PCRB for Positioning in GSM Networks

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Abstract: The posterior Cramér-Rao lower bound (PCRB) for estimating the position and velocity of a Mobile Station (MS) by integrating two different kinds of measurement from GSM networks, Timing Advance (TA) and Received Signal Strength (RSS), is derived. The bound shows that theoretically the data fusion approach yields better accuracy than using a single kind of measurement. The performance of an extended Kalman filter (EKF) is compared with the PCRB.

1 Introduction

Firstly driven by the requirement of localizing emergency calls, positioning in mobile cellular networks has become an exciting research area. However, due to the diverse environments and the communication signals which are usually not designed for positioning, it is difficult to get an accurate position estimation relying on a single type of measurement. Therefore, data fusion solutions have been proposed to provide position estimation with better accuracy, reliability and coverage [McGuire05]. Recently, we addressed a data fusion approach integrating two different types of measurement, Timing Advance (TA) and Received Signal Strength (RSS) from a Global System for Mobile communications (GSM) network, using an extended Kalman filter (EKF) [Zhang06]. In this paper, we derive the posterior Cramér-Rao lower bound (PCRB) for this nonlinear filtering problem to evaluate the estimation performance.

As is well known, Cramér-Rao lower bound (CRLB) is extremely useful to predict the best achievable performance before designing any estimator and to provide a benchmark of assessing the estimation algorithms. In the context of positioning, the estimated parameter vector should be considered random, and thus PCRB that is analogous to the CRLB for random parameters [Tichavsky98], [Bergman99] is used.

The remaining part of the paper is organized as follows. Section 2 describes the state transition and the measurement model for positioning in GSM networks. Section 3 derives the corresponding PCRB recursion formula. In section 4 some simulations are carried out to analyze the theoretical performance of the data fusion approach, and the error of the EKF is compared with the PCRB. Conclusions are drawn in section 5.

2 Positioning in GSM Networks

For the problem of estimating the position of a MS, we assume that the state vector $\mathbf{x}(k)$ is a 4×1 vector containing position and velocity of the MS in two dimensions: $\mathbf{x}(k)=\left[x(k)\quad \dot{x}(k)\quad y(k)\quad \dot{y}(k)\right]^T$. The state transition is described as a nearly constant velocity model [Bar-Shalom01]

$$\mathbf{x}(k+1)=\mathbf{A}\cdot\mathbf{x}(k)+\mathbf{G}\cdot\mathbf{w}(k)$$
$$\mathbf{A}=\begin{bmatrix}1 & T & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & T\\ 0 & 0 & 0 & 1\end{bmatrix},\mathbf{G}=\begin{bmatrix}\frac{T^2}{2} & T & 0 & 0\\ 0 & 0 & \frac{T^2}{2} & T\end{bmatrix}^T \tag{1}$$

where T is the sampling interval, $\mathbf{w}(k)$ is zero mean white Gaussian acceleration noise

$$E\left\{\mathbf{w}(k)\right\}=\mathbf{0},E\left\{\mathbf{w}(k)\cdot\mathbf{w}^T(j)\right\}=\tilde{\mathbf{Q}}\cdot\delta(k-j)$$
$$\tilde{\mathbf{Q}}=diag({\sigma_x}^2,{\sigma_y}^2),\mathbf{Q}=E\left\{\mathbf{G}\mathbf{w}(k)\mathbf{w}^T(k)\mathbf{G}^T\right\}=\mathbf{G}\tilde{\mathbf{Q}}\mathbf{G}^T \tag{2}$$

$\mathbf{Q}$ is the covariance matrix of $\mathbf{Gw}(k)$, the acceleration noise multiplied by the gain.

The measurements, $\mathbf{y}\in\Re^6$, are the distances between the Base Transceiver Stations (BTSs) and the MS according to the TA, and the signal strength losses received from three BTSs a, b, and c, whose position coordinates are (x_a,y_a), (x_b,y_b) and (x_c,y_c), respectively [Zhang06]. We define

$$d_i'=\sqrt{(x-x_i)^2+(y-y_i)^2},i=a,b,c \tag{3}$$

Then the measurement model can be written as

$$\mathbf{y}(k)=\mathbf{h}\left[\mathbf{x}(k)\right]+\mathbf{v}(k)$$
$$\mathbf{h}\left[\mathbf{x}(k)\right]=\begin{bmatrix}d_a'(k)\\ d_b'(k)\\ d_c'(k)\\ 132.85+38\log_{10}(d_a'(k))\\ 132.85+38\log_{10}(d_b'(k))\\ 132.85+38\log_{10}(d_c'(k))\end{bmatrix} \tag{4}$$

where $\mathbf{h}\left[\mathbf{x}(k)\right]$ describes the nonlinear mapping of the states into the observation space, and $\mathbf{v}(k)$ is zero mean white Gaussian noise

$$E\left\{\mathbf{v}(k)\right\}=\mathbf{0},E\left\{\mathbf{v}(k)\cdot\mathbf{v}^T(j)\right\}=\mathbf{R}\cdot\delta(k-j),\mathbf{R}=diag(\sigma_d^2,\sigma_d^2,\sigma_d^2,\sigma_l^2,\sigma_l^2,\sigma_l^2) \tag{5}$$

3 Derivation of the PCRB

Let $p_{\mathbf{y},\mathbf{x}}(\mathbf{y},\mathbf{x})$ be the joint probability density of $(\mathbf{y},\mathbf{x})$, i.e. of observation and state vector, and $\mathbf{g}(\mathbf{y})$ is an estimate of $\mathbf{x}$. The PCRB on the estimation error has the form

$$\mathbf{P} \triangleq E\left\{[\mathbf{g}(\mathbf{y})-\mathbf{x}][\mathbf{g}(\mathbf{y})-\mathbf{x}]^T\right\} \geq \mathbf{J}^{-1} \tag{6}$$

where $\mathbf{J}$ is the Fisher information matrix (FIM). Since $p_{\mathbf{y},\mathbf{x}}(\mathbf{y},\mathbf{x}) = p_{\mathbf{y}|\mathbf{x}}(\mathbf{y}|\mathbf{x}) \cdot p_{\mathbf{x}}(\mathbf{x})$, the FIM $\mathbf{J}$ can be decomposed into the information obtained from the data, and the *a priori* information. Considering the FIM for estimating $\mathbf{x}(k)$ as a submatrix of the FIM of $(\mathbf{x}(0),\cdots,\mathbf{x}(k))$, a recursion formula of the sequence $\{\mathbf{J}(k)\}$ for estimating the state vector $\{\mathbf{x}(k)\}$ can be derived [Tichavsky98]. According to the state space model (1) and the measurement model (4), the corresponding PCRB recursion is

$$\mathbf{J}(k+1) = \left(\mathbf{Q} + \mathbf{A}\mathbf{J}(k)^{-1}\mathbf{A}^T\right)^{-1} + E\left\{\mathbf{H}^T(k+1)\mathbf{R}^{-1}\mathbf{H}(k+1)\right\} \tag{7}$$

where the Jacobian matrix $\mathbf{H}(k+1)$ of the nonlinear measurement function $\mathbf{h}$ is

$$\mathbf{H}(k+1) = \frac{\partial \mathbf{h}[\mathbf{x}(k+1)]}{\partial \mathbf{x}(k+1)} = \begin{bmatrix} \frac{(x(k+1)-x_a)}{d'_a(k+1)} & 0 & \frac{(y(k+1)-y_a)}{d'_a(k+1)} & 0 \\ \frac{(x(k+1)-x_b)}{d'_b(k+1)} & 0 & \frac{(y(k+1)-y_b)}{d'_b(k+1)} & 0 \\ \frac{(x(k+1)-x_c)}{d'_c(k+1)} & 0 & \frac{(y(k+1)-y_c)}{d'_c(k+1)} & 0 \\ \frac{38(x(k+1)-x_a)}{\ln 10(d'_a(k+1))^2} & 0 & \frac{38(y(k+1)-y_a)}{\ln 10(d'_a(k+1))^2} & 0 \\ \frac{38(x(k+1)-x_b)}{\ln 10(d'_b(k+1))^2} & 0 & \frac{38(y(k+1)-y_b)}{\ln 10(d'_b(k+1))^2} & 0 \\ \frac{38(x(k+1)-x_c)}{\ln 10(d'_c(k+1))^2} & 0 & \frac{38(y(k+1)-y_c)}{\ln 10(d'_c(k+1))^2} & 0 \end{bmatrix} \tag{8}$$

It is evaluated at the real state $\mathbf{x}(k+1)$, and then the expectation is taken.

4 Simulations and Results

The simulations were carried out in a simulated urban square area as described in [Zhang06]. We assume a car equipped with a MS travels at nearly constant velocity with zero mean Gaussian acceleration noise with $\sigma_x = \sigma_y = 0.1 m/s^2$. The initial state is $\mathbf{x}(0) = [0m \quad 15m/s \quad 3000m \quad 0m/s]^T$, and the measurements update is $T = 0.48s$.

For comparing the PCRB with the Root-Mean-Square-Error (RMSE), the PCRBs were calculated by the root of the PCRBs in x direction plus the PCRBs in y direction. 1000 Monte Carlo trials were run. Fig. 1 shows the PCRBs of the data fusion approach described in section 2, and of the alternatives using only TA or only RSS measurements. The measurement noise has the standard deviation $\sigma_d = 288m$, $\sigma_l = 4dB$. The initial FIM is given by the inverse of $\mathbf{P}(0) = diag(5000^2 m^2, 50^2 m^2 / s^2, 5000^2 m^2, 50^2 m^2 / s^2)$, which is the covariance matrix of the initial state assumed to be Gaussian. As expected, the comparison of the PCRBs reveals that theoretically the integration of these two kinds of measurement yields better accuracy than using only one type of measurement.

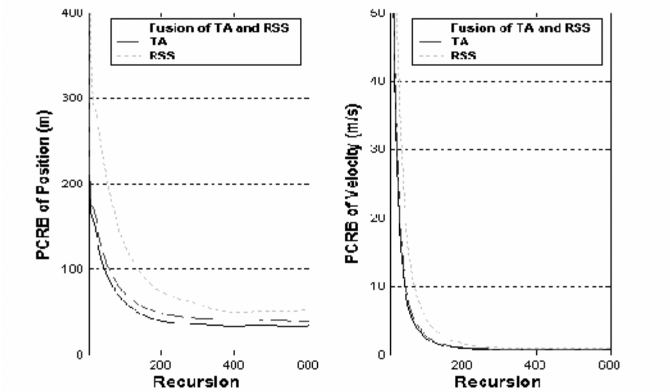

Fig. 1: Comparison of PCRBs using different measurements

Another comparison is done to analyze how the measurement noise influences the results of the data fusion and the single measurement method. We fixed the assumptions as above, but used various standard deviation of the RSS measurement noise $\sigma_l = 0.4dB$, $4dB$, $40dB$. The PCRBs of position using both two measurements and using only RSS measurements are given in Fig. 2. The plots indicate that for more noisy measurements, the data fusion approach will yield results with remarkable improved accuracy.

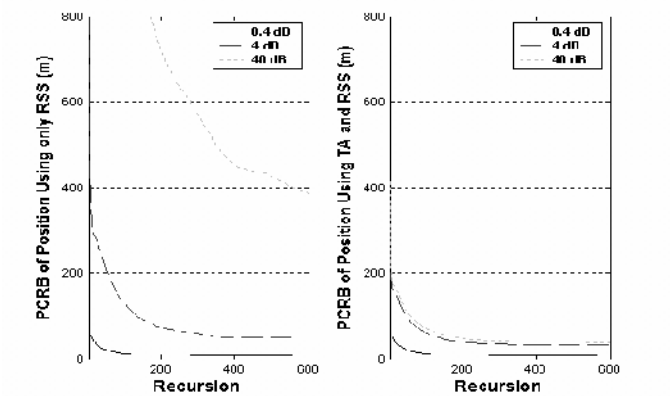

Fig. 2: PCRBs of position for $\sigma_l = 0.4dB$, $4dB$, $40dB$

Fig. 3 shows the RMSEs of the EKF using both kinds of measurement, using only TA measurements, and using only RSS measurements against the corresponding PCRB of the data fusion method. A single TA measurement was used to initialize the EKF. We observe that the RMSEs related to the EKF achieve the bound in this simple scenario, and among these three methods, the data fusion method yields the highest accuracy.

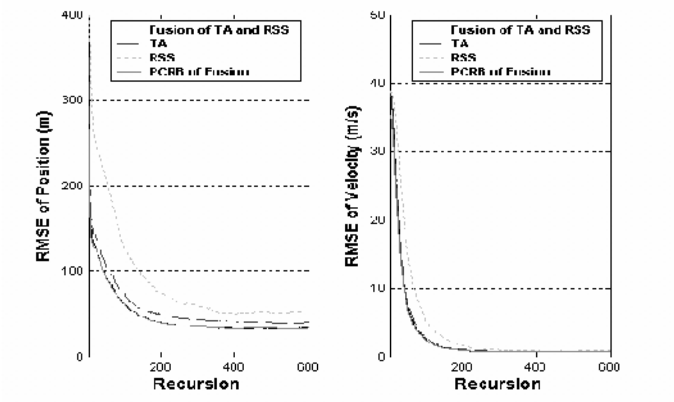

Fig. 3: RMSEs of EKF against PCRB $\sigma_d = 288m$, $\sigma_l = 4dB$

5 Conclusions

We derived the PCRB for estimating the position of a mobile station in GSM networks by integrating two kinds of measurement, TA and RSS. The comparisons show that the data fusion approach theoretically yields much better accuracy than using single measurement. And the RMSE performance of an appropriate EKF is bounded by the corresponding PCRB. In the future, the performance of the data fusion approach should be tested in complex simulation scenarios, and compared with the PCRB.

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