Automatic Transformation and Enlargement of Similarity Models for Case-Based Reasoning∗

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1 Introduction

In this paper, we present a new approach how similarity models for Case-Based Reasoning can be extended by means of operators for relations. A second contribution of the relations approach is that the mathematical model can be transformed into an OWL notation as well as in the proprietary format that is needed to build a Case Retrieval Net1

Inside a case-based system, a case $c$ is represented by a set of information entities $e_i \in E$: $c = \{e_1, e_2, ..., e_n\}$. It is compared with a query $q$ that is as well just a set of information entities. A composite similarity function for $q, c \subseteq E$ determines a numeric value for the degree of similarity $SIM(q, c)$. $SIM$ is computed by means of a local similarity function $sim : E \times E \rightarrow \mathbb{R}$. A very simple example of $SIM$ is the sum of the local values:

$SIM(q, c) = \sum_{e_i \in q} \sum_{e_j \in c} sim(e_i, e_j)$.

To provide the values for $sim$, we need a similarity model. In small domains, this might be a table of manually specified similarity values for the pairs of information entities like, for instance, $sim(MPEG-4, XviD) = 0.3$, $sim(XviD, Videocodec) = 0.7$. In real world applications, the similarity model is often too complex to be clearly arranged in a table. We employ more abstract relations instead.

A similarity type is a binary relation between two information entities $S \subseteq E \times E$. Each similarity type has a qualitative or quantitative identifier, e.g. “IS_SUCCESSOR”, “IS_ABSTRACTION”, “LOW_SIMILARITY”. A set $\mathcal{S}$ of similarity types is called a similarity dictionary. A weighting function $g : \mathcal{S} \rightarrow \mathbb{R}$ assigns a numeric value to a similarity

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2 Extending the similarity model by operators

Whether a pair of information entities belongs to a similarity type is empirically motivated. We define operators for relations with the aim to generate new relations that fulfill some formerly specified properties.

Definition (operators for relations)
An operator \( op : S \rightarrow S' \) for a relation \( S \subseteq E \times E \) generates a new relation \( S' \). We cover only some special cases of operators in the following. For further operators, we refer to future work:

- **Identity**: \( id(S) = S \)
- **Inverse**: \( inv(S) = S^{-1} \)
- **Transitive closure**: \( t(S) = \langle S \rangle \) (The transitive closure \( \langle S \rangle \) of a relation \( S \) is the smallest transitive relation including \( S \).)
- **Symmetrical closure**: \( s(S) = S \cup S^{-1} \)
- **Enlargement by the inverse of a partner relation**: \( e(S) = S \cup f(S)^{-1} \) (\( f : \mathcal{G} \rightarrow \mathcal{G} \) is a function that assigns a partner relation to a relation.)
- **Induced brother relation**: \( b(S) = (S^{-1} \circ S) \ \setminus \ I \) (\( I \) is the identity relation, i.e. \( b(S) = \{ [b, c] \mid b \neq c, \exists a \in E : [a, b], [a, c] \in S \} \))

\( t, s, \) and \( e \) are supposed to produce relations with the following properties: \( t \) extends a relation, e.g. the "IS_ABSTRACTION" relation, to a new transitive relation. \( s \) results in a symmetric relation. \( e \) can be applied to pairs of relations, e.g. the "IS_PART" and the "HAS_PART" relation, to generate a pair of real inverse relations.

Definition (Self-reflectivity) \( f \) is called self-reflexive if it holds: \( f(S) = R \) iff \( f(R) = S \).

Conclusion 1 Is \( f \) self-reflexive then holds: \( e(S) = [e(f(S))]^{-1} \)

The users specify which chains of operators will be applied to which relations (see Table 2). We consider only the eight special cases \{te, tse, se, e, bte, btse, bse, be\}.

Chain of operators \( \omega_S \)
\( todo : \mathcal{G} \rightarrow \{te, tse, se, e\} \) is given by the user. We use the abbreviations \( \omega_S = todo(S) \),

\[
\begin{align*}
  h_S &= \left\{ 
    \begin{array}{ll}
      t, & \text{if } \omega_S = te \\
      ts, & \text{if } \omega_S = tse \\
      s, & \text{if } \omega_S = se \\
      \text{id}, & \text{if } \omega_S = e \\
    \end{array}
  \right.
\end{align*}
\]

Specification of \( f \) (to be used by the operator \( e \))
<table>
<thead>
<tr>
<th>Identifier</th>
<th>Weight</th>
<th>$h_S$</th>
<th>$pf$</th>
<th>$\mathcal{E}, \text{Id.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO_SIMILARITY ID</td>
<td>0.0</td>
<td>$id$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>$id$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ABSTRACTION SPECIFICATION</td>
<td>0.3</td>
<td>$t$</td>
<td>SPECIFICATION</td>
<td>+, BROTH</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>$t$</td>
<td>ABSTRACTION</td>
<td>-</td>
</tr>
<tr>
<td>HAS_PART</td>
<td>0.5</td>
<td>$t$</td>
<td>IS_PART</td>
<td>-</td>
</tr>
<tr>
<td>IS_PART</td>
<td>0.5</td>
<td>$t$</td>
<td>HAS_PART</td>
<td>-</td>
</tr>
<tr>
<td>USES_AS_MEANS</td>
<td>0.3</td>
<td>$t$</td>
<td>IS_MEANS_FOR</td>
<td>+, MBROTH</td>
</tr>
<tr>
<td>IS_MEANS_FOR</td>
<td>0.7</td>
<td>$t$</td>
<td>USES_AS_MEANS</td>
<td>-</td>
</tr>
<tr>
<td>STRONG_SIM</td>
<td>0.75</td>
<td>$id$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>WEAK_SIM</td>
<td>0.25</td>
<td>$id$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>MEDIUM_SIM</td>
<td>0.5</td>
<td>$id$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>BROTH</td>
<td>0.5</td>
<td>$id$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>MBROTH</td>
<td>0.3</td>
<td>$id$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: Sample specification of the similarity types from the ExperienceBook project.

The user specifies the set of partner relations $\mathcal{S}_P \subseteq \mathcal{S}$ and the partner function $pf : \mathcal{S}_P \rightarrow \mathcal{S}_P$. With those, the function $f$ is built as follows:

$$f(S) = \begin{cases}  
pf(S), & \text{if } S \epsilon \mathcal{S}_P \\
S^{-1}, & \text{else.}
\end{cases}$$

**Parent relations** The set of parent relations $\mathcal{S}_E \subseteq \mathcal{S}$ is specified by the user.

**Definition (Enlargement of a similarity dictionary)**

Be $\mathcal{S}$, $\mathcal{S}'$ two similarity dictionaries. The operator $Ext : \mathcal{S} \rightarrow \mathcal{S}'$ is defined as:

$$Ext(\mathcal{S}) = \{\omega_S(S) \mid S \epsilon \mathcal{S}\} \cup \\
\{b\omega_S(S) \mid S \epsilon \mathcal{S}_E\}$$

**Definition (Consistency of $\omega_S$ with $f$)** $\omega_S$ is consistent with $f$ if holds: $\omega_S = \omega_{f(S)}$

**Proposition (Properties of the relations generated by $\omega_S$)**

(i) $\omega_S \epsilon \{tc, tse\}$ ⇒ $\omega_S(S)$ is transitive

(ii) $\omega_S \epsilon \{tse, se\}$ ⇒ $\omega_S(S)$ is symmetric

(iii) Under the conditions $S \epsilon \mathcal{S}_P$, $f$ self-reflexive, and $\omega_S$ consistent with $f$ holds:

$$\omega_S(S) = (\omega_{f(S)}(f(S)))^{-1}$$

**Lemma 1** (i) $t \circ inv = inv \circ t$ (ii) $s \circ inv = inv \circ s$ (iii) $ts \circ inv = inv \circ ts$

**Proof of the proposition** ad (i) und (ii): as one can see, ad(iii):

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\[ \omega_S(S) = h_S \circ e(S) = h_S \circ (e(f(S)))^{-1} \]
because of Conclusion 1
\[ = h_{f(S)} \circ inv \circ e(f(S)) \]
because \( \omega_S \) is consistent with \( f \)
\[ = inv(h_{f(S)} \circ e(f(S))) \]
because of Lemma 1
\[ = (\omega_{f(S)}(f(S)))^{-1}. \]
\[ \square \]

The mathematical description of the similarity model has been transformed into two alternative representations in computer files:

1. A proprietary notation that is helpful to build a Case Retrieval Net (see Fig. 1) lists each pair of information entities explicitly in both directions.
2. An OWL notation can be visualized by ontological tools like Protégé.

The first benefit of the proposed approach is that we can import and export knowledge about similarity from and to domain ontologies in the classical sense. We have implemented this by means of Perl scripts. The danger of ontology pollution is obviously given, but the impacts on the case-based retrieval are relatively small: In case a single local similarity value is too strong or too weak, it is compensated by the other values in the composite similarity function. A critical mass of pollution is reached when an empirically matching case is squeezed out of the first positions of the ranking for a query by overestimated cases. It is future work to investigate empirically when this critical mass is accomplished.

The second and major benefit of our approach is that we can make the modeling process more comfortable for the knowledge engineers. Specifying a desired property of a relation like transitivity is less time-consuming than verifying manually whether all members of a family tree are properly connected. The “NO_SIMILARITY” relationship allows to model exceptions. Protégé is able to generate a visualisation of the similarity model.