Algorithms for Computational Mechanism Synthesis and Analysis

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Abstract: This paper proposes algorithms for mechanism synthesis and analysis, including Rotation, Center, Connection, Numeric Differentiation, Animation and Index. These algorithms are used to transfer behaviors into structures, and to transfer structures into kinematic and dynamic behaviors of the structures. The algorithms are valid for the synthesis and analysis of different mechnisms. These algorithms can be used iteratively to produce mechanisms that can fulfill simple and complex motions. The paper contributes simple algorithms for the design and analysis of different mechanisms.

1 Introduction

In design process, behaviors are bridges to link functions and structures. There are different definitions of the function, behavior and structure. Here, we use a simple definition. The structure is a physical entity, or object, represented as sketch and drawing in the design stage, e.g., a drawing of a washing machine or a clamp device. The function is the purpose for which the physical entity is used. The behavior is an ascription of the working state of the object, which can be described using symbols and measured quantitatively or qualitatively. Once the main function is decomposed into a group of functions, the next step is to determine the behaviors to fulfill the functions. What kinds of behaviors are selected depends on what working principles are used. When existing working structures are selected, the design and analysis procedures follow a routine steps: drawing a sketch of the structures, calculating the structural parameters and predicting the behaviors. Specific equations are required to calculate the structural parameters of specified types of mechanisms and specific rules are required to draw the structures, which do not cause lots of computing complexity but lack generic rules linking the behavior and the structure. This paper presents simple algorithms that can be used to generate structures and to predict behaviors of mechanisms. The paper analyzes the design process of mechanisms and generalize simple algorithms in the second section, testifies the rules by using same rules for the syntheses and analyses of different types of mechanisms in the third section, and finally concludes the contribution of the paper.

2 Mechanism Synthesis: Case studies

In mechanical design, mechanisms are used to fulfill specific functions. A mechanism is a device designed to transform a set of input forces and movement into a desired set of output forces and movement. A mechanical device involves at least one mechanism. The synthesis of a mechanism means to find the dimensional parameters of the mechanisms. Existing methods of synthesis of mechanisms highly rely on the types of the mechanisms.

Current design methods of cam mechanisms include analytical methods and graphic methods. In the case of analytic methods, different equations are required for both design and analysis. In the case of graphic methods, the basic method is a kinematic inversion principle that adds an angular velocity $-\omega$ to the whole cam mechanism so that the cam stays stationary and that the follower and the frame rotate in the opposite direction than the cam does in reality, which method is termed "Inversion". In this case, the follower's motion is a combination of a relative turning motion around the cam center and a relative translating motion along a guide on the frame. The trace of the knife edge of the follower draws the profile of the cam. Different equations and design rules are used for different types of cam mechanisms. Also, different equations are used to descript the motions of the followers. However, a same rotation strategy is used for both analytical methods and graphic methods to establish equations in the analytical methods, and to generate the trace points of the knife edge during a motion period in the graphic methods. Different equations are required for computing the kinematic curves of displacements, velocity motions and acceleration motions. Current synthesis methods of linkage mechanisms also include analytical methods and graphic methods. The "Inversion" rotation strategy is also used. Besides, the center of the trace points of the motion path of the moved bars is produced to decide joint positions.

The equations used to calculate the dimensional parameters of a mechanism are different in the analytical methods, but the inversion method and the rotation strategy are used for both cam mechanisms and linkage mechanisms in the graphic methods. Graphic methods can produce both parameters and drawing, while analytical methods can only produce parameter values. However, the disadvantage of graphic methods is low dimensional precision. Thanks to computer science and technology, this problem can be solved by using computers.

Using computers to do drawing, the inversion methods are not necessary for the design of the cam profiles. The translating motion $s_i(t_i)$ of the knife edge of the follower can be directly transformed to the cam profile by rotating the points $s_i(t_i)$ with angles of $\delta_i(t_i)$. Furthermore, using software tools, the kinematic analysis can be generated by numerical differentiation, and the drawing can be executed by connecting points (drawing lines).

Analysis reports can be also produced easily by software. Hence, simple algorithms can be used to map behaviors onto structures, or vice versa.

3 Algorithms transforming behaviors to structures

Based on analysis of the design methods in the last section and our experience in engineering design, six simple algorithms are formulated. These algorithms are used iteratively to produce mechanisms.

(1) **Rotation for producing the profile of cams and linkages**

Rotation is a basic algorithm to form the structure. Any motion of a rigid body can be thought to rotate relative to another body and the center of the rotation is called the instant center. This is a principle of mechanics and it is the foundation of graphic methods in the synthesis of mechanisms. The rotation operation is used for the design of cam profiles and four bar mechanisms. This rule can be used to produce cam profiles without using the inversion method. By consequently rotating the discrete points $P_i(x,y)$ of the knife end of followers with step $\delta(i)$, cam profiles can be generated. Different kinds of cam profiles can be obtained by changing the working positions x_i and the rotational centre of the cams. Most important, this rule is also used to generate the profile of four bar mechanisms. The mathematical representation of the Rotation is

$$P(x,y)=\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \cdots & \cdots \\ x_n & y_n \end{bmatrix},\ T_R=\begin{bmatrix} \cos(\delta_i) & -\sin(\delta_i) \\ \sin(\delta_i) & \cos(\delta_i) \end{bmatrix},\ P'(x,y)=P(x,y)\cdot T_R \qquad (1)$$

(2) **Centre for joints of linkage mechanisms**

The algorithm Center is used to find the position of joints of linkage mechanisms as well as cam mechanisms. The relative motion of two bars is rotation and the joint is the center of the rotation. The Center algorithm is conducted after the Rotation operation. The relative displacement of two joints is obtained firstly through the Rotation operation and then the center of the rotation can be obtained. In the case of producing cam profiles, the center determines the type of a cam. A planar cam is produced when taking a point as the rotational center of the cam, while a cylindrical cam is produced when taking a line as the rotational center for same $s(\delta)$ curve. The mathematic representation of Center algorithm is $x^2+y^2=r^2$.

(3) Connection for the profile of mechanisms

The algorithm Connection means linking points to form curves or lines. Equations can produce parameter values but not the structure itself. Connection play a role of producing forms. The profiles of the mechanisms are produced by linking points to form curves or lines.

(4) Numeric Differentiation for producing kinematic curves

The kinematic analysis is conducted based on "Numerical Differentiation" rather than different equations for different mechanisms. In this case, both cam mechanisms and planar linkage mechanisms can use same equations. The mathematic representation is

$$y' = \frac{y_{x+h} - y_{x-h}}{2h}, \; y'' = \frac{y_{x+h} + y_{x-h} - 2y_x}{h^2} \tag{3}$$

where, y` and y`` are the first and second derivative, x is the value at which the derivative is calculated, h is a small number, y_{x+h} is function evaluated at x+h, and y_{x-h} is function evaluated at x – h.

(5) Index for generation of design and analysis report

Analysis reports are required in most cases. The structural parameters of the products, e.g. the length of bars and the profile of the cams in the context of mechanism synthesis, and kinematic and dynamic curves are listed on the reports. This is tedious work if to do it manually. Fortunately, quite a few of software provide the "report generator". To run "report" and establish a report file, user can establish an index to list required tables, figures and texts on a file with specified format (pdf, html, word, etc.) Then the report can be produced automatically.

(6) Animation for motion simulation

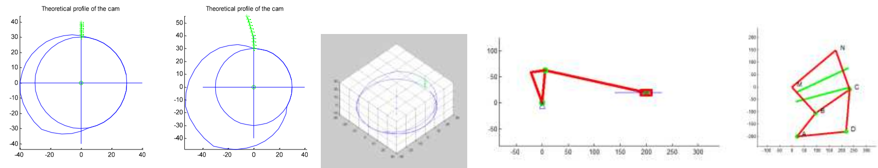
To simulate the motion of the structures, the Animation algorithm is required. Thanks to the computer technology, the animation of the mechanisms can be generated by playing the figures continuesly.

4 Examples: formation of structures for reciprocating motions

These six algorithms are tested by running examples on computers. Design examples are designing mechanisms to achieve reciprocating motions. The input data is working states of the followers or the bars, represented as a sequence of points. Other input data include the types of the mechanisms and basic parameters, such as base circle radius r_0 of cams. The whole process is automatically conducted.

Example 1: Mechanical synthesis. The result is shown in Figure 1. The rotation center is points in 1 (a), (b), (d) and (e), whihle the center is a line in (c). The input points in (a) , (c), (d) and (e) are on straight lines, while they are on an arc in (b). The outputs are slide cam mechanism, swing follower cam mechanism, cylindrical cam mechanism, crank rocker mechanism and swing follower cam mechanism in (a), (b), (c), (d) and (e) respectivily. Rotation and Connection algorithms are used for all cases and Center algorithm is used for case (d) and (e).

Example 2: Analysis of swing follower cam mechanism. The Numerical Differentiation, Centre, Connection, Index and Animation algorithms are used. The results are shown in the Figure 2, including kinematic curves produced by Numerical Differentiation and Connection, static and dynamic analyses produced by equations and Connection, the animation of the mechanism produced by Animation, and other analysis and drawing. Figure 3 shows a report generated by the Index. The report includes tables listing parameters of the mechanism and figures generated in the design and analysis process.



a b c d e

Figure 1: Profiles generated by the simple rules. a. planar cam with translating follower; b. planar cam with swing follower; c. cylindrical cam; d. crank slide mechanism; e. crank rocker mechanism

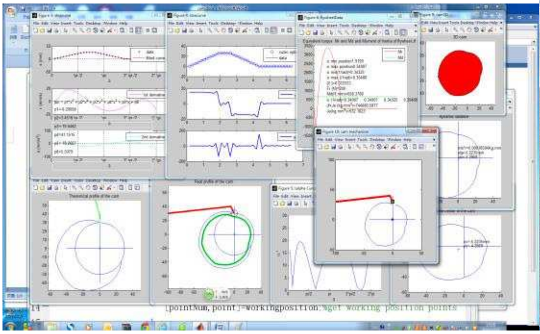


Figure 2: Output of analysis

Figure 3: The report generated automatically

5 Related works

Many methods and algorithms have been used for mapping functions to structures. These methods include shape grammar [OM11], design grammar [Ru05], graph based representation and design rules [MHS13, ECSL13], first-order logic and Boolean operation [MHS13], Physics-Based Reasoning for grammar creation etc.. Some design methods combine many mathematic algorithms and AI and IT technology to automate the design process. These methods are used for creating structures step by step. Among these methods, KBE is a key technology to automate the design process of complex mechanical systems. The principle of this kind of design system is recording the design process on computers. For example, the knowledge and design procedure of aircrafts, cars, small satellites and mold parts are recorded as software code so that repetitive design can be produced on computers [LV07]. The core at these methods is using software technology to build product templates including geometric description and the bill of material. The basic idea behind these methods is a predefined-template strategy, so it lacks of flexibility adapting to major structural modification and different design tasks. Current methods provide rules to assemble components into structures, while simple algorithms proposed here directly transform behaviours into structures step by step.

6 Conclusion and further works

Six simple algorithms are generated and presented. They are Rotation, Center, Connection, Differential, Animation and Index. They are used to transfer behaviors into structures, and to transfer structures into kinematic and dynamic behaviors of the structures. These algorithms can be used iteratively to produce mechanisms. Design experiments show that different mechanisms can be produced automatically from input behaviors with slight difference in working states based on these algorithms. The difference between the current mapping methods and the algorithms presented here is that they directly transfer behaviors into structures, while the current methods mainly focus on rules selecting components and assembling components into structures. The further work is using these algorithms to produce complex systems.

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