

About the Use of the TI-92 for an Open Learning Approach to Power Functions

A Teaching Study

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Abstract: In this report we present the results of a teaching study introducing the concept "power function" using a graphing calculator. The focus of our attention is on the development of the understanding of 15-16 year-old mathematics students. In the centre of our interest is their learning through graphs of power functions by discovering the properties of graphs. Our report presents the mathematical and social constructivist background together with a new deliberately constructivist approach beginning the teaching experiment with an open question. The students' cognitive and intuitive strategies and their attitudes towards computer algebra are described.

Kurzreferat: *Potenzfunktionen mit dem TI-92 bei einer offenen Unterrichtsgestaltung.* In diesem Beitrag präsentieren wir die Ergebnisse einer Unterrichtsstudie zum Begriff der Potenzfunktionen mittels Präsentation der "Potenzblume" beim Gebrauch eines graphikfähigen Taschenrechners. Hauptaugenmerk liegt auf der Entwicklung des Verständnisses der 15-16-jährigen Schüler durch das Entdecken der Eigenschaften von Graphen bei Potenzfunktionen. Die Studie beruht auf einer offenen Unterrichtsgestaltung, deren theoretischer Hintergrund der Soziale Konstruktivismus bildet. Die kognitiven und intuitiven Strategien der Schüler zusammen mit ihrer Haltung zur Computeralgebra werden präsentiert.

ZDM-Classification: I20, U70

1. Preliminaries

For the learning and teaching of functional concepts the benefits of the use of computer algebra are often indicated (e.g. Heugl et al. 1996). Mathematics teachers highly appreciate the use of these new possibilities because they believe it offers students a deeper understanding of the functional concepts. Specifically the learning of the relationships between the different representations of functions (e.g. equation, graph, table) are in the centre of their educational interests.

Questions like "Try to bring this graph on the screen!" are geared towards making students aware of the connection between the equation and the graph of a function. Drijvers (1993) was the first to ask this specific question as an open approach to let the students discover these connections themselves. Many examples and teaching experiences are published (e.g. Barzel et al. 1999) using this question mostly at the end of a teaching sequence to exercise or to apply gained knowledge.

In contrast, our study focuses on the use of this question at the beginning of a teaching sequence about power functions. We are interested in how well it promotes the understanding of a new topic, that is a new class of functions.

A typical introduction of the power functions on the lower secondary level emphasizes the aspect of applications. However there are only few applications for power functions that seem obvious and easy enough for students. Therefore most of the teachers use the graphical aspect to introduce this type of functions.

Since the curriculum suggests the use of graphing calculators (or computers) there are instructions of how to use it in class in order to enhance the students' understanding. It is our idea to start the teaching sequence with the "power flower" of graphs of power functions which gives the students the task to discover the connections between the graphs and the functional equations. Beside the benefits of a constructive approach on the part of the students for the specific subject matter at hand this teaching procedure can further in general the students' cognitive and intuitive strategies.

Since the learning of functions by discovering properties of graphs leans mostly on school knowledge Fischbein distinguishes here: "Secondary intuitions ... are those which are acquired, not through natural experience, but through educational interventions" (Fischbein 1987, p. 71). Our approach is therefore also a measure on how well students have learnt secondary intuitions.

In our teaching study the students' learning and their use of the TI-92 lies in the centre of our interest and the methodical arrangement was geared towards leaving room for interactions and communication.

The following questions served as chief guidelines of our investigation:

1. *Functional perspective:* What kind of understanding do students reach with respect to different representations of power functions?
2. *Technological perspective:* How does the use of a graph calculator help the understanding processes of the students?
3. *Social constructivist perspective:* How does the use of a graphing calculator improve a social constructivist procedure in math teaching? What role do students' interactions and communication play in an open learning environment?

2. Theoretical framework

The theoretical background of the study encompasses the three perspectives according to the research questions.

2.1 Functional perspective

Research in mathematics education has consistently stressed the students' difficulties in discovering functional relationships which are the

- idea of the one-to-one correspondence,
- idea of systematic change
- the idea of observing a function as a mathematical object (Vollrath 1989, p.8ff.)

These ideas emphasize the conception of a dependency between two (sets of) values together with methodical variations and an overall view of the properties of a function.

The necessary knowledge to recognize functions belongs to functional thinking and also depends on the concept images of functions (Dreyfus and Vinner 1985).

Concept images depend mostly on typical examples and their representations and not so much on their formal definitions.

The use of graphs of functions – stressing the idea of systematic change – is one of the main goals on the secondary I level. These functional illustrations offer a particular helpful feature: “Properties and peculiarities ... can more easily be recognised in the graphical picture than in formulae, and this is one of the reasons why graphs are so important.” (Freudenthal 1983, p. 578)

However, understanding graphs correctly presents another task for the students. By studying graphs Janvier discriminates between local and global properties of graphs. Students have special difficulties when changing from one to the other (Janvier 1987).

2.2 Technological perspective

In this teaching study we applied the TI-92 which easily offers the adaptation of the “window shuttle-principle”. This term, used within the theory of computer algebra, describes the possibility that parallel representations of different functions can be managed. This possibility allows for the idea of a function to become more noticeable (Heugl, Klinger, Lechner 1996). In computer algebra (e.g. TI-92) all the needed representations of a function (equation, graph, table) are implemented and the switching between them is easily done.

The readiness with which these representations are possible calls also for a new perspective of learning. Köhler (1995) emphasizes that the use computer algebra imposes on the students to learn to “see” these connections also between different types of functions. Once the students learned the subject matter the technology helps them to come to a more general meaning of a function.

The use of computer algebra in teaching functions additionally offers a new way to work with functions. Vom Hofe (1995) indicated that in traditional teaching the functions, mostly their graphs, were the results of mathematical, *that is mostly computational* activities. The possibilities of the software, which is more or less interactive, makes the students use functions as objects of their activities which has more of a dynamic aspect. This is also the way, the functions are used in our teaching sequence.

2.3 Social constructivist perspective

The theoretical background for our teaching experiment originates from a “social constructivist” orientation. Deadlock situations provide students with the opportunity to develop ideas and suppositions and support or revise their thinking as initial presumptions are being modified and refined.

Constructivism emphasises the individual’s unique knowledge schemata (Davis et al. 1990) and also the role of interaction in the learning process. The inclusion of a social dimension (i.e. linguistic and cultural factors and the way one talks about mathematics and the way one thinks about the use of computers in and out of class), teachers’ conceptions (their opinion about their and their experience with the computer algebra and their

instruction role in math class) of the constructivist learning theory combined in social constructivism.

Collaborative groups in the classroom promoted the students’ communication and interaction which allows to investigate these social constructivist aspects (e.g. Edwards, Jones 1999). The new view of finding unaccustomed methods of teaching – starting with an open question, granting time for exploring in groups, opening space for the documentation of the students’ learning procedure – offers an alternative to the method of stating facts and demonstrating procedures while introducing mathematical concepts.

3. Teaching study method

The methodological framework of our teaching study comprises several items. The data collection follows the sequence of the teaching study: at the beginning the students worked in groups. After this phase the students had to write down individually their spontaneous impressions about their way of finding the solution: first ideas, arising difficulties and unsolved questions. The posters by which the students had to document their understanding were another part of the data collection. A questionnaire concerning the contents, the method and the documentation (poster) was given to the pupils in a later lesson, when the topic “power functions” has been dealt with.

A test on the understanding of the connection between term and graph of several power functions was put to the students in the following lesson. In addition to all that a video recording of the lesson and our actual observations during the study form the basis for resulting conclusions.

4. Teaching study

4.1 Planning

Since the concept of a power function is new for the students, they can only begin with their preliminary knowledge:

- Linear and quadratic functions and their graphs. This subject was repeated during the previous lessons.
- The square root-function was taught about one year ago.
- Subject of one of the previous lessons was the reflection of a graph along the x - and y -axes and the changing of the equation of the function.

The students are accustomed to the TI-92 for about six weeks. Most of them do not have problems of handling anymore. The students are not yet used to “open teaching”.

The research study consists of two phases:

1. Group work

The first part of our teaching study comprises the introduction of the power function through the presentation of the “power flower” (Barzel et al. 1999). Five groups of students were geared towards discovery learning by investigating the graph.

Which functions create the following picture and how do they connect ? (Figure 1)

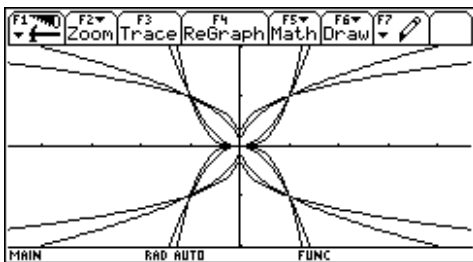


Figure 1

Window fixing (Window-Editor):
 $x_{min}=-4.7$; $x_{max}=4.7$; $xscl=1$;
 $y_{min}=-2$; $y_{max}=2$; $yscl=1$

Each group had to document their findings on a poster.

2. Presentations

In the second phase the groups presented their posters to the others and a conclusion was drawn and common results were made known.

4.2 Realization and findings

The students worked in groups together for about one lesson (40 minutes) using the worksheet as a guideline. They worked in groups of four, which was determined by the seating arrangement. The pupils were free to choose their partners.

The open question at the beginning had a motivation force and the students eagerly committed themselves to finding the solution. Some immediately discussed the problem with their neighbours, some worked with the TI-92 and others used paper and pencil first.

The phase of group activity was terminated by the presentation of their posters in the second phase. Six of the eight groups created a poster. Questions of differences and similarities and the necessary evaluations of the posters filled a lively discussion.

Three essential points had to be realised. These were also mentioned by the students in their description of their way and manner of solving the problems:

- All the graphs in the "power flower" pass through the points (0|0) and (1|1), therefore some of them cannot represent the normal parabola. The strategy of adding a factor to the equation in a quadratic function does not help.
- Some of the graphs are reflections of each other on each of the axes. According to that the equations of those functions differ only in the sign (before x or y).
- In order to create those parts of the graphic, which lean to the left or right side, the exponent k in $f(x) = x^k$, $k \in \mathbb{Q}$ needs to be a fraction.

All students found the first two aspects, six of the eight groups also discovered the third aspect. These groups also recorded their results in a poster. All posters show a listing of the needed functional equations.

The original power flower is created by $f(x)=a \cdot x^k$, $a \in \{1, -1\}$ and $k \in \{2, 3, \frac{1}{2}, \frac{1}{3}\}$.
 Some groups used for k the numbers 2, 4, $\frac{1}{2}$, $\frac{1}{4}$. Because

of the quality of the screen, this was acceptable.

There were mainly two different types of documentations. One type shows only the listing of the needed functional equations. In some posters the listing was attached by graphs (the power flower or rough drafts), see Figure 2.

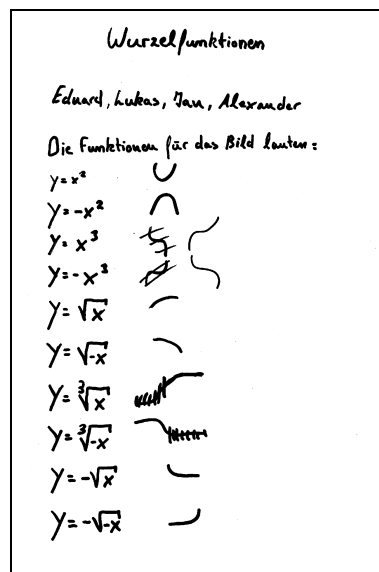


Figure 2

The other type of documentation additionally contains questions and assumptions (Figure 3).

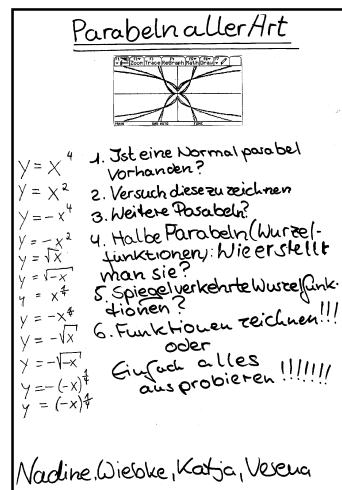


Figure 3

5. Conclusions and reflections

In our considerations of the teaching study the findings lead to many questions, which require further research. For example:

- We found it noteworthy that the students used decimal and rational numbers for the exponents and that the numbers differed only slightly (see above). What available knowledge leads to the choice of numbers for exponents?
- The students were very inventive with the titles of the posters: "Pictures with parabolas", "Root functions", "Functions", "Parabolas of all kind". With what

mathematical topics do the students associate the question asked at the beginning of the teaching sequence?

- The answers to the questionnaire after the sequence show evident differences between boys and girls. Do they differ in their ways of working with each other and are there divergent perceptive faculties responsible for varying learning processes?

This teaching study concentrated only on the three research questions mentioned above.

5.1 Functional perspective

The introduction of the concept of the power function with the “power flower” is a successful alternative to using an application. This evaluation can be drawn from different factors:

- The observations during the respective lessons show that the pupils could operate with different representations of the power functions: the connection between equation and graph depending on the exponent was clear to the students.
- A test in the subsequent period showed that most of the pupils succeeded in finding the equations belonging to the graphs on Figure 4.
- In the questionnaire the students were asked to give a self-evaluation of their concept understanding. 66% of the pupils think that they are able to explain the main characteristics of a power function and all of them are able to explain the specific standard example $f(x)=x^3$.
- In the sense of the spiral principle, repeating basic ideas on different levels, it was interesting to realize that the pupils used former knowledge of the meaning of parameters when modifying graphs. They remembered the stretching and screening processes from the quadratic parabolas and realized the reasons why this way cannot help to obtain the power flower picture. So it seems to be very easy for them to transfer basic ideas from one class of functions to another.

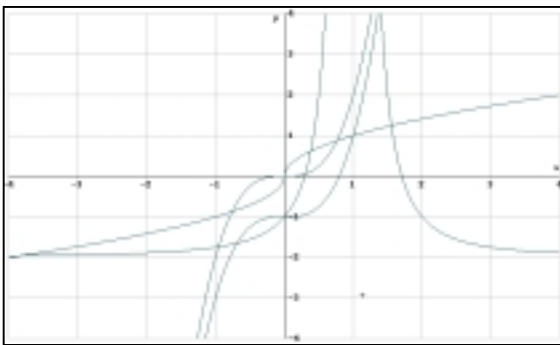


Figure 4

5.2 Technological perspective

One conspicuous result of the questionnaire was that the use of the TI-92 was very much accepted by the pupils and they felt highly motivated. Although all the pupils pointed out that their enthusiasm of using technology in class was unbroken, the answers of the boys in this respect were even more positive. For us it was important to see that the pupils found out for themselves how often and in which manner they used this medium. The benefit

of having such a little device for plotting the graphs is that it does not take so much space – not even in their school bags.

All of them employed the graphic capabilities of the TI-92. They utilized it to prove their assumptions and their applied strategies and took it afterwards as a basis to discuss their ideas with their group partners. The tool was good for a “Trial-and-error strategy”. It was interesting that some of them did not highly estimate this procedure though they belonged to those with good results (see Figure 1). Above all this may only prove they are not used to such a discovery learning method, a question that need intensive investigations.

The change between the two representations of a function (equation and graph) seemed to be indisputable to the students. They switched from one window to the other quite easily and the connection between equation and graph was obvious to them.

An unexpected result of the questionnaire was that the pupils shared the opinion of the necessity of drawing the graph of a function by hand although the calculator could do it faster. This result reminded us that Bruner's idea of enactive, iconic and symbolic representations of concepts should always be remembered in teaching even though the students are past the age of always needing concrete handling.

5.3 Social constructivist perspective

All aspects of our teaching study confirm the assumption that the social constructivist approach is convincing:

- During the process the students were always motivated and highly committed, they even forgot to take their break. They enjoyed solving the problem, which was not only the teachers' observation, but was also asserted by the pupils themselves.
- The preliminary knowledge was sufficient for the task.
- Only one of the students would have preferred an explanation of the concept of the power function first before handling such a task.
- The pupils pointed out that working in collaborative groups was efficient and interesting. Especially the opportunity to talk informally about mathematics was a big benefit for them and led them to good results.
- The manner of documentation by a poster was also estimated.

For the teachers, the results of our instruction study were encouraging and prompted to transfer this social constructivist approach and its technology also to other mathematical topics.

For the researchers a strong necessity was felt to further investigations into the role of technology in math education. The teachers also strongly need reassuring support to find a meaningful way of applying new media in their teaching endeavours and discussions of possible changes in the goals and contents of math classes.

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