

Steinbring, H.; Bartolini Bussi, M. G.; Sierpiska, A. (Eds.):

Language and Communication in the Mathematics Classroom

Reston, VA: NCTM, 1998. – 361 p.
ISBN 0-87353-441-7

Anna Sfard, Haifa (Israel)

Communicating to Learn or Learning to Communicate?

Mathematics Education in Quest for New Answers to Old Questions

Even if not all the authors of *Language and Communication in the Mathematics Classroom* share Falk Seeger's and Harold Brodkey's "distrust of summaries", most of them probably do. As a group, the creators of this important book show clear signs of being "bored" with the long tradition of neat, rigorously structured accounts of reality that aspire to be "in control of what [they] recount" and "pretend to be superhuman". Most of the voices that can be heard on the 350 pages of this volume disclose disillusionment with educational research molded in the image of scientific laboratory study, and vote for something new.

Like many other scholars these days, the contributors of *Language and Communication* are tired of the sterility and objectivist pretences of positivist science. During the last few decades we have had an ample opportunity to become disappointed by research that attempts to enclose real-life phenomena in enticingly simple, artfully crafted networks of cause-effect explanations. As has been becoming increasingly obvious due to the long series of cross-cultural and cross-situational studies on mathematical thinking (see e.g. Scribner, 1983, Lave, 1988, Saxe, 1991, Nunes et al. 1993, and a summary in Cole, 1996), only too often does the neatness and aesthetics of educational research come at the expense of the profoundness and usefulness of its insights. One cannot see the gist of things while looking through too selective a lens; or, to use the Falk Seeger's metaphor, one cannot feel the taste of a sophisticated dinner just by reading its menu (p. 86).

Rather than trying to force classroom realities into theoretical straitjacket, those who wish to phantom the intricacies of learning should be entitled to "being on [their] knees in front of the event" (Harold Brodkey, quoted by Seeger on p. 85). Today, most of educational researchers regard as their first duty documenting the everyday and the common in all its ordinary manifestations and in all its unyielding complexity. Of course, nothing of significance will be noticed as long as "the common" is watched in the everyday, casual ways. Like an ethnographer studying a foreign culture, the observer must first try "to make the familiar strange" (Erickson, 1986, p. 121; cf. Seeger's chapter, pp. 97-98).

Although far from easy, such eye-opening de-familiarization is not altogether impossible. Like in any cultural analysis, the unprejudiced scrutiny should be accompanied by "guessing at meaning, assessing the guesses, and drawing explanatory conclusions from the better guesses" (Geertz, 1973, p. 20).

All this is being done along the twenty chapters of *Language and Communication*. Conceived almost ten years ago in a set of ICME-7 papers¹, the book documents the point in history when the conceptual unrest is at its peak: The rich and promising landscape revealed through the authors' collective effort is still rather inchoate, with its vast areas mostly under construction. The old infrastructure has already been shaken, but the new foundations are not yet fully shaped. Different architects wish to size the special opportunity and each one of them brings his or her own perspective. Since the new discourse has only begun, they did not yet have a chance to agree on much, not even on a vocabulary. As the Editors note in the Epilogue, "The book leaves us with many issues that need to be addressed" (p. 344). For many readers this openness is likely to count as an advantage rather than shortcoming. Most of them may be expected to enjoy the rare opportunity to watch a whole new research paradigm in making. The clearly visible uncertainties and dilemmas of the beginnings make for a thought-provoking reading. In this review, rather than trying to summarize or criticize the book, I focus on the message it brings as to the current state of research and practice of mathematics education, as to the most urgent needs of this domain, and as to the goals towards which we may wish to direct our further efforts.

1. On joys and sorrows of trailblazing

Doing research in the new ways means, above all, being attentive to the activity of learning as such and not just to its outcomes. In school, verbal communication is the principal manifestation of this activity. No wonder, then, that for the new researcher, studying learning becomes tightly related, or even tantamount, to studying classroom communication. It is here that Geertz's methodological principle of "guessing at meaning" becomes relevant and needs to be enacted. But methodology is not the only issue at stake in these times of change. The new awareness of the centrality of communication comes as a theoretical, methodological and pedagogical package deal, with its roots deep in the fertile soil of sociocultural approach to human cognition. Different chapters in the book emphasize different ingredients of this package.

The first group of articles, those by Falk Seeger, Heinz Steinbring, and Clive Kanen, grapples with the weighty question of conceptual and theoretical infrastructure for research that puts communication in the center. The diversity of possible approaches outlined on less than one hundred pages is certainly impressive, if not overwhelming. Not surprisingly, all three authors draw lavishly from contemporary philosophy, ethnography, anthropology, and linguistics, the domains where the

¹ The 7th International Congress of Mathematics Education took place in 1992 in Quebec, Canada.

disillusionment with the classical “scientific method” began and where the steep increase in the interest in issues of communication take its roots. Although the crucial contribution of Vygotsky and his followers is repeatedly stressed, many other philosophical and social theories are proposed as possible sources of useful conceptualization. The authors seek inspiration in places as diverse in character and as remote in origins as the Russian “dialogical” school initiated by Bakhtin (Kanes); Bruner’s essays on “modes of thought” and language acquisition (Seeger, Steinbring); new approaches to culture and cultural studies (Seeger), the French structuralists, post-structuralists, and post-modernists thought (Kanes), and, above all, the late work of the Austrian-British philosopher Ludwig Wittgenstein (Kanes). Closer to home, the German group of mathematics education researchers, headed by Heinrich Bauersfeld and inspired by the work of Alfred Schutz and Harold Garfinkel, is repeatedly mentioned as playing the pioneering role in shifting communication to the center of the educational scene (Steinbring, Wood, Krummheuer). Some other research communities, such as that of the British “discursive psychologists” (e.g. Edwards, Harre, Potter, Mercer) or of the American ethnomethodologists and applied linguists (e.g. Mehan, Cazden, O’Connor, Michaels, Forman) are mentioned only marginally, if at all, although they also deserve attention in the present context. But even if not exhaustive, the variety of conceptual frameworks offered in the book may be more than some readers would ask for. Of necessity, this dazzling mosaic is presented in rough strokes and in a rather breathless manner. Indeed, this is only understandable, considering the constraints of space and the overall amount of work to be done. And yet, uninitiated readers, and even those who do know a thing or two, may find this hasty, dense, and somewhat fragmented account a bit confusing. If this hurried trip through the vast theoretical area is not altogether overwhelming, it is in a great measure thanks to the helpful organizing framework offered by Anna Sierpinski in her introductory chapter.²

Methodological issues confront a beginner with yet another difficulty. If “making familiar strange” is the slogan the new researchers put on their banner, the question they have to answer is how to look at something as familiar as everyday classroom events so as to see beyond the obvious. Reacting to what other people say with an immediate intuitive interpretation is the skill which we learn since our birth as this is what makes communication possible. The new research requires that we inhibit this spontaneous interpretive impulse and bracket our own understandings. We thus have to unlearn what has become the basis of our ability to deal with, and make sense of, our human environment. This may be the most difficult of tasks, and the greatest of the new researcher’s challenges. No wonder then, that like in the

case of theory, the book’s authors seek methodological help wherever possible, mostly in places other than mathematics education. Once again, the reader is exposed to a confusing variety of possibilities. Conversational analysis and discourse analysis are repeatedly mentioned in the book, although not much explanation is given to either (see e.g. the chapters by Mariolina Bartolini Bussi and by Maria Luiza Cestari). One can hardly view this omission as weakness. It is not in the responsibility of the book’s contributors to teach well-known, well-developed approaches, detailed introductions to which may be found in many other places. And yet, not knowing the principles underlying the methods can make reading difficult. What renders the situation even more complex is the fact that, as rightly noted by Bartolini Bussi and as echoed by all the Editors in the Epilogue, being adept in the well established all-purpose techniques of analysis is not enough. Mathematics education researchers cannot satisfy themselves with ready-made methods, supposed to fit any discourse and any subject matter. The need for special ways of analyzing, carefully crafted to fit the unique type of data, the specific content, and the particular questions faced by mathematics education researchers is an additional difficulty, likely to boggle the minds of both the authors and the readers.

One can conclude and say that by rejecting the relative safety of the well-trodden traditional research routes, the new researchers doom themselves to a life more difficult than ever. No more can they enjoy the luxury of structuring the reality with the help of universal tools and pre-designed patterns. No more can they cede most substantial parts of their work to mechanical number crunchers. From now on, every study, be it restricted as it may, would demand high level of attention and creativity all along. Designing the ways of looking at data will now require as much inventiveness as deciding what and why should be looked at. The whole process can only be accomplished by an intricate, painstaking dialectic between the conceptual and methodological aspects. So demanding this new type of research seems to be that its being frequently disappointing should surprise nobody. Indeed, only too often does it happen these days to see studies that fail to deliver the promised eye-opening effect. A researcher may summon a sophisticated, usually quite eclectic, theoretical-methodological apparatus only to apply it to an amount of data so scarce and so restricted in content that none of the loudly announced advantages of the intricate method gets a chance to show its strength. One finishes reading such paper and asks herself about the emperor’s new clothes. Another often witnessed affliction of empirical studies at this early stage of the new research approach is the pervasive lack of clarity of the focus of analysis and a vagueness surrounding the questions asked.

Fully aware of the danger of these childhood diseases, I have been reading *Language and Cognition* with some misgivings and with an eye to the typical symptoms. To my relief and satisfaction, I found out that although not entirely free from common weaknesses, and although quite uneven in its quality, the empirical research presented here has much to offer. The papers in the book may be divided into two groups, according to the type of

² More generally, the Editors should be applauded for the exceptionally thorough work invested in this volume. The unified format of the chapters, with the summarizing “Final remarks” concluding all of them and with thoughtful cross-referencing in between chapters makes the reading much easier and helps in seeing this book as a logical whole.

classroom interaction that is being studied. While Mariolina Bartolini Bussi, Falk Seeger, Heinz Steinbring, Albrecht Abele, Maria Luiza Cestari, and Terry Wood focus mainly, although not exclusively, on exchanges between teacher and a class or between teacher and a small group of students, Frances Curcio and Alice Artz, Kaye Stacey and Anne Gooding, and Marta Civil investigate interaction between students working in teams. Interested in the phenomenon of argumentation, Goetz Krummheuer takes a close look at both these types of interaction.

Another distinction between the different studies can be made along the methodological axis. While some of the researchers use specially designed analytical techniques, the majority of them looks at the data in rather commonsensical ways. Since it would be impossible to mention all the studies, let me limit myself to those which I found particularly appealing and rich in insights. There is a number of chapters in this book that may provide advocates of the new research paradigm with a strong evidence for their claims. First, let me mention the studies that document students' difficulties stemming from invisible disparities between everyday and mathematical discourses, and above all, from differences between spontaneous and scientific use of words (with the terms "spontaneous" and "scientific" being used here in the sense of Vygotsky, 1987). While a number of authors deal with this theme (e.g. Susan Pirie, Ferdinando Arzarello, Mollie MacGregor), the chapter by Susan Pirie is particularly rich in truly illuminating examples.

Specially crafted, elegant, and evidently quite effective methods of analysis can be found in the chapters of Mariolina Bartolini Bussi, Frances Curcio and Alice Artz, and Kaye Stacey and Anne Gooding. Together, these three pieces of work show with clarity how wide is the spectrum of possible approaches to the study of classroom interactions. Bartolini Bussi's purely qualitative analysis uses Leont'ev's interrelated notions of activity, action, and operation to present classroom events with different "zooms of lens" (coarse-grained vs. fine-grained analyses) and at different scales (long-term and short-term analyses). True to herself, in these analyses the author refuses to separate the issues of interaction from those of content, or those of affect and attitudes from those of cognition. In her study of kindergarten children trying to explain the notion of "point of view", she aims at the double goal of designing instruction and modeling learning. The aims and methods proposed in the chapters by Curcio and Artz and by Stacey and Gooding may seem quite far removed, but they are equally worth attention. Curcio and Artz set to themselves as a goal to chart cognitive and meta-cognitive behaviors in order "to categorize the problem-solving behaviors of the students in a way that would give a global picture of dynamics of the communication" (p. 184). Stacey and Gooding develop special coding methods as well, but rather than distinguishing between content-related and "navigational" activities, they classify students' small group interactions according to the occurrences of different types of problem solving behaviors, such as reading, analyzing, etc., and according

to degrees of interactivity. This is an exemplary piece of work, containing all that is needed in an empirical study aspiring to be convincing (this feature would hardly deserve mentioning if not the fact that it is absent only too often from published research reports). After specifying at the outset what will count as an evidence of learning, the authors look for a correlation between patterns of interaction and effectiveness of learning. Not surprisingly, learning turns out to be most effective in teams which are interactive in a genuine way, that is, in those that show the highest number of turn-taking, the most elaborate and focused explanations by students, and the most responsive attitudes of the participants. This is true both at the group level and at the level of individual learning. This study provides much needed and still much too scarce information about factors that render students' interactions effective and, while making it clear that the effectiveness is not a default property of such interactions, it adds to, rather than undermines, the claim about the overall advantages of this collective mode of learning.

As the Editors of the book say in the Epilogue, "there is ... a growing need for theories, analytical tools, concepts, and research methodologies that would allow one to capture the subtleties and complexities" (p. 342) of what is happening in the classroom (p. 344). The above brief survey of the main themes tackled in *Language and Communication* confirms this statement. In the remainder of this review I use what I was able to learn from the book to identify and to analyze those theoretical, methodological, and practical issues that seem to require most urgent attention. Let me stress that what follows uses the book but as a representative example of what is going on in the domain of mathematics education in general, these days. As a community, we did not yet have time to deal with all that needs to be dealt with, and thus none of the problems I will be talking about is an exclusive domain of *Language and Communication*. Since I feel a part of this developing community, and since I see its problems as my own, whatever criticism or precautions I will be presenting on these next pages should be read as grounded also in my personal experience, and as regarding my work not less than anybody else's.

2. On the need for conceptual stocktaking

The immaturity of the field expresses itself most clearly in a certain conceptual vagueness that can be felt in great many recent writings on classroom interactions. *Language and Communication* is no exception, and the Editors are well aware of this fact. "Research is now at the stage of trying to understand the complex phenomena of language and communication in the domain of mathematics", they say in the Epilogue (p. 341). I would add that even before trying to understand the phenomena, those who wish to build in the new field must do some serious conceptual groundwork. Without it, they remain exposed to the danger of circularities and entanglements that may undermine the value of the whole endeavor. Let me explain this claim and substantiate it with a few

examples.

The words *communication*, *language*, *speech*, *discourse*, *practice*, *culture*, and *meaning* feature prominently in all the chapters. These terms, when they come in clusters and turn into keywords, can be read as identification marks of the new discourse on human thinking and learning which I choose to call *participationist* since it views learning as becoming a skillful participant in certain well-defined types of activities. This new discourse is built on the deep distrust of the assumptions of biological determinism and of cross-cultural and cross-situational cognitive invariants that have been the central ideas of the traditional cognitive science since the time of Piaget (this traditional approach may be called *acquisitionist* as it views learning as the acquisition of something – knowledge, concepts, mental structures)³. Thus, on the face of it, by speaking of discourse, practice, and culture rather than about subject-matter, and by focusing on how students talk to each other rather than trying to map their mental schemes, the authors signal their membership in the community of participationist thinkers. And yet, at a closer look, this impression may turn out misleading. The words are like Trojan horses: once we let them in, we unknowingly let a whole army of concepts, indeed, a whole philosophical outlook, get in as well. The problem begins when such conceptual invasion is unintended and the word-users remain unaware of entailments and interpretations others are likely to ascribe to their words. Since the possibilities of interpretation are usually diverse and many, situation like this invites confusion, if not incoherence and contradictions. This is what is only too likely to happen when the words-users are never explicit either about the meanings of the words they use or about relations between them.

Like in all the other major recent publications I am aware of, the keywords of *Language and Communication* are never explicitly clarified or related one to another. Although in the case of the book at hand the insightful survey by Anna Sierpiska (chapter 1) helps to introduce some conceptual order, the lack of a straightforward analysis of the term as central as *communication* may be highly consequential. The chapter by Clive Kanen, which deals with Wittgenstein's late philosophy and thus seems to be an ideal setting for such analysis, does not tackle the notion of communication directly. This, in spite of the fact that the author opens his chapter with questions that read like a promise of a thorough conceptual examination: "What is the nature of communication in mathematics teaching? How does communication relate to the workings of language?" (p. 120).

At this point, one may object to my insistence on explicit conceptual clarifications saying that terms like communication or language, being a regular part of our everyday vocabulary, do not require any special explanations. To this let me respond: What is good enough for everyday discourse may become unhelpful in

a discourse that aspires to scientific clarity and usefulness (and Susan Pirie's observations are a powerful confirmation of this claim!). Our everyday usage of the words reflects our "folk models" of the world. To get some idea of the one that underlies our understanding of the world *communication* suffices to have a look at its definitions in any popular dictionary. Thus, according to *Collins English Dictionary* communication is "the imparting or exchange of information, ideas, or feelings," whereas *Encyclopedia Britannica* defines communication as "the exchange of meanings between individuals through a common system of symbols." These definitions conceptualize communication as transporting some entities – and have all the unhelpful entailments of this metaphor. Indeed, these descriptions imply that information, ideas, and feelings, which are inherently private, can be objectified and become subject to external measurement and inter-personal comparisons. Accordingly, to find out whether communication works would require checking whether the entity "sent" by a speaker – an idea or feeling – is "the same" as the one "received" by the listener. Since the only way to judge the sameness of human experiences is to compare the discourses they produce, we would be estimating this sameness by assessing the effectiveness of communication. And so, we would end up entangled into a logical circularity.

This logical entanglement was certainly Wittgenstein's central concern when he launched the revision of his own ideas about language presented in the famous *Tractatus*. As his late work shows with clarity, the dilemma of the circularity cannot be easily resolved. And yet, even if we cannot solve the problem right away, we may at least make ourselves aware of the dangers of naive conceptualizations. As a minimum, we should take certain linguistic precautions so as to avoid the pitfalls of unhelpful metaphors. For example, we should beware of such everyday expressions as "passing information" or "communicating knowledge", which reinforce the "transport" model of communication. And yet, these phrases can be found in the book even in the chapter that presents Wittgenstein's late stance on language! (see e.g. pp. 121, 250).

Let me look at another group of concepts: *discourse*, *practice*, and *culture*. In many of the book's chapters these three terms feature intermittently without any clear hint as to the similarities in meaning that in some contexts make them interchangeable, and as to the differences that in other contexts set them apart. Throughout the participationist literature on learning, these terms are often used in comparable roles (as, e.g., in conceptualizing learning as peripheral participation in a certain practice or in a particular discourse, or as enculturation/socialization), but since by choosing any one of them the writer subscribes to a particular outlook with its special emphases and concerns, using all three of them in one text without taking necessary precautions may have a confusing effect.

Let me illustrate this with an example. Whereas some writers propose to define learning as "legitimate peripheral participation" in a certain *practice* (Lave & Wenger, 1991; Wenger, 1998), Falk Seeger speaks about

³ As I was arguing elsewhere (Sfard, 1998), the metaphors of participation and acquisition are not incompatible and elements of both can be found in many theories. The suggested categorization is supposed to be made according to the metaphor that dominates in a given theory.

“learning as an introduction into *culture*” (p. 85, my emphasis). Why is the notion of culture preferable to that of practice? This question cannot be answered without thorough clarification of the difference in the meaning of the two terms. That the term *practice* is understood by Seeger differently than by Lave and Wenger is never stated explicitly but is rather obvious from the fact that while for its conceivers the expression “peripheral participation in a practice” is a general definition of human learning, including all its possible forms, Seeger restricts its applicability to “mimetic” learning, that is one “based on observing and performing gestures and the like” (p. 90). A similar idiosyncrasy exists in Seeger’s use of the term *discourse*. While having recourse to the concept of culture to define learning, Falk Seeger remarks that this perspective allows to reconcile two “basically conflicting ideas: the idea that the classroom discourse is a principal target area for research in teaching and learning mathematics and the idea that giving a complete picture of teaching and learning mathematics may require more than reconstructing it as discourse” (p. 85). This implies that Seeger’s understanding of the notion “discourse” is more restrictive than that of many other authors, for most of whom this word signifies activity of communication with absolutely everything that goes in such activity and makes it possible (see e.g. Gee, 1997; Harre & Gillet, 1994; Sfard, 2000 a, 2000b; Sfard & Kieran, 2001). Seeger seems to represent the more traditional school for which the term discourse refers only to verbal text produced in oral personal exchange. All this comes to say that while every author may be allowed to use words in his or her own ways, specifying these ways and making clear the reason for their idiosyncrasies is necessary as a means of preventing confusion and misunderstanding.

Finally, one can only regret that in the new educational discourse that often presents classroom interactions as a “collective effort to construct a shared (or taken-as-shared) meaning”, the concept of *meaning* does not receive any direct attention. Disturbing vagueness, as well as inconsistencies and even incompatibilities, may follow. While this seems to be true about the decisive majority of educational publications, the contemporary philosophical writings might help.

In the context of conceptual dilemmas such as those discussed above, and especially when the notion of meaning is mentioned, there is no better source than Wittgenstein’s late writings. Indeed, for Wittgenstein the concept of meaning was the principal concern and his critiques, analyses, and caveats can be hold responsible for, or rather credited with, the “discursive turn” we are witnessing these days. In the light of this, Clive Kanes’s chapter devoted to Wittgenstein’s late theory is very much in place. Alas, the survey offered on less than twenty pages is much too concise to make justice to the ideas of a thinker so deep and at times so esoteric that until this very day, after libraries were filled with exegetic literature, no two writers seem to agree in their interpretations of Wittgenstein’s texts. In addition, as Kanes aptly observes himself, not much Wittgensteinian research has been done in mathematics education so far, and thus one cannot assume readers’ familiarity with

Wittgenstein’s complex ideas. As a result, I find it difficult to believe that the deep meaning and the relevance of Wittgenstein’s work for our project will be fully appreciated by the intended audience on the basis of this brief chapter alone.

The more so, as Wittgenstein’s main message – the one according to which meaning should be understood not as a private experience but as a *public* affair – has not been conveyed with all the clarity and emphasis it deserves. The famous definition, according to which “the meaning of a word is its use in the language” (Wittgenstein 1967, p. 20, §43) does not appear at all. If it did, perhaps some inconsistencies in the use of the term “meaning” along the book could be prevented. This definition would be an important contribution to the debate on the possibility of “negotiating meaning” that goes implicitly throughout the chapters. While most of the authors speak of meaning as negotiable and “shareable”, and thus seem to imply that although socially constructed, it ends up as private, Bartolini Bussi and Falk Seeger disagree: mathematical meanings are not really negotiable in classrooms. Each of these two writers has his or her own reason, however, and only the one given by Bartolinin Bussi may be seen as a derivative of Wittgenstein-like approach (“no meaning can be a matter of negotiation” says the author on p. 83, “since scientific concepts are not to be created anew at school ... but are to be assimilated as the product of centuries of development by humankind”; Seeger’s criticism stems from his assumption that “‘Negotiating’ suggests an interaction between equals” (p. 96), whereas the different participants of classroom discourse – the teacher and the students – are not at equal footing).

While saying all this I don’t try to imply that Wittgenstein’s vision of meaning should become an agreed basis for the new research, even though it has been widely accepted among philosophers, anthropologists, and ethnographers (see e.g. Geertz’s statement: “The generalized attack on privacy theories of meaning is, since early Husserl and late Wittgenstein, ... a[n unquestionable] part of modern thought”, (Geertz, 1973, p. 12). In the context of mathematics educational research, Wittgenstein’s analysis may be more important than his conclusions. If not the conclusions, then at least the reasons for the philosopher’s doubts and concerns should be taken seriously by all those that try to put some order into the still inchoate aggregate of ideas.

The above survey of the conceptual dilemmas pervading today’s literature on learning and classroom interactions, while certainly not exhaustive, may be already too long. Let me thus summarize by saying that all the quandaries briefly reviewed in the preceding paragraphs are related to the time-honored question of the relation between thought and language. Generations of thinkers proposed a whole spectrum of possibilities, delineated by two extreme doctrines (cf. Vygotsky, 1987). According to one of them, thought and language are separate, although related and often concomitant, types of human activity, with language playing an auxiliary role of a tool for expressing thought. On the other end of the spectrum there are approaches that culminate in Wittgenstein’s critical philosophy that deny the primacy of thought over speech and reject the idea that there is

any such thing as “pure thought” that can be given different linguistic (or non-linguistic, for that matter) expressions while remaining basically “the same”. This latter position finds its moderate expression in the work of Vygotsky, who illustrated the inseparability of thought (meaning, in his case) and words by saying that attending to words and thought separately is like trying to find out the properties of water by looking at those of hydrogen and oxygen. The most extreme doctrine in this group is generated by the Whorf hypothesis which, in its radical version, says that all our conceptualization of the world, and thus all our thinking, is dictated by the language we use.

Almost all the possibilities delineated by these two extremes seem to be represented in our field these days. The majority of those that can be found in *Language and Communication* seem to be closer to the language-as-tool conceptualization. The positions endorsed by the authors are often conveyed by their language rather than by any explicit declarations. Since I don't want to be too long, I will not bring the many examples with which this claim could be illustrated. Let me only remark that the most indicative of the author's stance is usually his or her use of those very terms that were identified above as the most problematic. Let me also add that my own personal preference would be for approaches closer to the other end of the spectrum, that is those that deny the possibility of viewing thinking and the activity of communication as in any way separate. On the contrary, following Vygotsky and accepting even those consequences of his ideas which he himself did not explicitly formulate, I choose to view thinking as a special kind of communication – communication of a person with herself (cf. Harre & Gillett, 1994; Edwards, 1997; Sfard, 2000 a, 2000 b). This simple tenet has many far reaching consequences and, if taken seriously, can become a basis for a comprehensive outlook at human cognition. Above all, it implies that thinking may be regarded as molded in the image of inter-personal communication (which is phylogenetically and ontogenetically primary), and as untreatable separately from “the means of communication” such as language or any other symbolic system that is being used as a mediating tool.

It seems that taking a well-defined, explicitly presented position on the issue of the relationship between language and thought is necessary in order to ease the conceptual vagueness pervading the growing field, and vice versa, conceptual groundwork that consists in deciding about relationships between concepts is, in fact, theory-building activity that includes taking a stance on the issue of thought and language. Needless to say, easier said than done. Let me thus be more clear about what, in my opinion, would be the most helpful, and still feasible, thing to do at the present stage of our emerging new approach to thinking and learning. Once again, we might take Wittgenstein's lead. Wittgenstein's own work can be summarized as a life-long attempt at cleansing the philosophical discourse from the linguistic vagueness which, according to this philosopher, is usually the primary source of philosophical dilemmas. So intense was Wittgenstein at tracing down conceptual circularities and theoretical untenabilities that he did not hesitate to

revise and refute his own ideas whenever these seemed to him afflicted with the same disease he was trying to cure. And yet, as nicely summarized by Clive Kanen, although Wittgenstein was chasing linguistic ambiguities, he was not necessary pursuing the “purest crystal” of clear-cut definitions. On the contrary, he deeply appreciated the creative potential of the “blurred edges” and the possibilities of movement that come with conceptual friction. So, what is it that Wittgenstein, and people like ourselves, could be asking for? Rather than looking for ultimate definitions, we might try to become more aware of dangers that one faces in their absence. Rather than seeking guarantees of a risk-free use of words we might try to remove as many hurdles as possible. Rather than asking what is the meaning of a given word we may often satisfy ourselves with finding out what it is not. Finally, when it comes to conceptual dilemmas, rather than attempting to solve them we might sometimes make them disappear just by purging our language from misleading metaphors and hidden assumptions.

3. On the need for methodological open-mindedness, inventiveness – and restraint

Research in mathematics education can be seen as a special case of the inquiry into human thinking and learning. This latter research is, and has always been, torn between two complementary, but not necessarily compatible, goals. On the one hand, the intention of the researcher is to fathom the phenomenon of human thinking in all its uniqueness and with all its ramifications. On the other hand, it is imperative to find a way to do it properly, that is, in a manner that would put this inquiry on a par with any other scientific endeavor with respect to cogency, trustworthiness, and, above all, usefulness. These two goals create an essential tension that fuels the incessant change. While the request of scientificity (whatever this term means at a given moment) pushes toward simplicity and feeds the belief in cross-contextual invariants, the wish for an all-encompassing, true-to-life picture of learning implies that the complexity of the phenomenon, and in particular its only too often ignored sensitivity to contextual factors, should never disappear from the researcher's sight. Not surprisingly, bridging these two conflicting needs – the need for scientific rigor and simplicity versus the need for real-life fidelity and usefulness – turns out an extremely difficult task.

As transpires from *Language and Communication*, we are paying today the methodological cost of our decision to put premium on the goal of capturing the intricacies of learning in all their specificity and uniqueness. When it comes to tools and techniques that would fit this endeavor, we certainly have still a very long way to go. Much of the skepticism that surrounds the new research on classroom interactions may be ascribed to the absence of well-defined, convincing methods. Unlike in the pre-participationist era, we have to craft our ways of analyzing data each time anew, appropriately to the questions we are asking and in accord with the data we were able to collect. Of course, we may still try to create a reservoir of ready-made methods, but at the present

stage this seems a rather remote target (once again, I alert the reader to the chapters by Bartolini Bussi and by Stacey & Gooding where one may find elements of promising beginnings).

Since I am one of those researchers who feel compelled to pave new methodological pathways simply because the conceptual revision has left us methodologically empty-handed, I am only too familiar with all these trials and tribulations of the research that has to create its own tools. Let me thus summarize these musings on method with a warning which I try to keep in mind whenever entering classrooms armed with video cameras, audio-recorders, and notebooks, and determined to document “absolutely everything” that is going on in this classroom in the hope to “catch the gist of things”. These sentiments and this hope can be felt in many chapters, but they are particularly explicit in Seeger Falk’s rejection of the culinary recipes that don’t give an access to the experience of eating, as well as in his observations on the need for a “complete picture” of learning (see the passage from p. 85 quoted above). While it is true that we all long for such “complete picture”, we might feel much better if we pause for a moment and ask ourselves whether there is such thing as “complete picture”. And if we do, we may discover that in fact, there is an inherent contradiction between our critique of the positivist science and this essentially positivist longing for the description that conveys “the gist of things”. Those who criticize traditional research for its aspirations to bring a faithful, unquestionable picture of reality have no reasons to be more forgiving with other types of research. It is time, it seems, to reconcile ourselves to the idea that whatever the amount of the registered details about the world, the resulting picture of the fleeting moments that constitute this world will always have the temporary status of “the best interpretation so far”.

At this point, I cannot resist the temptation to reinforce the message of the last paragraph with Jorge Luis Borges’s story of Funes the Memorious, the person who remembered all and forgot nothing, and who was thereby “the solitary and lucid spectator of multiform, instantaneous and almost intolerably precise world”. As admirable as Funes’s perfect retention might have been, it did have its price. This infallibly memorious person “was not very capable of thought.” “To think is to forget differences, generalize, make abstractions” explains Borges. Alas, “[i]n the teaming world of Funes, there were only details, almost immediate in their presence” (p. 94). In the “teaming presence” of too many details, there is little hope for the kind of understanding that the new researchers are seeking. Let us, therefore, beware of the dangers of analyses that try to achieve too much. Lest our greed for detail and fidelity makes us throw away the little insights we have already managed to attain, and lest our understanding be lost in the hopeless chase after the phantom of the “complete picture” of learning.

4. On the need for pedagogical second thought

We have yet a long way to go also when it comes to becoming wiser about classroom practice. Pedagogical issues go like a thread of scarlet throughout *Language*

and *Communication*, if sometimes only implicitly. The overarching theme is that of the transition from traditional, “univocal” ways of teaching to multilogue-based instruction, where the voices of the learners can be heard not less, and usually much more, than that of the teacher. The authors use names as different as *recitation*, *IRE script* (Wood) or *funneling* (Steinbring, Wood) to describe the patterns of interaction predominant in the traditional classroom; and they refer to ideas as diverse as *constructivism* (Cestari), *dialogism* (Wood), and *Socratic methods* (Loska) as sources of inspiration for instruction based on the increased participation of the learner. In the latter type of classroom, the teacher refrains from explicit telling or showing, and the rules of the game that is being played are believed to be negotiated by all the participants. Thus, while in the traditional classroom drama the teacher plays the triple role of scriptwriter, director and the main character, in the non-traditional setting she is merely an unobtrusive guide and observer.

This pedagogical shift is yet another manifestation of the same increased attention to mathematical communication that has been noted in research. What strikes the reader’s eyes is that the authors do not make any attempt to stay scientifically neutral on the pedagogical issues. All of them, almost without exception, explicitly condemn the traditional mode and have only the best to say on the non-traditional approach. The consensus with respect to the advantages of the newer approach is so overwhelming that the readers may ask themselves whether the authors really need any empirical evidence to make their case. Group work is regarded as one that “offers fertile environment” (p. 189) for learning. All the examples brought here are claimed to be an evidence for the advantages of this form of learning. It is supposed to be effective by default, and barely anybody deals with the question of the conditions that have to be fulfilled in order for the communication to work. (One exception is the Stacey & Gooding paper, where one reads that, according to former studies, “the factors that make peer interaction a valuable device for learning are not automatically present when the classroom is organized around learning in groups”, p.192). As a result, while reading one report after another one cannot escape the impression that this is a chronicle of conclusions foretold. While this feature is certainly not unique to the new research, the new beginnings may be a good opportunity for becoming more balanced and more careful.

The pedagogical message that seemed to precede the empirical study often interfered with my reading and stood in the way of my getting fully convinced by its results. This difficulty was particularly acute in those cases in which the authors failed to provide information that one needs in order to be able to judge the success of the given pedagogical approach for herself. Indeed, when I read that “students learn mathematics with greater understanding in classrooms in which they are allowed to explore, investigate, reason, and communicate about their ideas” (p. 169), I feel intuitively confident that this is true (who wouldn’t?). And yet, if I am also to be convinced by an empirical study, I need to know what is understood by learning and by understanding, what counts as an

evidence thereof and how I am to look for this evidence. While endowed with this information, I now would ask the authors to show that in the “non-traditional” setting, so unanimously favored by everybody, some learning did occur and that this learning can, indeed, count as superior to the one occurring in traditional environments according to the criteria given in advance.

But the roots of my partial skepticism about the pedagogical message of the book go deeper. If one accepts the idea of learning as an initiation to a practice, culture or discourse, then one must be aware of the fact that there is more to learning that has been taken into the traditional cognitivist account. It is now clear that it is not enough to look at the “acquisition of mathematical content” – on the way students master operating mathematical concepts and performing mathematical operations – in order to understand the change the learning is generating. Certain patterns of interaction and, more generally, certain meta-discursive rules that render mathematical discourse its unique identity, must be learned as well. This point seems to be agreed in this volume on the majority of authors, whether they speak of *sociomathematical norms and regularities of behavior* (Wood), *patterns of interaction* (Steinbring, especially while quoting Voigt) or *formats of argumentation* (Krummheuer). The great majority of these meta-rules can only be learned implicitly, and thus there is no point here in talking about learning-with-understanding, as traditionally interpreted. It is not to say that the learning of meta-rules occurs without understanding, in a mechanistic manner. Neither it is to say that the student is kept on purpose without the possibility of “building meaning”. Rather, it means that it may be useful to try to look at learning through lens other than the clear-cut meaningful learning/rote learning dichotomy. Alternatively, if this dichotomy is to be kept as a basis for a useful categorization, the idea of meaning and understanding must be revised along with all the others. If we do that, and if we are faithful to the new conceptualization of learning and communication, then we may arrive at a reappraisal of the pedagogical approaches which, at this moment, seem so very much in consensus. One of the tacit agreement that should be first submitted to critical scrutiny is that we can always make distinction between the genuine, meaningful learning on the one hand, and a learning that is but a mechanical imitation and is “empty from content”.

All this has not yet been done. The existence of distinctively mathematical meta-discursive rules is generally acknowledged by all the writers, but the status of these rules as all-important contributors to content-level understanding is denied. “The students learn the implicit conventions of such apparent mathematical communication, but these conventions are irrelevant from the mathematico-epistemological point of view,” say the Editors in the Epilogue (p. 343). This, in my opinion, is a good example of a belief that seems to be on its way toward reappraisal within the new conceptual framework. The authors of the book admit that “learning mathematics becomes an initiation to a very special culture in which quite a few things are taken for granted and very many things cannot be taught explicitly and directly” (p. 342),

but they – as all of us, at the moment – may be still quite far from seeing all the consequences of this statement. It seems that the insistence on culture-like quality of what is being learned promotes “conventions” to a much more prominent and significant place than we have been ready to admit so far. The conception of thinking as communicating brings home to us that there is an intricate, dialectic relation, and thus mutual dependence, between object-level, content-related elements, traditionally regarded as determined by extra-discursive factors, and meta-level rules of mathematical discourse, at least some of which have always been regarded as a matter of convention. It also implies that one element does not exist without the other. If the Platonic temptation is consistently resisted, the distinction between conventionally given and externally imposed disappears. Conventional is tantamount to human-made, and socially constructed. The most innovative message of the fully-fledged participationist framework is that not just meta-level, but also object-level rules are a matter of human construction, and thus of convention, and none is justifiable in a fully rational way.

That even what seems to be the very gist of mathematics is a matter of convention is stated in a powerful way by Wittgenstein (1967) who, to make this point, chooses perhaps the least expected type of discursive activity – the activity of mathematical proving:

“For it is a peculiar procedure: I *go through* the proof and then accept its results. – I mean: this is simply what I *do*. This is use and custom among us, or a fact of our natural history.” (p. 61).
....

Wittgenstein is even more explicit in his claim on inherent unjustifiability of meta-rules in the context of mathematical calculations:

“The danger here, I believe, is one of giving a justification of our procedure where there is no such thing as justification and we ought simply to have said: *That’s how we do it*. (p. 199).”

On the face of it, these are rather surprising statements. After all, nothing could be more rational than mathematical proof. And yet, the claim of unjustifiability does not refer to the proof as such but rather to the meta-discursive rules that govern the activity of constructing it. It is the justifiability of the meta-discursive conventions which is questioned, and not the inner consistency of object-level inferences. Moreover, saying that the meta-discursive rules cannot be justified does not mean that there are no reasons for their existence. It only means that, contrary to the Platonic view of mathematics, reasons that can be given have to do with human judgements, choices and conventions rather than with an “objective necessity”.⁴ It also means that one should not be too quick in dismissing conventions and in condemning “learning how to participate successfully in

⁴ The strict meta-rules of modern mathematical discourse are the result of unprecedented efforts of 19th- and 20th-century mathematicians to reach maximally effective communication. Evidently, their undeclared hope was to create a discourse that would leave no room for personal idiosyncrasies and would therefore lead to unquestionable consensus. Such consensus would imply certainty of mathematical knowledge.

patterns of interaction” (a quote from Voigt, p. 107), because in this way he or she may be throwing away the concept of learning as such.

It is my belief that a penetrating, unprejudiced look at classroom interactions, grounded in a more consistent conceptual infrastructure, will eventually bring a balanced view of learning and instruction, one that will re-confirm the need for a blend of methods and where open “negotiations” and students’ own doing will be regularly reinforced with teachers’ more direct interventions. As Maggie Lampert once beautifully put it, it will be the kind of teaching that, “like teaching someone to dance” will consist in “some telling, some showing, and some doing it [by the teacher] with [the students], along with regular rehearsals” (Lampert, 1990, p. 58).

More generally, it seems that putting communication in the center of our thinking about learning mathematics must amount, sooner or later, to not less than paradigm shift. With the long inventory of theoretical, methodological and practical needs that must yet be fulfilled if such shift is to occur, we may be certain that our hands will remain full of work for many years to come. For me, pursuing the many lines of study that emerge from *Language and Communication* it is a truly fascinating, worthwhile challenge. The Editors and the authors of this important book should be congratulated on their work and thanked for providing so much food for thought.

References

- Cole, M. (1996): *Cultural Psychology*. – Cambridge, MA: The Belknap Press of Harvard University Press.
- Edwards, D. (1997): *Discourse and cognition*. – London: Sage.
- Gee, J. P. (1997): Thinking, learning, & reading: The situated sociocultural mind. – In: D. Kirshner & J.A. Whitson (Eds.), *Situated cognition: Social, semiotic, and psychological perspectives* (pp. 235-260). Mahwah, NJ: Lawrence Erlbaum Associates.
- Harre, R. and Gillet, G. (1994): *The discursive mind*. – Thousand Oaks: SAGE Publications.
- Erickson, F. (1986): Qualitative methods in research on teaching. – In: M. Wittrock (Ed.), *Handbook of research on teaching*. Third Edition (pp. 119-159). New York: Macmillan.
- Geertz, C. (1973): *The interpretation of cultures*. – Basic Books.
- Lampert, M. (1990): When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. – In: *American Educational Research Journal* 27, pp. 29-63.
- Lave, J. (1988): *Cognition in practice*. – Cambridge, MA: Cambridge University Press.
- Lave, J. and Wenger, E. (1991): *Situated Learning: Legitimate Peripheral Participation*. – Cambridge, MA: Cambridge University Press.
- Nunes, T., Schlieman, A. D. & Carraher, D. W. (1993): *Street mathematics and school mathematics*. Cambridge, MA: Cambridge University Press.
- Saxe, G. B. (1991): *Culture and cognitive development: Studies in mathematical understanding*. – Hillsdale, NJ: Lawrence Erlbaum Associates.
- Scribner, S. (1983/1997): Mind in action: A functional approach to thinking. – In: M. Cole, Y. Engstrom, & O. Vasquez (Eds.), *Mind, culture, and activity: Seminal papers from the Laboratory of Comparative Human Cognition* (pp. 354-368). Cambridge, MA: Cambridge University Press.
- Sfard, A. (1998): On two metaphors for learning and on the

dangers of choosing just one. – In: *Educational Researcher* 27(2), pp. 4-13.

- Sfard, A. (2000a): Steering (dis)course between metaphor and rigor: Using focal analysis to investigate the emergence of mathematical objects. – In: *Journal for Research in Mathematics Education* 31(3), pp. 296-327.
- Sfard, A. (2000b): On reform movement and the limits of mathematical discourse. – In: *Mathematical Thinking and Learning*, 2(3), pp. 157-189.
- Sfard, A. & Kieran, C. (2001): Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students’ mathematical interactions. – To appear in: *Mind, Culture, and Activity*.
- Vygotsky, L. S. (1987): Thinking and speech. – In: R. W. Rieber, & A. C. Carton (Eds.), *The collected works of L. S. Vygotsky* (Vol. 1, pp. 39-285). New York: Plenum Press.
- Wenger, E (1998): Practice. – In: E. Wenger, *Communities of practice: Learning, meaning, and community* (pp. 43-102). New York: Cambridge University Press.
- Wittgenstein, L. (1967): *Philosophical investigations* (G. E. M. Anscombe, Trans.). – Oxford, UK: Blackwell. (Original work published 1953)

Author

Sfard, Anna, Dr., The University of Haifa, Faculty of Education, Haifa 31905, Israel.
E-mail: Sfard@netvision.net.il