in sixth forms and universities to students of risk and finance.


ZDM-Classification: A34, A35, M34, M35

Introduction
None of us knows when we will die. The range of uncertainty is very wide.

Some of the relevant factors are: sex, age, hereditary influences, present state of health, lifestyle, accidents, smoking habits, disease, medical advances, wars etc.

However, given a large group of people, predictions can confidently be made about how many of them will normally die this year, how many next year, and so on. If an insurance scheme were set up for the group, providing future payments on each person’s death for the benefit of his or her family, we could work out nowadays how much money each person in the group would have to contribute in advance to a common fund to pay for the scheme, provided we knew the rate of interest which would be obtainable on the invested fund.

This article outlines how a number of influences came together in the late seventeenth century to create actuarial science and enable such calculations to be made with confidence. It is suggested that the history of actuarial science might usefully be taught in sixth forms and universities, as a background to the present-day study of risk and finance.

There were four separate developments which led up to the creation of actuarial science:

- the prior existence of a well established framework of pensions, insurance and leases on lives
- a good understanding of compound interest techniques
- the emergence of probability theory
- some pioneering statistical analysis of Bills of Mortality, which showed for the first time how real data could be obtained and applied to practical uses. All these developments came together in 1693, when Edmond Halley, the famous mathematician, published a crucially important paper, using real life data from the Breslaw Bills of Mortality to construct a life table and describing for the first time how to use such a table to work out the purchase price of a life annuity.

Halley’s paper had been preceded during the previous twenty years or so by one or two pieces of similar work by others, but his was the influential paper which created actuarial science. His paper was widely read and led, half a century and more later, to the establishment of pension funds and life assurance companies funded on scientific principles.

The need for insurance
The basic need for insurance and pension arrangements stems from individual risk and uncertainty. If you go on a journey or voyage, there is the risk of losing any goods entrusted to you, or your own possessions, or even your life. Your house may catch fire and leave you and your family without a roof over your heads. If you are a breadwinner, you run the risk of dying too soon and leaving your dependants destitute. Alternatively, you may live too long after retirement and fall into poverty when your savings are exhausted.

These risks existed from the earliest times and the traditional method of relieving the resulting poverty was by charity. This was never very satisfactory, however, because it often provided inadequate relief and it had a social stigma. It was natural, therefore, to look for some form of insurance whereby the necessary sums could be provided as of right at an adequate level, out of funds earmarked for the purpose.

We shall now look briefly at the beginnings of marine insurance, life insurance, pensions and leases for lives, long before actuarial science was conceived.

Marine insurance
Originally the finance for a sea voyage was commonly put up by one or more wealthy individuals who agreed not to seek repayment if the cargo was lost. If the ship arrived safely, the money was repaid with a heavy rate of interest, which covered both the risk premium and the true interest on the loan.

This kind of transaction can be traced as far back as the ancient Greeks. In one case quoted by Demosthenes (who was born about 384 BC), 3000 drachmas were advanced in respect of a cargo of wine. The rate of interest was to be 22½%, which was to be increased to 30% if the return voyage was delayed until the stormy season. (Kennedy 1880).

However, not everyone who was prepared to provide finance would have been happy to take the risk of loss, even if compensated by a high rate of interest. It is thought that the separation of the protection element from the financing came during the period 1300-1350 AD. This presumably happened because an expansion of world trade meant that more finance was needed than could be provided from traditional sources. Thus

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Insurance was born.

The first genuine insurance policy we know of was dated 1350. A cargo of wheat from Sicily to Tunis was assured for 300 florins at a premium of 18%. The insurer, Leonardo Cattaneo, undertook to assure all risks from act of God and from perils of the sea. (Raynes 1964).

Marine insurance was well established in London by 1559, when Sir Nicholas Bacon said: „Doth not the wise merchant, in every adventure of danger, give part to have the rest assured?„.

However, when claims arose the insurers did not always pay up, and this gave rise to numerous disputes. In 1575 the English Privy Council acted to establish the Office of Assurances at the Royal Exchange in London. (Lewin 1987). A copy of every policy had to be copied into the policy register held there. There was a set of detailed rules, which still survive, covering the circumstances in which the sum assured had to be paid. (Lewin 1991).

Life assurance

Although marine insurance was the main business of the Office of Assurances, there were also some insurances on lives. (Lewin 1988a). These contracts were for one year only. If continued cover were required, for example to protect a lender until the borrower had repaid the loan, it was necessary to take out a succession of one year contracts. The disadvantages of this arrangement were that the premium would rise as the life got older and there was always the possibility that his health might deteriorate to such an extent that he became uninsurable at the time of renewal.

Pensions

The idea of a pension for life is very old indeed. In the Bible we read that Jehoiachin, King of Judah, was released from prison in his thirty-seventh year of exile in Babylon, treated kindly, and given a seat at table and „lived as a pensioner of the king for the rest of his life...“ (New English Bible 1970). This was in 562 BC.

Around 400 BC Lysias, a Greek orator, found that his State pension was discontinued. (Lamb 1930). He had been granted the pension when he was younger, possibly because of war injuries. The reasons now given for its cessation were that he was able-bodied and not classed as disabled, and he was skilled in a trade, could mount a horse, and was well-off financially. He protested vigorously about having his pension withdrawn in his old age. He also put forward the subtle argument that, when the city voted a pension for him and others like him, this was on the basis of „regarding the chances of evil and of good as the same for all alike... In other words, the goal posts should not now be moved, just because things had turned out relatively well in his own case, just as they would not have been moved in the other direction if he had fallen into poverty.

The earliest British occupational pension we know about was awarded in 1286 to Philip de Harwodelme, Rector of Bigby, near Exeter. (Cutts 1898). He was so worn down by disease and old age that he was awarded a pension of about £13 per annum, payable out of the parish tythes.

Despite the absence of insurance companies, it was sometimes possible to buy yourself a pension in the Middle Ages. You paid a lump sum to a monastery, or granted them a piece of land, and in return they would let you live there for the rest of your life, with board and lodging, and often a small regular cash payment as well. The income and other benefits derived from these transactions was referred to as a corody. (Harvey 1993).

In addition it was common practice for the King of England to grant pensions to those who had done him good service or had served in the royal household.

Several European towns, including Amsterdam in the seventeenth century, issued life annuities in order to raise capital for wars or property development.

Leases for lives

One of the most common transactions in the seventeenth century was the granting of leases for lives. Henry Phillipes, a British writer in 1654, gives us details of these transactions. The purchaser of the lease paid a lump sum and could occupy the house rent-free or at a low rent until the death of a named life (often his own). It was the custom to calculate the lump sum as the amount which would be paid for a lease of 7 years.

Sometimes the lease continued until the death of the last survivor of two or more lives. The lump sum was calculated as the amount which would be paid for a lease of 14 years (for 2 lives) or 21 years (for 3 lives) and so on.

Phillipes felt that the use of 7, 14 or 21 years as the terms equivalent to one, two or three lives, was unfair and should be replaced by 12, 23 or 33 years respectively – a big difference. This illustrates the unsatisfied need at that time for a proper scientific basis to determine an appropriate purchase price for a financial transaction depending on the chances of human survival.

Phillipes appears to be the first writer, at least in English, to discuss in any depth which is the most appropriate rate of interest to use for valuing different kinds of transactions. For example, he points out that land is considered a safe investment and should be valued at 5% per annum, whereas houses are more risky and should be valued at 10% per annum. The use of different interest rates as a tool to reflect differing risks has come down to us and is still often practised today when valuing investments, properties and capital projects. Modern actuaries are only just starting to point out the weaknesses in this method and the advantages to be gained from identifying and then mitigating the specific risks in the transaction.

But how had interest itself come about?

Interest

There is some evidence that interest may have been charged on loans in the ancient civilisations of Babylon, Greece and Rome. There are a few Biblical references to usury, for example in Exodus: „If thou lend money to any of my people that is poor by thee, thou shalt not be to him as an usurer, neither shalt thou lay upon him usury..."

In the Middle Ages the Jewish money lenders in
England often charged a rate of twopence in the pound per week, ie about 43% per annum. Debts often mounted up quickly at this rate and many theological debates ensued about the justification for charging interest and whether a maximum rate should be fixed. (Lewin 1990a).

However, not all money lenders were Jewish and in 1179 AD a Papal law ordained „that manifest usurers shall not be admitted to communion, nor, if they die in their sin, receive Christian burial, and that no priest shall accept their alms„.

The mathematical basis of compound interest was certainly understood by some people in the Middle Ages – a surviving fourteenth century manuscript includes tables showing accumulations at compound interest. (Pegolotti 1936).

In England the line between permitted genuine commercial transactions involving risk and forbidden usurious transactions was often a fine one. Usury was characterised by a specific interest charge and the taking of security, so that risk was virtually eliminated.

By the seventeenth century, however, it was generally accepted that money-lending was both legally and morally acceptable in some circumstances. As Malyne put it in 1622:

„There are three sorts of men, the stark beggar, the poor household and the rich merchant or gentleman. To the first you ought to give freely, not only lend freely. To the second you ought to lend either freely or mercifully, and not to feed him with excessive usury. But with the third you may deal straightly, and ask your own with gain, especially when he gaineth by your money„.

Simon Stevin of Bruges published a detailed treatment of interest in 1585, in his textbook „La Pratique d’Arithmetique„. He had many worked examples, and tables showing the present values of future payments.

In 1613 Richard Witt, a 44-year old London mathematical practitioner, published a landmark work in the history of compound interest, entitled „Arithmetical Questions, touching the Buying or Exchange of Annuities„. The book delved deeply into the subject in a very practical way, with many tables and examples. (Lewin 1970). The solutions demonstrated a knowledge of the equation of payments, which was later to become a fundamental concept in actuarial work.

An example from the book is as follows:

„A oweth to B £1200 to be paid in 6 years, in 12 equal payments, viz. at the end of each half year £100. They agree to clear this debt in 3 years, in equal payments, viz. at the end of each half year, one payment. The Question is, what each payment ought to be, reckoning interest after the rate of 10 per cent per annum and interest upon interest„.

Witt gives the extremely elegant solution that each payment is (a) the £100 that would have been paid anyway plus (b) the £100, due to be paid 3 years later, discounted for 3 years. This gives £175.2s.7d, with hardly any arithmetic being needed.

Witt made some remarks about the usefulness of arithmetic, pointing out that, although some businessmen managed to get by without such knowledge, „it is as true that the blind drink many a fly„.

There is a possibility that compound interest tables may have inspired the invention of logarithms, published by Napier in 1614. Napier tells us in another of his books that he studied Stevin’s book on arithmetic, mentioned above as including compound interest tables. Stevin’s discount table at 1% per annum was in almost the same form as the tables used by Napier in working out his logarithms. Is it possible that this is where Napier got the idea? (Lewin 1990b).

**Probability theory**

The emergence of probability theory was the third of the four factors which led to the creation of actuarial science. Pioneering work was done by Cardano in the sixteenth century, on games of chance and the probabilities involved in dice throws. (Ore 1965). However, this material was not published until 1663, long after his death.

The influential work which introduced the theory of probability to the world was written by Christian Huygens and published in 1657. His third proposition can be summarised thus: if the number of chances of getting a is \( p \) and the number of chances of getting \( b \) is \( q \), assuming always that every chance occurs equally, then this is worth \( (p+q)/(p+q) \) to me. Now this proposition is particularly relevant to the valuation of the instalments of a life annuity (ignoring interest). Suppose one has a group of lives of a given age and one puts \( p \) as the number of them who survive to a later age to receive a particular instalment, with \( a \) as the amount of the instalment. If \( q \) is the number of lives in the original group who do not survive until that later age, and \( b \) equals nil (since the deceased lives do not get the instalment), the expression simplifies to \( p/(p+q) \). In other words, the value at the outset of that particular instalment of the annuity for one person equals the amount of the instalment times the proportion of lives in the original group who will survive to receive it.

Although Huygens’ third proposition might seem an elementary truth today, it was new at the time and it is very likely that the methods of thought to which it gave rise helped a little later, in the creation of methods by which life annuities could be valued.

**Statistical data**

The final factor leading to actuarial science was the analysis of statistical data relating to life and death.

The principal source of this data was the Bills of Mortality, which had been kept weekly in London and large cities on the Continent of Europe to warn the upper classes when the incidence of plague was starting to increase, so that they could leave the city for the supposed safety of the countryside.

The first reference I have seen to the possibility of using Bills of Mortality for statistical purposes is contained in James Howell’s „Letters„, of 1645, when he writes of Amsterdam:

„This City, notwithstanding her huge trade, is far inferior to London for populousness; and this I infer out of their Weekly Bills of Mortality, which come not at most but to fifty or thereabout; whereas in London the ordinary number is ‘twixt two and three hundred, one week with another„.
However, the major development in this field was undertaken some years later by John Graunt, whose classic book, "Natural and Political Observations made upon the Bills of Mortality", was published in 1662. Graunt, a London shopkeeper who belonged to the drapers’ company, investigated the London Bills of Mortality for a long series of past years, with a view to drawing conclusions about the city’s population.

One of the book’s achievements was the publication of the first known life table to be based, in part, on real mortality data. (Lewin 1988b). Although the Bills did not record the age at death, Graunt observed that about one-third of all deaths occurred from childish ailments and guessed that some of the other illnesses also caused deaths among children. From this he estimated that about 36% of all deaths related to children under 6 years old. Hence he argued that, of every 100 children conceived, only 64 of them would reach age 6. He then used a mathematical projection to obtain estimates of the numbers of the original 100 who would reach ages 16, 26, 36 etc as follows:

<table>
<thead>
<tr>
<th>Age</th>
<th>Number living</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>64</td>
</tr>
<tr>
<td>16</td>
<td>40</td>
</tr>
<tr>
<td>26</td>
<td>25</td>
</tr>
<tr>
<td>36</td>
<td>16</td>
</tr>
<tr>
<td>46</td>
<td>10</td>
</tr>
<tr>
<td>56</td>
<td>6</td>
</tr>
<tr>
<td>66</td>
<td>3</td>
</tr>
<tr>
<td>76</td>
<td>1</td>
</tr>
<tr>
<td>80</td>
<td>0</td>
</tr>
</tbody>
</table>

One of Graunt’s main purposes was to estimate the London population, which common talk often asserted ran into millions. He approached the question in several ways and produced an estimate of 384,000 people, which is nowadays considered reasonably accurate.

Graunt’s book was the starting point for both statistics and demography. He was the first person to analyse a considerable body of social data carefully, considering possible recording inaccuracies, checking from other sources where possible, and drawing conclusions directly from the data. He discovered the regularities of the patterns of life and death in groups of people.

The book was widely circulated on the Continent. It was certainly known in Holland, where the Dutch Prime Minister, Jan de Witt, produced some pioneering calculations in 1671 to work out the price which should be paid for Dutch Government life annuities. This work was probably influenced by the mortality investigations which had been carried out by John Hudde at about that time. However, all this work remained in obscurity, despite the important advances it made, and did not influence the mainstream development of actuarial science. We shall therefore pass over it here.

**The creation of actuarial science**

We come now to the creation of actuarial science. All the conditions were right. Primitive forms of insurance, pensions and leases on lives had existed for a long time.

The mathematical techniques of compound interest were well-known. Probability theory had made an appearance. Data relating to life and death had been analysed, and life tables constructed.

The trigger was supplied by Caspar Neumann, a pastor of Breslaw, who was interested in scientific matters. He collected some data from the Bills of Mortality of Breslaw, a town devoted to linen manufacture, and sent them to the Royal Society in London in case they might be useful.

The 36-year old Halley was at this time the editor of the Philosophical Transactions of the Royal Society. It appears that he decided to make some use of the data which had arrived from Neumann, by using it as the basis for a paper. Since he was an expert in quite a different field – astronomy – it is quite remarkable that he was able to produce such an excellent treatise, which laid the foundation for actuarial science. (Halley 1693).

After stating the numbers of deaths at each age, according to the Breslaw data, Halley produced his famous life table, an extract from which follows:

<table>
<thead>
<tr>
<th>Age</th>
<th>Persons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>855</td>
</tr>
<tr>
<td>3</td>
<td>798</td>
</tr>
<tr>
<td>4</td>
<td>760</td>
</tr>
<tr>
<td>5</td>
<td>732</td>
</tr>
<tr>
<td>6</td>
<td>710</td>
</tr>
<tr>
<td>7</td>
<td>692</td>
</tr>
<tr>
<td>8</td>
<td>680</td>
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<tr>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>84</td>
<td>20</td>
</tr>
</tbody>
</table>

He used the life table to estimate the total number of people in Breslaw at 34,000, with a subdivision into 13 age groups.

Halley then used his life table in the same way as an actuary would today. Suppose that one wishes to value a life annuity payable to someone aged 3 years. The first payment will fall due at the age of 4 and the chance of being alive to receive it is 760/798. Assuming the payment amounts to £100 and is actually received, the present value of it (using an interest rate of 4% pa) would be 100/1.04, ie £96.15. Allowing for the chance of surviving to get it, the present value is £96.15 multiplied by 760/798, ie £91.57. One can then proceed similarly for the value of the second instalment, which is worth 100/1.04/1.04 times 732/798, ie £84.81. By proceeding similarly one can evaluate the present values of each of the future instalments of the annuity. Summing these then gives the total present value of the annuity for the whole of life.

People then started to think about the practical applications of Halley’s method. There was one great obstacle – the amount of labour required to make the calculations. In 1725 the mathematician Abraham de Moivre published a method for reducing the work. This method assumed that the numbers of those living, according to the life table, decreased in arithmetical
progression with increasing age. Other books appeared before 1750, giving new approximate methods or calculated values of life annuities.

In 1743 the first long-term financial institution to be based on scientific principles was set up. This was the Scottish Ministers Widows’ Fund, which survived for 250 years until its closure in 1993. Estimates of the growth of the fund each year were made in the early years and these proved amazingly accurate.

A notable contribution to the theory of insurance appeared in 1747, when a pamphlet by Coryn Morris demonstrated that an insurer’s `probability of ruin’, decreases as he spreads his available wealth between more and more policies at one time. His conclusions were arrived at by simple probability theory, using the binomial theorem.

In 1762 the first life assurance company based on actuarial science was founded – the Equitable, which still exists today, though its recent history has been troubled and it has stopped accepting new business.

The teaching of actuarial history
Many of those who nowadays work in risk, finance, insurance or pensions have no idea of how long and respectable a history the subject has. Those who work in pensions, for example, are very often unaware that people were awarded pensions in the Middle Ages, let alone in ancient Greece. A limited historical perspective can sometimes be a hindrance when considering the future, whereas those who know about the past can not only avoid reinventing the wheel but also consider today’s decisions in an overall context: important, for example, when it comes to setting the terms today for pensions which will not be paid for another 50 years.

It is suggested that present-day students in risk, finance and related subjects might therefore find it useful to learn a little about the beginnings of pensions and insurance, and how actuarial science came about. The history is very interesting, particularly when one examines the wealth of detail available, which is far more than it has been possible to cover in this article. This is hardly surprising in view of the fact that the subject covers two of the most fascinating aspects of life - people and money. It also embraces some quite advanced theory and techniques, which have engaged the minds of many of the greatest mathematicians for the last three hundred years. For example, one intriguing question has been whether the underlying force of mortality follows a mathematical law. Another has been whether it can be inferred from mortality statistics that there is an age beyond which a human being cannot live.

Perhaps the best way forward would be to make history modules available to sixth form students or undergraduates before they become too deeply immersed in the intricacies of the subject as it exists today. A new text-book would probably be needed. Sufficiently knowledgeable lecturers might be hard to find initially, so there could be a place for distance learning through videos, etc.

It should not be thought that it is only actuarial students who will be interested in the history of the subject. As the world becomes more complex, many of today’s undergraduates will find themselves studying risk and finance, even if they are not taking actuarial degrees. Insurance and pensions have provided the early examples of fields in which the mathematical techniques of probability, statistics and compound interest may usefully be applied in combination. However, today the actuarial profession has realised that these techniques, and modifications of them, have much wider applications in the modern world, for example in capital project appraisal, risk management methodologies and corporate finance. Actuaries are starting to become actively involved in those wider fields, particularly in Britain and Australia, alongside professionals from other disciplines.

As a new chapter in the history is beginning, it is appropriate to turn attention to the teaching of the history itself.

References
Pegolotti, F.B.; Evans A. (Ed.) (1936): La Pratica della Mercatura. – Mediaeval Academy of America (No.24).
Phillipps, H. (1654): The Purchasers Pattern. (Other editions in ZDM 2001 Vol.33 (2) Information
1663, 1676 and 1719).

Author
Lewin, Chris, Institute of Actuaries, Staple Inn Hall, High Holborn, London, WC1V 7QJ, Great Britain.
E-mail: chris.lewin@unilever.com.